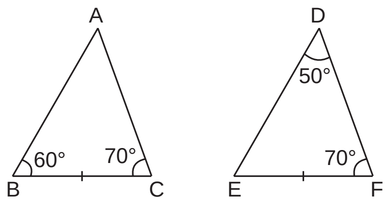
CHAPTER 7

Triangles

EXERCISE 7A

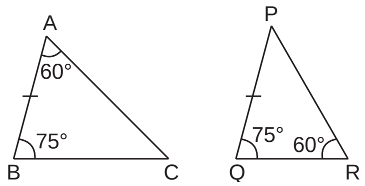
1. (*i*) **Yes,** in ΔDEF,

$\angle E = 180° - (50 + 70)°$
$= 60°$



Yes, using ASA congruence $\Delta ABC \cong \Delta DEF$

(*ii*) **No** $\angle P = 180° - 75° - 60°$ [Sum of $\angle$s of a Δ]
$= 45°$



In ΔABC and ΔPQR,

$\angle B = \angle Q$ [Each is 75°]
$BA = QP$ [Given]
$\angle A = 60° \neq \angle P = 45°$

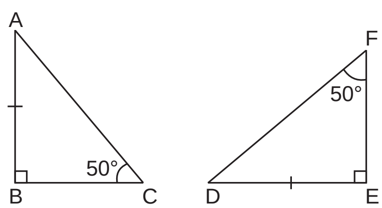
$\therefore$ ΔABC is not congruent to ΔPQR.

(*iii*) **Yes**, in ΔABC,

$\angle A = 40°$ $(90° - 50° = 40°)$

and in ΔDEF,

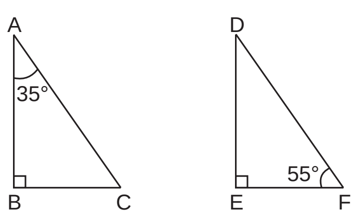
$\angle D = 40°$



Also, $AB = DE$

Yes, $\Delta ABC \cong \Delta DEF$. [By AAS congruence]

(*iv*) **No**

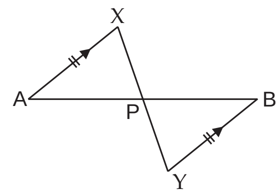


In ΔABC,

$\angle C = 90° - 35° = 55°$

But none of the corresponding sides are equal.

2. (*i*) In ΔAPX and ΔBPY,



$\angle A = \angle B$
[$AX \parallel BY$ $\Rightarrow$ Alternate angles are equal]
$\angle APX = \angle BPY$ [Vert. opp. angles]
$AX = BY$ [Given]

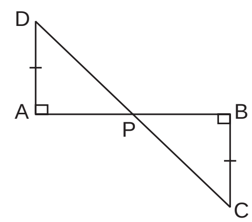
$\Rightarrow$ $\mathbf{\Delta APX \cong \Delta BPY}$ [Using AAS congruency]

(*ii*) Since $\Delta APX \cong \Delta BPY$

$\therefore$ $AP = BP$ and $XP = YP$ [CPCT]

$\Rightarrow$ AB and XY bisect each other at P.

3. (*i*) AB and CD intersect at P.



$\therefore$ $\angle APD = \angle BPC$ [Vert. opp. $\angle$s] … (1)

In ΔADP and ΔBCP,

$\angle A = \angle B$ [Each = 90°]
$\angle APD = \angle BPC$ [From (1)]
$AD = BD$ [Given]

$\Rightarrow$ $\Delta ADP \cong \Delta BCP$ [Using AAS congruency]

$\therefore$ $AP = BP$ [CPCT]

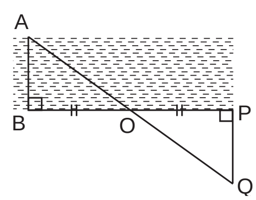
i.e. P is mid-point of AB.

Also, $DP = CP$ [CPCT]

i.e. P is mid-point of DC.

Thus, **P is the mid-point of AB as well as that of CD.**

(*ii*) Let AB be the breadth of the river.



Mark a point O on the bank of the river.

Mark B and P on the bank such that

$OB = OP$

Draw $PQ \perp BP$. Join AQ.

Now, in ΔABO and ΔQPO,

$\angle B = \angle P$ [Each = 90°]
$BO = PO$ [Given]

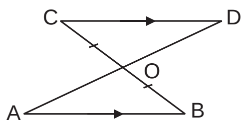
$\angle AOB = \angle QOP$ [Vert. opp. angles]

$\therefore$ $\mathbf{\Delta ABO \cong \Delta QPO}$ [Using ASA congruency]

$\therefore$ $AB = QP$ [CPCT]

Thus, we measure QP, which is equal to the breadth of the river.

4. In ΔAOB and ΔDOC, we have



$\angle AOB = \angle DOC$ [Vert. opp. $\angle$s]

$BO = CO$

[Given that O is the mid-point of BC]

$\angle ABO = \angle DCO$ [Alt. $\angle$ as $AB \parallel CD$]

$\Rightarrow$ $\Delta AOB \cong \Delta DOC$ [ASA congruency]

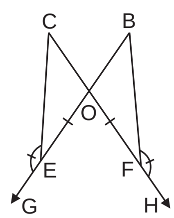
$\therefore$ $AO = OD$ [CPCT]

$\Rightarrow$ **O is the mid-point of AD.**

5. $\angle CEG = \angle BFH$

$\Rightarrow (180° - \angle CEG) = (180° - \angle BFH)$

$\Rightarrow$ $\angle CEO = \angle BFO$ … (1)



In ΔCOE and ΔBOF,

$\angle CEO = \angle BFO$ [From (1)]

$EO = FO$ [Given]

$\angle EOC = \angle FOB$ [Vert. opp. $\angle$s]

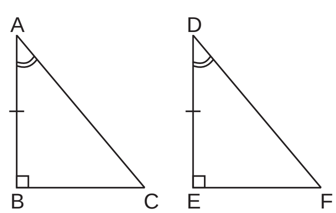
$\Rightarrow$ $\Delta COE \cong \Delta BOF$

$\therefore$ $\mathbf{OC = OB}$ [CPCT]

6. We have ΔABC and ΔDEF such that

$\angle B = 90°$

and $\angle E = 90°$



Side AB of ΔABC and side DE of ΔDEF are equal.

Also, $\angle A = \angle D$

Now, in ΔABC and ΔDEF,

$\angle A = \angle D$ [Given]

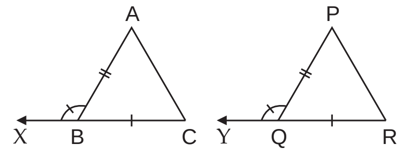
$AB = DE$ [Given]

$\angle B = \angle E$ [Each = 90°]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [Using ASA congruency]

7. $\angle ABX + \angle ABC = 180°$

and $\angle PQY + \angle PQR = 180°$ [Linear pair]



$\therefore$ $\angle ABX + \angle ABC = \angle PQY + \angle PQR$

But $\angle ABX = \angle PQY$ [Given]

$\therefore$ $\angle ABC = \angle PQR$ … (1)

Now, in ΔABC and ΔPQR,

$AB = PQ$ [Given]

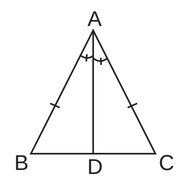
$\angle ABC = \angle PQR$ [From (1)]

$BC = QR$ [Given]

Using SAS congruency,

$\mathbf{\Delta ABC \cong \Delta PQR}$

8. AD is the bisector of $\angle A$.



$\therefore$ $\angle BAD = \angle CAD$ … (1)

Now, in ΔABD and ΔACD,

$AB = AC$ [Given]

$\angle BAD = \angle CAD$ [From (1)]

$AD = AD$ [Common]

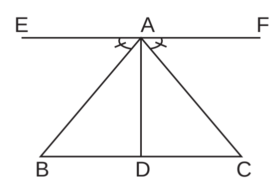
$\therefore$ $\mathbf{\Delta ABD \cong \Delta ACD}$ [By SAS congruency]

9. We have, $\angle EAB = \angle FAC$ … (1)

But $\angle EAB + \angle BAD = 90°$ [$\because$ $AD \perp EF$]

And $\angle FAC + \angle CAD = 90°$

$\Rightarrow$ $\angle EAB + \angle BAD = \angle FAC + \angle CAD$ … (2)



From (1) and (2),

$\angle BAD = \angle CAD$ … (3)

Now, in ΔABD and ΔACD,

$\angle BAD = \angle CAD$ [From (3)]

$AD = AD$ [Common]

$\angle ADB = \angle ADC$

[Each = 90° as $AD \perp BC$]

$\therefore$ $\mathbf{\Delta ABD \cong \Delta ACD}$ [By ASA congruency]

10. In ΔPMQ and ΔQNP

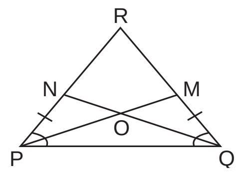
$MQ = NP$ [Given]

$\angle PQM = \angle QPN$ [$\because$ $\angle PQR = \angle QPR$]

$PQ = QP$ [Common]

$\therefore$ $\Delta PMQ \cong \Delta QNP$ [By SAS congruency]

$\angle PMQ = \angle QNP$ [CPCT] … (1)



Now, in ΔOPN and ΔOQM,

$\angle PON = \angle QOM$ [Vert. opp. $\angle$s]

$\angle ONP = \angle OMQ$

[$\because$ $\angle QNP = \angle PMQ$, From (1)]

PN = QM [Given]

$\therefore$ $\Delta OPN \cong \Delta OQM$

[By AAS congruency]

$\Rightarrow$ **OP = OQ** [CPCT]

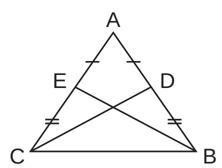
11. AB = AD [Given] … (1)

CE = BD [Given] … (2)

Adding (1) and (2),

AE + EC = AD + DB

$\Rightarrow$ AC = AB … (3)



Now, in ΔAEB and ΔADC,

AB = AC [From (3)]

$\angle A = \angle A$ [Common]

AE = AD [Given]

$\therefore$ **$\Delta AEB \cong \Delta ADC$** [By SAS congruency]

12. AB = AC [Given]

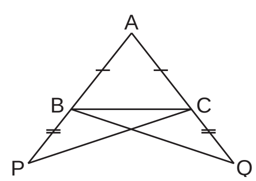
$\therefore$ $\angle ABC = \angle ACB$

$\Rightarrow$ Ext. $\angle PBC$ = Ext. $\angle QCB$ … (1)

AP = AQ [Given]

$\therefore$ (AP − AB) = (AQ − AC)

$\Rightarrow$ BP = CQ … (2)



Now, in ΔBCP and ΔCBQ,

$\angle PBC = \angle QCB$ [From (1)]

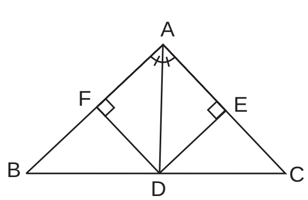
BP = CQ [From (2)]

BC = BC [Common]

$\Rightarrow$ $\Delta BCP \cong \Delta CBQ$ [By SAS congruency]

$\therefore$ **PC = QB** [CPCT]

13. In ΔAFD and ΔAED,



AD = AD [Common]

$\angle FAD = \angle EAD$

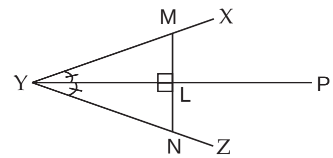
[$\therefore$ AD is bisector of $\angle A$]

$\angle AFD = \angle AED$ [Each = 90°]

$\therefore$ $\Delta AFD \cong \Delta AED$ [By AAS congruency]

$\Rightarrow$ **AF = AE** [CPCT]

14. In ΔYLM and ΔYLN,



YL = YL [Common]

$\angle YLM = \angle YLN$ [Each = 90°]

$\angle MYL = \angle NLY$

[$\because$ YP is bisector of $\angle Y$]

$\therefore$ $\Delta YLM \cong \Delta YLN$ [By ASA congruency]

$\Rightarrow$ **LM = LN** [CPCT]

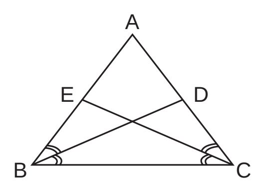
15. In ΔABC, AB = AC

$\Rightarrow$ $\angle ABC = \angle ACB$

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow$ $\angle ABD = \angle ACE$ … (1)

($\because$ BD and CE are bisectors of $\angle B$ and $\angle C$ respectively)



In ΔABD and ΔACE,

$\angle A = \angle A$ [Common]

AB = AC [Given]

$\angle ABD = \angle ACE$ [From (1)]

$\therefore$ $\Delta ABD \cong \Delta ACE$ [By ASA congruency]

$\Rightarrow$ **BD = CE** [CPCT]

16. $\because$ BD = EC [Given]

$\therefore$ BD + DE = EC + DE

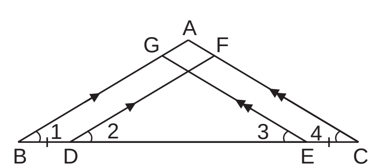
$\Rightarrow$ BE = DC … (1)

$\therefore$ BA $\parallel$ DF

$\Rightarrow$ $\angle 1 = \angle 2$ [Corr. $\angle$s] … (2)

CA $\parallel$ EG

$\Rightarrow$ $\angle 3 = \angle 4$ [Corr. $\angle$s] … (3)



In ΔGBE and ΔFDC,

$\angle 1 = \angle 2$ [From (2)]

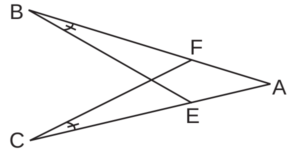
BE = DC [From(1)]

$\angle 3 = \angle 4$ [From (3)]

$\therefore$ $\Delta GBE \cong \Delta FDC$ [By ASA congruency]

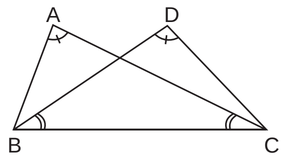
$\Rightarrow$ ***(i)* BG = DF** and ***(ii)* EG = CF** [CPCT]

17. In ΔABE and ΔACF, we have



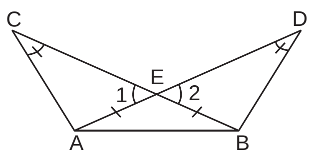
$\angle ABE = \angle ACF$ [Given]
$AB = AC$ [Given]
$\angle A = \angle A$ [Common]
$\therefore$ $\Delta ABE \cong \Delta ACF$ [By ASA congruency]
$\Rightarrow$ $\mathbf{BE = CF}$ [CPCT]

18. In ΔABC and ΔDCB, we have



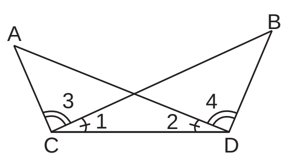
$\angle A = \angle D$ [Given]
$\angle ACB = \angle CBD$ [Given]
$BC = CB$ [Common]
$\therefore$ $\Delta ABC \cong \Delta DCB$ [By AAS congruence]
$\Rightarrow$ $\mathbf{AC = DB}$ [CPCT]

19. In ΔACE and ΔBDE, we have



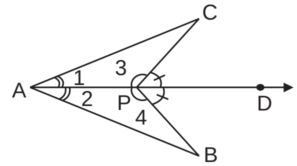
$\angle 1 = \angle 2$ [Vert. opp. $\angle$s]
$\angle C = \angle D$ [Given]
$AE = BE$ [Given]
$AE = BE$ [Given]
$\therefore$ $\Delta ACE \cong \Delta BDE$ [AAS congruency]
$\Rightarrow$ $\mathbf{AC = BD}$ [CPCT]

20. In ΔACD and ΔBDC, we have



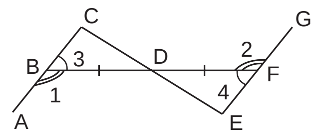
$\angle ACD = \angle BDC$
[$\because$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ $\Rightarrow$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$]
$CD = DC$ [Common]
$\angle 2 = \angle 1$ [Given]
$\therefore$ $\Delta ACD \cong \Delta BDC$ [ASA congruency]
$\therefore$ $\mathbf{AD = BC}$ [CPCT]
and $\mathbf{\angle A = \angle B}$ [CPCT]

21. $\angle 3 + \angle CPD = 180°$ [Linear pair]
$\angle 4 + \angle BPD = 180°$
$\Rightarrow$ $\angle 3 + \angle CPD = \angle 4 + \angle BPD$
$\Rightarrow$ $\angle 3 = \angle 4$
[$\because$ $\angle CPD = \angle BPD$] … (1)
Also, $\angle 1 = \angle 2$
[$\because$ AD is bisector $\angle A$] … (2)



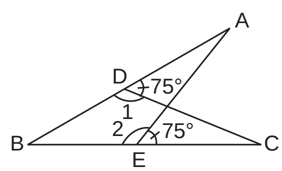
Now, in ΔACP and ΔABP, we have
$\angle 3 = \angle 4$ [From (1)]
$AP = AP$ [Common]
$\angle 1 = \angle 2$ [From (2)]
$\therefore$ $\Delta ACP \cong \Delta ABP$ [ASA congruency]
$\Rightarrow$ $\mathbf{CP = BP}$ [CPCT]

22. $\angle 1 + \angle 3 = 180°$ [Linear pair]
$\angle 2 + \angle 4 = 180°$ [Linear pair]
$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$
But, $\angle 1 = \angle 2$ [Given]
$\Rightarrow$ $\angle 3 = \angle 4$ … (1)



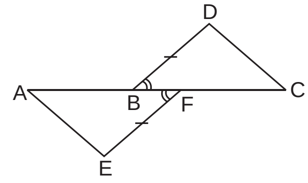
In ΔBCD and ΔFED, we have
$\angle BDC = \angle FDE$ [Vert. opp. $\angle$s]
$BD = FD$ [Given as CE bisects BF]
$\angle 3 = \angle 4$ [From (1)]
$\therefore$ $\Delta BCD \cong \Delta FED$ [By ASA congruence]
$\Rightarrow$ $\mathbf{\angle C = \angle E}$ [CPCT]

23. $\angle 1 + 75° = 180°$ [Linear pair]
$\angle 2 + 75° = 180°$ [Linear pair]
$\Rightarrow$ $\angle 1 = \angle 2$ … (1)



In ΔABE and ΔCBD, we have
$\angle B = \angle B$ [Common]
$\angle 2 = \angle 1$ [From (1)]
$AB = BC$ [Given]
$\Rightarrow$ $\Delta ABE \cong \Delta CBD$ [AAS congruency]
$\Rightarrow$ $AE = CD$ [CPCT]
We can say: **Yes, AE = CD.**

24. We have, $AB = CF$



Adding BF to both sides, we get,
$AB + BF = CF + BF$
$\Rightarrow$ $AF = CB$ … (1)
Now, in $\Delta AFE \cong \Delta CBD$, we have
$AF = CB$ [From (1)]

$\angle AFE = \angle CBD$ [Given]

$FE = BD$ [Given]

$\therefore$ $\mathbf{\Delta AFE \cong \Delta CBD}$ [By SAS congruency]

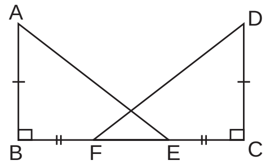
25. We have,

$BF = EC$

Adding EF on both sides,

$BE + EF = EC + EF$

$\Rightarrow$ $BE = CF$ … (1)



Now, in ΔABE and ΔDCF, we have

$\angle B = \angle C$ [Each = 90°]

$AB = DC$ [Given]

$BE = CF$ [From (1)]

$\therefore$ $\Delta ABE \cong \Delta DCF$ [By SAS congruence]

$\mathbf{AE = DF}$ [CPCT]

26. $\angle AYZ = 90°$

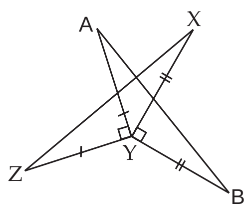
and $\angle XYB = 90°$

$\angle AYZ = \angle XYB$

Adding $\angle AYX$, on both sides, we get

$\angle AYZ + \angle AYX = \angle XYB + \angle AYX$

$\Rightarrow$ $\angle ZYX = \angle AYB$ … (1)



Now, in ΔAYB and ΔZYX, we have,

$\therefore$ $AY = ZY$ [Given]

$\angle AYB = \angle ZYX$

$BY = XY$ [Given]

$\therefore$ $\Delta AYB \cong \Delta ZYX$ [SAS congruence]

$\therefore$ $\mathbf{AB = ZX}$ [CPCT]

27. In ΔPOQ and ΔPOS, we have

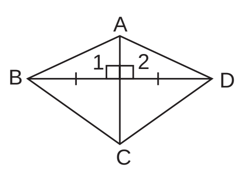
$PO = PO$ [Common]

$\angle 1 = \angle 2$ [Each = 90°]

$QO = SO$ [Given]

$\Rightarrow$ $\Delta POQ \cong \Delta POS$ [SAS congruence]

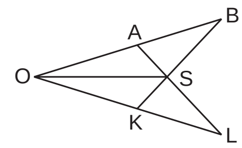
$\therefore$ $\mathbf{PQ = PS}$ [CPCT]



Similarly, $\mathbf{\Delta QOR} \cong \mathbf{\Delta SOR}$ and

$\mathbf{QR = SR}$ [CPCT]

28. (*i*) In ΔOKB and ΔOAL, we have



$OK = OA$ [Given]

$\angle BOK = \angle AOL$ [Common]

$OB = OL$ [Given]

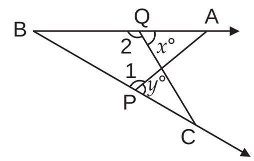
$\Rightarrow$ $\mathbf{\Delta OKB \cong \Delta OAL}$ [SAS congruency]

(*ii*) $x° + \angle 2 = 180°$ [Linear pair]

$y° + \angle 1 = 180°$ [Linear pair]

$\Rightarrow$ $x° + \angle 2 = y° + \angle 1$

$\Rightarrow$ $\angle 2 = \angle 1$ [$\because$ $x° = y°$ (Given)]



Now, in ΔABP and in ΔCBQ, we have

$\angle 1 = \angle 2$ [From (1)]

$AB = CB$ [Given]

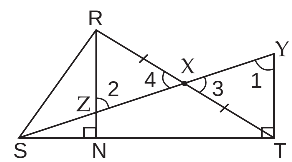
$\angle B = \angle B$ [Common]

$\therefore$ $\Delta ABP \cong \Delta CBQ$ [AAS congruency]

$\Rightarrow$ $\mathbf{AP = CQ}$ [CPCT]

29. $YT \parallel RN$

[$\perp$ to same line segment ST]



In ΔXYT and ΔXZR, we have

$\angle 1 = \angle 2$ [Alt $\angle$s, $YT \parallel RN$]

$\angle 3 = \angle 4$ [V. opp. $\angle$s]

$XT = XR$

[X is the mid-point of RT]

$\therefore$ $\Delta XYT \cong \Delta XZR$ [By AAS congruence]

$\Rightarrow$ $\mathbf{YT = ZR}$ [CPCT]

30. Produce XY to meet the circle at P.

Produce XZ to meet the circle at Q.

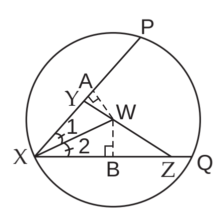
Here, W is the centre of its circle.

Draw $WA \perp XP$

$\Rightarrow$ $\angle XAW = 90°$ … (1)

Draw $WB \perp XQ$

$\Rightarrow$ $\angle XBW = 90°$ … (2)



From (1) and (2),

$\angle XAW = \angle XBW$ … (3)

In ΔWXA and ΔWXB, we have

$\angle XAW = \angle XBW$ [From (3)]

$\angle 1 = \angle 2$ [Given]

$XW = XW$ [Common]

$\therefore$ $\Delta WXA \cong \Delta WXB$ [By AAS congruency]

$\Rightarrow$ $WA = WB$

$\Rightarrow$ Chords XP and XQ are equidistant from the centre W of the circle

$\Rightarrow$ $\mathbf{XP = XQ}$

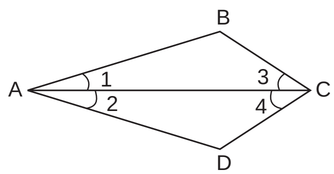
[Chords of a circle equidistant from the centre are equal]

31. AC bisects $\angle A$.

$\therefore$ $\angle 1 = \angle 2$ … (1)

AC bisects $\angle C$.

$\therefore$ $\angle 3 = \angle 4$ … (2)



In ΔABC and ΔADC, we have

$\angle 1 = \angle 2$ [From (1)]

$AC = AC$ [Common]

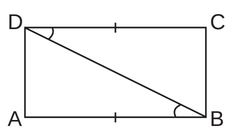
$\angle 3 = \angle 4$ [From (2)

$\therefore$ $\Delta ABC \cong \Delta ADC$

$\therefore$ $\mathbf{AB = AD}$ [CPCT]

and $\mathbf{CB = CD}$ [CPCT]

32. Diagonal DB divides the quadrilateral into two ΔABD and ΔDBC.



Now, in ΔABD and ΔCDB, we have

$AB = CD$ [Given]

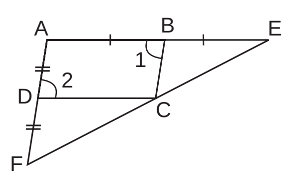
$\angle ABD = \angle CDB$ [Given]

$BD = DB$ [Common]

$\Delta ABD \cong \Delta CDB$ [SAS congruency]

$\Rightarrow$ $\mathbf{AD = CB}$ [CPCT]

33. $\because$ ABCD is a parallelogram.



$\therefore$ $\angle B = \angle D$

$\Rightarrow$ $\angle 1 = \angle 2$ [opp. $\angle$s of a $\parallel$gm]

$\angle 1 + \angle CBE = 180°$ [Linear pair]

$\angle 2 + \angle FDC = 180°$ [Linear pair]

$\Rightarrow$ $\angle FDC = \angle CBE$ … (1)

$BC = AD$ [Opp. sides of a $\parallel$gm]

$BC = DF$ [$\because$ AD = DF]… (2)

Similarly, $BE = DC$ … (3)

Now, in ΔBEC and ΔDCF

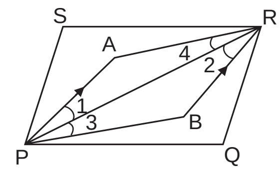
$BC = DF$ [From (2)]

$\angle CBE = \angle FDC$ [From (1)]

$BE = DC$ [From (3)]

$\therefore$ $\mathbf{\Delta BEC \cong \Delta DCF}$ [SAS congruency]

34. (*i*) In ΔAPR and ΔBRP,



$AP = BR$ [Given]

$\angle 1 = \angle 2$

[$\therefore$ PA $\parallel$ BR and $\angle 1$ and $\angle 2$ are alternate angles]

$PR = RP$ [Common]

$\Rightarrow$ $\mathbf{\Delta APR \cong \Delta BRP}$ [SAS congruency]

(*ii*) $\because$ $\Delta APR \cong \Delta BRP$

$\therefore$ $\mathbf{AR = BP}$ [CPCT]

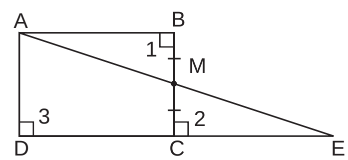
(*iii*) $\because$ $\Delta APR \cong \Delta BRP$

$\therefore$ $\angle 4 = \angle 3$ [CPCT]

But they form a pair of alternate angles.

$\therefore$ $\mathbf{RA \parallel BP}$

35. In ΔABM and ΔECM,



$\angle 1 = \angle 2$

[$\because$ ABCD is a rectangle, $\angle 1 = \angle 3 = 90°$, $\angle 3 = \angle 2$ corr. angles]

$BM = CM$ [M is mid-point of BC]

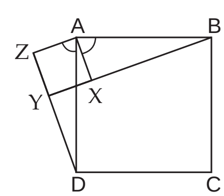
$\angle BMA = \angle CME$ [Vert. opp. angles]

$\therefore$ $\Delta ABM \cong \Delta ECM$ [By ASA congruence]

$\Rightarrow$ $AB = EC$ [CPCT]

or $\mathbf{EC = AB}$

36. ABCD is a square.



$\therefore$ $AB = AD$ … (1)

AXYZ is a square.

$\therefore$ $AX = AZ$ … (2)

$\therefore$ $\angle BAX = 90° - \angle XAD$

$\angle DAZ = 90° - \angle XAD$

$\Rightarrow$ $\angle BAX = \angle DAZ$ … (3)

Now, in ΔABX and ΔADZ, we have

$AB = AD$ [From (1)]

$\angle BAX = \angle YAZ$ [From (3)]

$AX = AZ$ [From (2)]

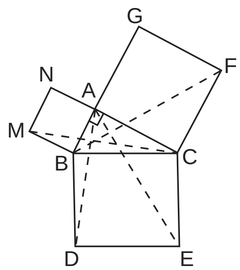
$\therefore$ $\Delta ABX \cong \Delta ADZ$ [SAS congruency]

$\Rightarrow$ $\mathbf{BX = DZ}$ [CPCT]

37. $\angle ABM = 90°$ [Angle of a square]

$\angle CBD = 90°$ [Angle of a square]

$\Rightarrow$ $\angle ABM = \angle CMD$



Adding $\angle ABC$ on both sides, we have,

$\angle ABM + \angle ABC = \angle CMD + \angle ABC$

$\Rightarrow$ $\angle MBC = \angle ABD$ … (1)

Now, in ΔMBC and ΔABD, we have

$MB = AB$ [Sides of square ABMN]

$\angle MBC = \angle ABD$ [From (1)]

$BC = BD$ [Sides of square BCED]

$\therefore$ $\Delta MBC \cong \Delta ABD$

[SAS congruency]… (2)

Similarly, we have

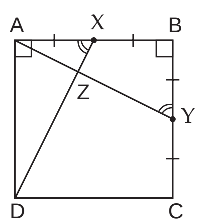
$\Delta FCB \cong \Delta ACE$ … (3)

From (2) and (3), we conclude that

$\mathbf{\Delta MBC \cong \Delta ABD}$

and $\mathbf{\Delta FCB \cong \Delta ACE}$

38. (*i*)



In ΔADX and ΔBAY,

$AD = BA$ [Sides of a square]

$\angle DAX = \angle ABY$ [Each 90°]

$AX = BY$

[Each = $\frac{1}{2}$ the sides of a square]

$\Rightarrow$ $\mathbf{\Delta ADX \cong \Delta BAY}$ [SAS congruency]

(*ii*) $\because$ $\Delta ADX \cong \Delta BAY$

$\therefore$ $\mathbf{\angle DXA = \angle AYB}$ [CPCT]

(*iii*) In ΔAXZ and ΔAYB, we have

$\angle AXZ = \angle AYB$

[$\because$ $\angle DXA = \angle AYB$, proved]

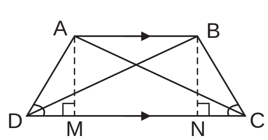
$\angle XAZ = \angle BAY$ [Common]

Remaining $\angle AZX$ = remaining $\angle ABY = 90°$

[$\angle$ of a square]

$\Rightarrow$ $\mathbf{DX \perp AY}$

39. Draw $AM \perp DC$ and $BN \perp DC$.



Now, in ΔAMD and ΔBNC,

$\angle AMD = \angle BNC$ [Each = 90°]

$\angle ADM = \angle BCN$ [Given]

$AM = BF$

[Opp. sides of rectangle ABNM]

$\therefore$ $\Delta AMD \cong \Delta BNC$ [By AAS congruence]

$\Rightarrow$ $\mathbf{AD = BC}$ [CPCT]

Again in ΔADC and ΔBCD, we have

$AD = BC$ [Proved above]

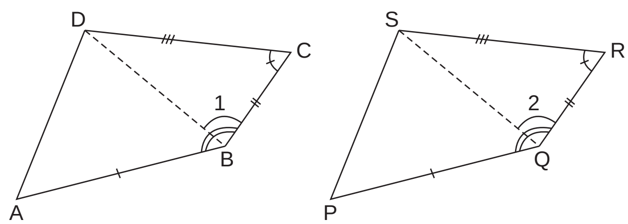
$\angle ADC = \Delta BCD$ [Given]

$DC = CD$ [Common]

$\therefore$ $\Delta ADC \cong \Delta BCD$ [By SAS congruence]

$\Rightarrow$ $\mathbf{AC = BD}$ [CPCT]

40. In ΔBCD and ΔQRS,



$BC = QR$ [Given]

$\angle C = \angle R$ [Given]

$CD = RS$ [Given]

$\therefore$ $\Delta BCD \cong QRS$ [SAS congruency] … (1)

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]

$BD = QS$ [CPCT]

Since $\angle B = \angle Q$ [Given]

$\therefore$ $\angle B - \angle 1 = \angle Q - \angle 2$

$\Rightarrow$ $\angle ABD = \angle PQS$ … (2)

Now, in ΔABD and ΔPQS, we have

$AB = PQ$ [Given]

$\angle ABD = \angle PQS$ [From (2)]

$BD = QS$ [proved above]

$\therefore$ $\Delta ABD \cong \Delta PQS$

[SAS congruency] … (3)

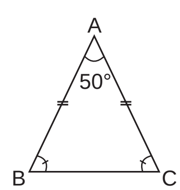
From (1) and (3), we have

$[\Delta ABD + \Delta BCD] \cong [\Delta PQS \pm \Delta QRS]$

$\Rightarrow$ **Quad (ABCD) $\cong$ Quad (PQRS)**

EXERCISE 7B

1. In ΔABC, the sides $AB = AC$.



$\Rightarrow$ $\angle B = \angle C$

$\therefore$ $\angle A = 50°$

$\angle A + \angle B + \angle C = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow$ $50° + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 180° - 50° = 130°$

$\Rightarrow$ $\angle B = \angle C = \frac{130°}{2} = 65°$

$\therefore$ $\mathbf{\angle B = \angle C = 65°}$

2. (*i*) $x = 50° + \angle B$

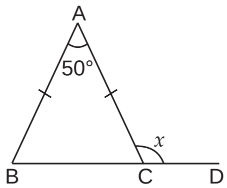
[Ext. $\angle$ = Sum of the opp. $\angle$]... (1)

$AB = AC$

$\Rightarrow \angle B = \angle C$

Also, $\angle B + \angle C = 180° - 50° = 130°$

$\therefore \angle B = \frac{130°}{2} = 65°$ ... (2)



From (1) and (2),

$x = 50° + 65°$

$\Rightarrow$ $\mathbf{x = 115°}$

(*ii*) $AB = BC$

$\Rightarrow \angle 1 = \angle 2$

But $\angle 1 + 130° = 180°$ [Linear pair]

$\Rightarrow \angle 2 + 130° = 180°$

$\Rightarrow \angle 2 = 180° - 130°$

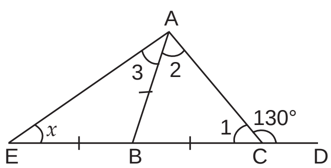
$= 50°$

$\angle 2 + \angle ABC = \angle 130°$

$\Rightarrow \angle ABC = 130° - \angle 2$

$= 130° - 50°$

$= 80°$



But in ΔABE,

$x + \angle 3 = \text{Ext. } \angle ABC$

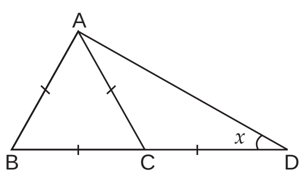
$\Rightarrow x + x = \text{Ext. } \angle ABC$

[$\therefore$ AB = AE $\Rightarrow \angle 3 = x$]

$\Rightarrow x = \frac{\angle ABC}{2} = \frac{80^0}{2} = 40°$

Thus, $\mathbf{x = 40°}$.

(*iii*) ΔABC is an equilateral triangle.



$\Rightarrow \angle B = 60° = \angle BAC$

Ext. $\angle ACB = 60 + 60° = 120°$

In ΔACD, $AC = CD$

$\Rightarrow x = \angle CAD$

$\Rightarrow 120° + x + \angle CAD = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow x + x = 180° - 120 = 60°$

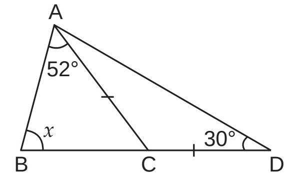
$\Rightarrow 2x = 60°$

$\Rightarrow$ $\mathbf{x = 30°}$

(*iv*) In ΔACD,

$AC = CD$

$\Rightarrow \angle CAD = 30°$ ... (1)



Now,

$\angle ACD + \angle CAD + 30° = 180°$

$\Rightarrow \angle ACD + 30° + 30° = 180°$ [From (1)]

$\angle ACD = 180° - 30° - 30° = 120°$

In ΔABC,

Ex. $\angle ACD = x + 52°$

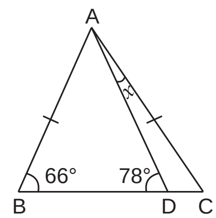
$\Rightarrow x + 52° = 120°$

$\Rightarrow x = 120 - 52°$

$= 68°$

Thus, $\mathbf{x = 68°}$.

(*v*) In ΔABC,



$AB = AC$

$\Rightarrow \angle B = \angle C$

$\therefore \angle C = 66°$ [$\because \angle B = 66°$]

In ΔACD,

Ext. $\angle ADB = x + \angle C$

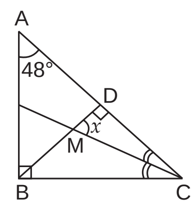
$\Rightarrow x + 66° = 78°$ [$\because \angle ADB = 78°$]

$\Rightarrow x = 78° - 66°$

$= 12°$

Thus, $\mathbf{x = 12°}$.

(*vi*) In ΔABC,



$\angle ABC = 90°$

$\Rightarrow \angle ACB = 180° - 90° - 48°$

[$\because \angle A = 48°$]

$\Rightarrow \angle ACB = 42°$

In ΔCDM, $\angle DCM = \frac{1}{2} \angle ACB$

$= \frac{1}{2} \times 42° = 21°$

$\therefore \angle CMD = 180° - (90° + 21°)$

$= 69°$

$= x$

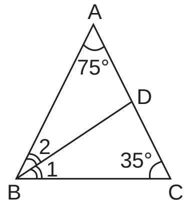
Thus, $\mathbf{x = 69°}$.

3. In ΔABC, $\angle ABC = 180° - (75° + 35°)$

[Sum of $\angle$s of a Δ is 180°]

$= 180° - 110°$

$= 70°$



$\therefore$ BD is bisector of $\angle B$.

$\therefore$ $\angle 1 = \angle 2 = \frac{70°}{2} = 35°$

In ΔBDC, $\angle 1 = \angle C = 35°$

$\Rightarrow$ **BD = CD**

[Sides opp. equal $\angle$s of a Δ]

4. (*a*) Measure of each angle of an equilateral $\Delta = 60°$

Each ext. $\angle$ = Sum opp. interior angle

$= 60° + 60°$

$= 120°$

$\Rightarrow$ Measure of exterior angle = **120°**

(*b*) (*i*) Let the measure of vertical angle = x

$\therefore$ Each of the base angle = $2x$

Now, $x + 2x + 2x = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow$ $5x = 180°$

$\Rightarrow$ $x = \frac{180°}{5} = 36°$

$\Rightarrow$ $2x = 72°$

$\therefore$ Angles of the Δ are **36°, 72°, 72°.**

(*ii*) Let vertical angle = x

Then, base angles are $(x + 15)°$ and $(x + 15°)$.

$\therefore$ Sum of the $\angle$s of the Δ

$= x + (x + 15)° + (x + 15)°$

$= 180°$

$\Rightarrow$ $3x + 30 = 180°$

$\Rightarrow$ $3x = 180° - 30°$

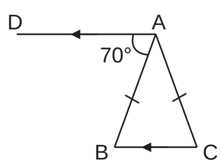
$= 150°$

$\Rightarrow$ $x = \frac{150°}{3} = 50°$

$\Rightarrow$ Each base angle = $x + 15° = 65°$

Thus, the measure of angles of the Δ are **50°, 65°, 65°.**

5. In ΔABC, AB = AC



$\Rightarrow$ $\angle B = \angle C$ … (1)

$\therefore$ AD $\parallel$ BC and BA is a transversal.

$\therefore$ $\angle B = 70°$ [Alt. angles] … (2)

$\Rightarrow$ $\angle C = 70°$ [$\therefore$ $\angle B = \angle C$]

Now, $\angle A + \angle B + \angle C = 180°$ [Sum of $\angle$s of a Δ]

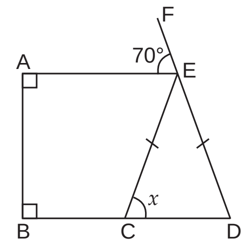
$\Rightarrow$ $\angle A = 180° - \angle B - \angle C$

$= 180° - 70° - 70°$

$= 40°$

Thus, $\angle$**BAC = 40°.**

6. (*i*)



$\angle EAB + \angle ABD = 90° + 90°$

$= 180°$

But $\angle EAB$ and $\angle ABD$ are corr. $\angle$s

$\therefore$ AE $\parallel$ BD

$\Rightarrow$ $\angle FDC = \angle FEA$

$= 70°$ [Corr. $\angle$s, AE $\parallel$ BD]

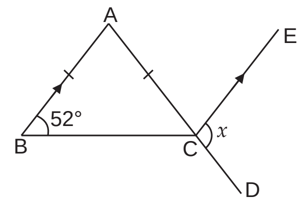
$\Rightarrow$ $\angle EDC = 70°$ … (1)

$x = \angle EDC$

[$\angle$s Opp. equal sides of ΔEDC]

$\Rightarrow$ $\boldsymbol{x = 70°}$ [Using (1)]

(*ii*) In ΔABC,



AB = AC

$\Rightarrow$ $\angle B = \angle C$

$\therefore$ $\angle B = 52°$

$\therefore$ $\angle C = 52°$

$\Rightarrow$ $\angle A = 180° - 52° - 52°$

$= 76°$

AB $\parallel$ CE and AC is a transversal.

$\Rightarrow$ $\angle A = \angle ACE$ [Alt. angles]

$\Rightarrow$ $\angle ACE = 76°$

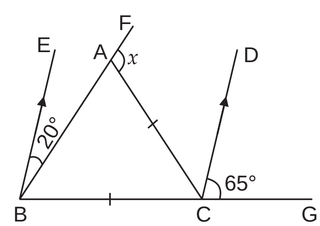
Now, $\angle ACE + x = 180°$ [Linear pair]

$\Rightarrow$ $76° + x = 180°$

$\Rightarrow$ $x = 180° - 76° = 104°$

Thus, $\boldsymbol{x = 104°}$.

(*iii*) BE $\parallel$ CD and BC is a transversal.



$\therefore$ $\angle EBC = \angle DCG = 65°$ [Corr. angles]

$\Rightarrow$ $\angle EBA + \angle ABC = 65°$ or $20° + \angle ABC = 65°$

$\angle ABC = 65° - 20°$

$= 45°$

In ΔABC, AC = BC

$\Rightarrow$ $\angle ABC = \angle BAC$

$\Rightarrow$ $\angle BAC = 45°$

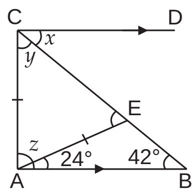
Now, $\angle BAC + x = 180°$ [Linear pair]

or $45° + x = 180°$

$\Rightarrow \quad x = 180° - 45°$
$= 135°$

Thus, x **= 135°**.

7. (*i*) AB ∥ CD and CB is a transversal.



$\therefore \quad x = 42°$ [Alt. angles]

$\angle CEA = y$ [$\because$ AC = AE, Given]

Ext. $\angle CEA = \angle Y$
$= 24° + 42°$
$= 66°$ [$\because$ AC = AE]

In ΔACE, we have

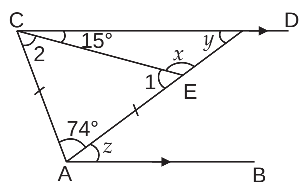
$Z + Y + Y = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow \quad Z + 66° + 66° = 180°$

$\Rightarrow \quad Z = 180° - 66° - 66°$
$= 48°$

Thus, x **= 42°**, y **= 66°** and z **= 48°**

(*ii*)



(*ii*)

In ΔACE, AC = AE

$\Rightarrow \quad \angle 1 = \angle 2$ … (1)

In ΔACE, we have

$\angle 1 + \angle 2 + 74° = 180°$

$\Rightarrow \quad \angle 1 + \angle 1 + 74° = 180°$ [Using (1)]

$\Rightarrow \quad 2\angle 1 = 180° - 74°$

$\Rightarrow \quad \angle 1 = \frac{106°}{2} = 53°$

$x + \angle 1 = 180°$ [Linear pair]

$\Rightarrow \quad x = 180° - 53°$
$= 127°$

$y = 180° - x - 15°$ [Sum of is a Δ]
$= 180° - 127° - 15°$
$= 38°$

AB ∥ CD and AD is a transversal.

$\therefore \quad z = y$ [Alt. angles]
$= 38°$

$\Rightarrow \quad z = 38°$

Thus, x **= 127°**, y **= 38°**, z **= 38°**

8. PQ = PR

$\Rightarrow \quad \angle Q = \angle PRQ$

Ext. $\angle XPQ = \angle Q + \angle PRQ = 128°$

$\therefore \quad \angle Q = \angle PRQ$
$= \frac{128°}{2}$
$= 64°$ … (1)

$\therefore$ RQ = RS

$\Rightarrow \quad \angle Q = \angle 1 = 64°$ [From (1)]

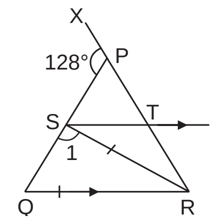
ST ∥ QR and QS is a transversal.

$\therefore \quad \angle TSQ + \angle Q = 180°$ [cointerior angles]

$\Rightarrow \quad \angle RST + \angle 1 + \angle Q = 180°$

$\Rightarrow \quad \angle RST + 64° + 64° = 180°$

or $\quad \angle RST = 180° - 64° - 64°$
$= 52°$ … (2)



From (1) and (2) in ΔRST,

$\angle RST + \angle STR + \angle TRS = 180°$

$\Rightarrow 52° + (180 - \angle PRQ) + \angle TRS = 180°$

$\Rightarrow \quad 52° + (180° - 64°) + \angle TRS = 180°$

$\Rightarrow \quad 52° + 116° + \angle TRS = 180°$

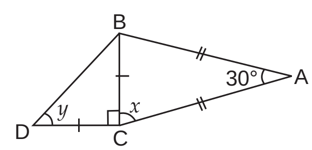
$\Rightarrow \quad \angle TRS = 180° - 168°$
$= 12°$ … (3)

From (2) and (3), we have

∠RST = 52°

and **∠TRS = 12°**

9. (*i*) In ΔABC,



AB = AC

$\Rightarrow \quad \angle ABC = \angle ACB = x$

$x + x + 30° = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow \quad x = \frac{180° - 30°}{2}$
$= \frac{150°}{2}$
$= 75°$

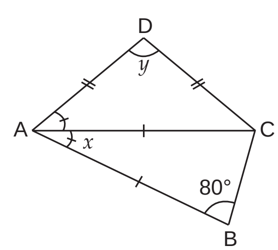
In Ext ΔBCD, BC = CD and ∠BCD = 90°

$\Rightarrow \quad \angle CDB = \angle CBD = y$

$\therefore \quad y = \frac{90°}{2} = 45°$

Thus x **= 75°** and y **= 45°**.

(*ii*)



In ΔABC, AB = AC

$\Rightarrow \quad \angle B = \angle ACB = 80°$

$\therefore \quad \angle BAC + \angle B + \angle ACD = 180°$

[Sum of ∠s of ΔABC]

$\Rightarrow \quad x + 80° + 80° = 180°$

or, $\quad x = 180° - 80° - 80°$

$= 20°$

In ΔADC, $\quad AD = CD$

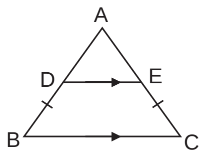
$\Rightarrow \quad \angle DAC = \angle DCA = x$

$\therefore \quad y = 180° - (20° + 20°)$

$= 140° \quad [\because \angle DAC = x]$

Thus, $\boldsymbol{x = 20°}$ and $\boldsymbol{y = 140°}$.

10.



In ΔABC, $\quad AB = AC$

$\Rightarrow \quad \angle B = \angle C$

DE ∥ BC and AB is a transversal.

$\therefore \quad \angle B = \angle ADE$ [Corr. angles]

Similarly, $\quad \angle C = \angle AED$ [Corr. angles]

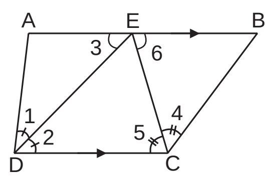
$\angle ADE = \angle AED$ [Using (1)] … (2)

Now, in ΔADE,

$\because \quad \angle ADE = \angle AED$ [From (2)]

$\therefore \quad$ **AD = AE** [Sides opp. equal ∠s]

11. AB ∥ DC and DE is a transversal.



$\therefore \quad \angle 3 = \angle 2$ [Alt. ∠s]

But $\quad \angle 1 = \angle 2$ [DE bisects ∠ADC]

$\therefore \quad \angle 1 = \angle 3$

$\Rightarrow \quad AE = AD$

[Sides opp. equal ∠s] … (1)

$\because \quad \angle 4 = \angle 5$ [CE bisects ∠DCB]

$\angle 6 = \angle 5$

[∵ AB ∥ DC and ∠5 and ∠6 are alt. ∠s]

$\Rightarrow \quad \angle 4 = \angle 6$

$\Rightarrow \quad EB = BC$

[Sides opp. equal ∠s] … (2)

Adding (1) and (2),

$EA + EB = AD + BC$

$\Rightarrow \quad$ **AB = AD + BC**

12. $\quad AB = AC$

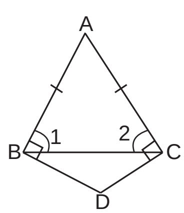
$\Rightarrow \quad \angle 1 = \angle 2$

[∠s opp. equal sides of a Δ]

But, $\quad \angle ABD = \angle ACD$ [Each = 90°]

$\Rightarrow \quad \angle ABD - \angle 1 = \angle ACD - 1$

$\Rightarrow \quad \angle DBC = \angle DCB$ … (1)



Now, in ΔDBC,

$\angle DBC = \angle DCB$ [From (1)]

$\Rightarrow \quad$ **CD = BD**

[Sides opp. equal ∠s of a Δ]

13. $\quad AB = AC$

$\Rightarrow \quad \angle ACB = \angle ABC$ … (1)

$\Rightarrow \quad 180° - \angle ACB = 180° - \angle ABC$

$\Rightarrow \quad \angle BCF = \angle CBE$

$\Rightarrow \quad \frac{1}{2}\angle BCF = \frac{1}{2}\angle CBE$

$\Rightarrow \quad \angle BCD = \angle CBD$ … (2)

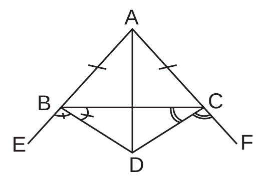
$\Rightarrow \quad BD = CD$

[sides of opp. equal ∠s] … (3)

Adding (1) and (2), we get

$\angle ACB + \angle BCD = \angle ABC + \angle CBD$

$\Rightarrow \quad \angle ACD = \angle ABD$ … (4)



In ΔACD and ΔABD, we have

$AC = AB$ [Given]

$\angle ACD = \angle ABD$ [From (4)]

$CD = BD$ [From (3)]

$\therefore \quad \Delta ACD \cong \Delta ABD$ [By SAS congruence]

$\Rightarrow \quad \angle CAD = \angle BAD$ [CPCT]

and $\quad \angle BDA = \angle CDA$ [CPCT]

$\Rightarrow \quad$ AD bisects ∠A

and $\quad$ AD bisects ∠D

Hence, **BD = CD** and **AD bisects ∠D**.

14. $\because \quad RT = RS$ [Given]

$\therefore \quad \angle RST = \angle RTS = x°$ [Say]

In right ΔLNT, we have

$\angle RLM = 90° - \angle RTS$

[Sum of ∠s a Δ is 180°]

$= 90° - x°$ … (1)

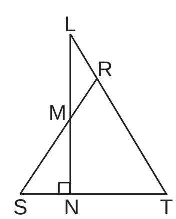
In right ΔMNS,

$\angle SMN = 90° - x°$

[Sides of ∠s of a Δis 180°] … (2)

Now, $\quad \angle SMN = \angle RML$ [Vert. opp. ∠s]

$\Rightarrow \quad \angle RML = 90° - x°$ [From (2)] … (3)



From (1) and (3), we have

$\angle RML = \angle RLM$

$\Rightarrow \quad$ **RL = RM**

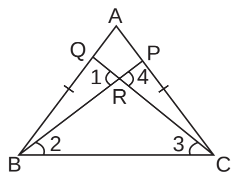
[Sides opp. to equal ∠s of a Δ]

15. $\because \quad \angle 1 = 2\angle 2$

and $\quad \angle 4 = 2\angle 3$

But $\quad \angle 1 = \angle 4$ [Vert. opp. ∠s]

$\Rightarrow \quad \angle 2 = \angle 3$



Now, in ΔBPA and ΔCQA, we have

$(\angle ABC - \angle 2) = (ACB - \angle 3)$

[$\because$ $\angle ABC = \angle ACB$ as, AB = AO (Given)]

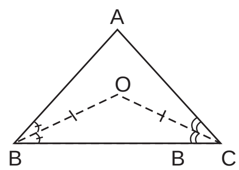
or, $\angle ABP = \angle ACQ$

$\angle A = \angle A$ [Common]

AB = AC [Given]

$\Rightarrow$ **ΔBPA $\cong$ ΔCQA** [ASA congruency]

16.



In ΔOBC, we have OC = OB [Given]

$\angle OBC = \angle OCB$

[$\angle$s. opp. equal sides of a Δ]

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

[$\therefore$ BC and CO are bisectors of $\angle$B and $\angle$C respectively]

$\Rightarrow \angle ABC = \angle ACB$

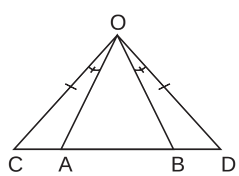
$\Rightarrow$ AB = AC

[Sides opp. equal $\angle$s of a Δ]

$\therefore$ **ΔABC is an isosceles triangle.**

17. In ΔOCD, OC = OD [Given]

$\Rightarrow \angle C = \angle D$ … (1)



In ΔOCA and ΔODB, we have

$\angle C = \angle D$ [From (1)]

OC = OD [Given]

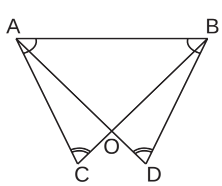
$\angle COA = \angle DOB$ [Given]

$\therefore$ $\Delta OCA \cong \Delta ODB$ [ASA congruency]

$\Rightarrow$ OA = OB [CPCT]

$\therefore$ **ΔOAB is an isosceles triangle.**

18.



In ΔACB and ΔBDA, we have

$\angle ACB = \angle BDA$ [Given]

$\angle CAB = \angle DBA$ [Given]

AB = BA [Common]

$\therefore$ $\Delta ACB \cong \Delta BDA$ [By AAS congruency]

$\Rightarrow \angle ABC = \angle BAD$ [CPCT]

$\Rightarrow \angle ABO = \angle BAO$

[$\therefore$ O lies on BC as well as on AD]

$\therefore$ AO = BO [Sides opp. to equal $\angle$s]

Hence, **ΔAOB is isosceles triangle.**

19. AD = AE

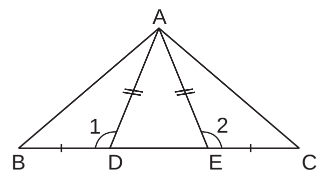
$\Rightarrow \angle ADE = \angle AED$ … (1)

$\angle 1 + \angle ADE = 180°$

$\angle 2 + \angle AED = 180°$

$\Rightarrow \angle 1 + \angle ADE = \angle 2 + \angle AED$

$\Rightarrow \angle 1 = \angle 2$ [From (1)]



In ΔADB and ΔAEC, we have

BD = EC [Given]

$\angle 1 = \angle 2$ [Proved above]

AD = AE [Given]

$\Rightarrow$ $\Delta ADB \cong \Delta AEC$ [SAS congruency]

$\therefore$ AB = AC [CPCT]

$\Rightarrow$ **ΔABC is an isosceles triangle**.

20. Let $\angle F = x$

$\therefore \angle HGF = 2\angle F = 2x$

$\angle FHG = 2\angle F = 2x$

$\Rightarrow \angle FHG = \angle HGF$

$\therefore$ FG = FH

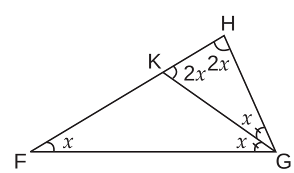
[Sides opp. to equal angles] … (1)

$\angle HGF = 2\angle F$

$= 2x$

$= \frac{1}{2}\angle HGF = x$

$\Rightarrow \angle KGF = x$



Now, Ext. $\angle HKG = \angle F + \angle KGF$

$= x + x$

$= 2\angle F$

Also, $\angle FHG = \angle HGF$

$= x + x$

$= 2x$

$\Rightarrow \angle HKG = \angle FHG$

$\Rightarrow \angle HKG = \angle KHG$

$\therefore$ HG = KG … (2)

Also, in ΔKFG, we have

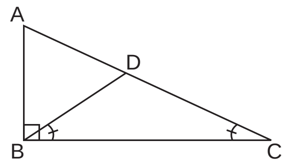
$\angle KFG = \angle KGF = x$

$\Rightarrow$ KF = KG … (3)

From (1), (2) and (3), we have

FG = FH, **HG = KG** and **KF and KG**.

21.



In ΔDBC, $DC = DB$ [Given]

$\Rightarrow$ $\angle DBC = \angle DCB = x$ (Say)

In ΔABC, we have

$\angle BAC + 90° + \angle C = 180°$ [Sum of $\angle$s of a Δ]

$\angle BAD + 90° + x = 180°$ [D lies on AC]

$\Rightarrow$ $\angle BAD = 90° - x$ … (1)

Also, $\angle ABD = 90° - \angle DBC$

$\Rightarrow$ $\angle ABD = 90° - x$ … (2)

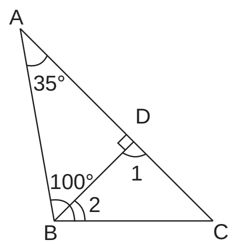
In ΔDAB, we have

$\angle BAD = \angle ABD$ [Using (1) and (2)]

$\Rightarrow$ **BD = AD**

[Sides opp. equal angles of a Δ]

22.



In ΔABC, $\angle A = 35°$

$\angle ABC = 100°$

$\therefore$ $\angle C = 180° - 35° - 100°$

$= 45°$ [Sum of $\angle$s of a Δ] … (1)

$BD \perp AC$

$\Rightarrow$ $\angle 1 = 90°$

In ΔBDC,

$\angle 1 + \angle 2 + \angle C = 180°$

$\Rightarrow$ $90° + \angle 2 + 45° = 180°$

$\Rightarrow$ $\angle 2 = 180° - 90° - 45°$

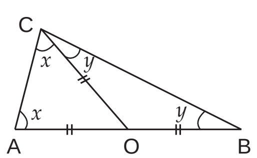
$= 45°$ … (2)

In ΔBDC, from (1) and (2)

$\therefore$ $\angle C = \angle 2 = 45°$

$\Rightarrow$ **DB = DC** [Sides opp. equal $\angle$s]

23.



In ΔACO, $OC = OA$ [Given]

$\therefore$ $\angle OAC = \angle OCA = x$ (Say)

Similarly, in ΔBCO,

$OC = OB$ [Given]

$\therefore$ $\angle OBC = \angle OCB = y$ (Say)

Now, in ΔACB,

$\angle A + \angle C + \angle B = 180°$ [Sum of $\angle$s of a Δ]

$x + (x + y) + y = 180°$

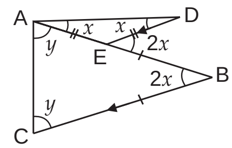
$\Rightarrow$ $2(x + y) = 180°$

or, $x + y = 90°$

$\Rightarrow$ $\angle ACB = 90°$

Thus, **$\angle$ACB is a right angle.**

24.



In ΔAED, we have $ED = EA$ [Given]

$\therefore$ $\angle EAD = \angle EDA = x$ (say)

[Angle opp. equal sides]

In ΔABC, we have

$BC = BA$ [Given]

$\Rightarrow$ $\angle BAC = \angle BCA = y$ (say)

[Angles opp. equal sides]

Now, ext. $\angle BED = x + x = 2x$

$\therefore$ $DE \parallel BC$

$\therefore$ $\angle EBC = \angle DEB = 2x$ [Alt. $\angle$s]

In ΔABC,

$y + y + 2x = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow$ $2y + 2x = 180°$

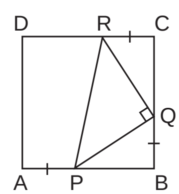
$\Rightarrow$ $2[x + y] = 180°$

or $x + y = 90°$

$\Rightarrow$ $\angle CAD = 90°$

Thus, **$\angle$CAD is a right angle**.

25. ABCD is a square.



$\therefore$ $AB = BC$ [Sides of a square]

$\Rightarrow$ $AB - AP = BC - BQ$ [$\because AP = BQ$, Given]

$\Rightarrow$ $PB = QC$ … (1)

In ΔPBQ and ΔQCR, we have

$PB = QC$ [From (1)]

$\angle B = \angle C$ [Each = 90°]

$BQ = CR$ [Given]

$\therefore$ $\Delta PBQ \cong \Delta QCR$ [By SAS congruency]

$\Rightarrow$ $PQ = RQ$ [By CPCT]

$\Rightarrow$ $\angle PRQ = \angle RPQ$

[$\angle$s Opp. sides of ΔQPR] … (2)

In ΔPQR,

$\angle PRQ + \angle RPQ + 90° = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow$ $2\angle RPQ + 90° = 180°$ [Using (2)]

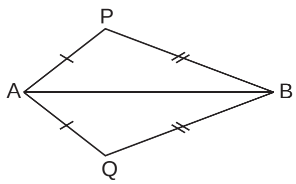
$\Rightarrow$ $2\angle RPQ = 180° - 90°$

$\Rightarrow$ $\angle RPQ = \frac{90°}{2}$

$\Rightarrow$ **$\angle$RPQ = 45°**

EXERCISE 7C

1.



In ΔAPB and ΔAQB, we have

$AP = AQ$ [Given]

$BP = BQ$ [Given]

$AB = BA$ [Common]

$\therefore$ $\Delta APB \cong \Delta AQB$ [By SSS congruency]

$\Rightarrow$ $\angle PAB = \angle QAB$ [CPCT]

and $\angle PBA = \angle QBA$

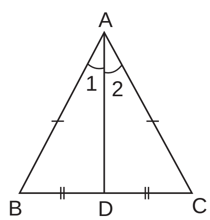
$\Rightarrow$ AB is bisector of $\angle PAQ$ and AB is bisector of $\angle PBQ$

Thus, **AB is the bisector of $\angle PAQ$ and $\angle PBQ$**

2. In ΔABC, $AB = AC$ [Given]

and AD is the median.

$\Rightarrow$ $BD = DC$ … (1)



(*i*) In ΔABD and ΔACD,

$AB = AC$ [Given]

$BD = CD$ [From (1)]

$AD = AD$ [Common]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [SSS congruency]

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]

i.e. AD bisects $\angle A$.

(*ii*) $\Delta ABD \cong \Delta ACD$ [Proved above]

$\therefore$ $\angle ADB = \angle ADC$ [CPCT]… (2)

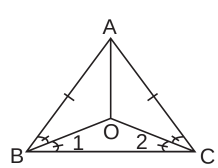
But $\angle ADB + \angle ADC = 180°$ [Linear pair]… (3)

$\Rightarrow$ $\angle ADB = \angle ADC$

$= 90°$ [Using (2) and (3)]

i.e. **AD is perpendicular to BC**.

3.



In ΔABO and ΔACO, we have

$AB = AC$ [Given]

$\Rightarrow$ $\angle ACB = \angle ABC$

$\Rightarrow$ $\frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow$ $\angle 2 = \angle 1$

[$\because$ CO and BO bisect $\angle C$ and $\angle B$ respectively]

$\Rightarrow$ **BO = CO**

[Sides opp. equal $\angle$s of a ΔOBC]… (1)

In ΔABO and ΔACO, we have

$AB = AC$ [Given]

$AO = AO$ [Common]

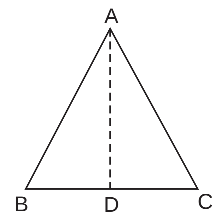
$BO = CO$ [From (1)]

$\therefore$ $\Delta ABO \cong \Delta ACO$ [By SSS congruence]

$\Rightarrow$ $\angle OAB = \angle OAC$

$\Rightarrow$ **AO bisects $\angle A$.**

4.



In ΔADB and ΔACD, we have

$AB = AC$ [sides of equal Δ]

$BD = CD$ [AD is median]

$AD = AD$ [Common]

$\therefore$ $\Delta ADB \cong \Delta ACD$ [By SSS congruence]

$\Rightarrow$ $\angle ADB = \angle ADC$ [CPCT] … (1)

But $\angle ADB + \angle ADC = 180°$ [Linear pair]

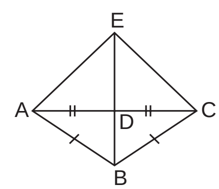
$\therefore$ $\angle ADC + \angle ADC = 180°$ [Using (1)]

$\Rightarrow$ $2\angle ADC = 180°$

or $\angle ADC = 90°$

i.e. **$\angle ADC$ is a right angle**.

5.



In ΔADB and ΔBDC, we have

$AB = BC$ [Given]

$AD = CD$ [Given]

$BD = BD$ [Common]

$\therefore$ $\Delta ADB \cong \Delta BDC$ [By SSS congruency]

$\Rightarrow$ $\angle ADB = \angle BDC$ [CPCT] … (1)

$\angle ADB + \angle BDC = 180°$ [Linear pair] … (2)

$\Rightarrow$ $\angle ADB = \angle BDC$

$= 90°$ [Using (1) and (2)] … (3)

$\angle ADE = \angle BDC$

and $\angle CDE = \angle ADB$ [Vert. opp. $\angle$s]

$\Rightarrow$ $\angle ADE = 90°$ [Using (3)]

and $\angle CDE = 90°$ … (4)

Hence, $\angle APE$ is a right angle.

In triangles AED and CED, we have

$AD = CD$ [Given]

$\angle ADE = \angle CDE$ [From (4)]

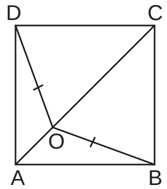
$DE = DE$ [Common]

$\therefore$ $\Delta ADE \cong \Delta CDE$ [By SAS congruence]

$\Rightarrow$ $AE = AE$ [CPCT]… (2)

Hence, **$\angle ADE$ is a right angle** and **AE = AC**.

6.



In ΔAOD and ΔAOB, we have

$AO = AO$ [Common]

$AD = AB$ [Sides of a square]

$OD = OB$ [Given]

$\therefore$ $\Delta AOD \cong \Delta AOB$ [SSS congruency]

$\Rightarrow$ $\angle AOD = \angle AOB$ [CPCT] … (1)

Similarly, $\Delta COD \cong \Delta COB$ [By SSS congruency]

$\Rightarrow$ $\angle COD = \angle COB$ [CPCT]… (2)

Adding (1) and (2), we get

$\angle AOD + \angle COD = \angle AOB + \angle COB$ … (3)

$\therefore$ Sum of all angles about a point is 360°

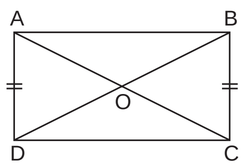
$\therefore$ $\angle AOD + \angle COD + \angle AOB + \angle COB = 360°$

$\Rightarrow$ $2(\angle AOD + \angle COD) = 360°$ [Using (3)]

$\Rightarrow$ $\angle AOD + \angle COD = 180°$

$\Rightarrow$ **AOC is a straight line**.

7.



In ΔADC and ΔBCD, we have

$AD = BC$ [Given]

$AC = BD$ [Given]

$DC = CD$ [Common]

$\therefore$ $\Delta ADC \cong \Delta BCD$ [By SSS congruency]

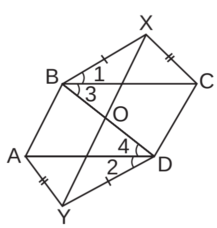
$\Rightarrow$ $\angle ACD = \angle BDC$ [CPCT]

$\Rightarrow$ $\angle OCD = \angle ODC$

In ΔCOD, $\angle OCD = \angle ODC$

$\Rightarrow$ **OC = OD** [sides opp. equal $\angle$s]

8. (*i*)



In ΔBCX and ΔDAY, we have

$XB = YD$ [Given]

$XC = YA$ [Given]

$BC = DA$ [Opp. sides of a $\parallel$gm]

$\therefore$ $\Delta BCX \cong \Delta DAY$ [SSS congruency]

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]… (1)

$\angle 3 = \angle 4$ [Alt. angles] … (2)

Adding (1) and (2), we get

$(\angle 1 + \angle 3) = (\angle 2 + \angle 4)$

$\Rightarrow$ $\angle XBO = \angle ODY$ … (3)

$\Rightarrow$ $\angle XBD = \angle BDY$

But $\angle XBD$ and $\angle BDY$ form a pair of interior alternate angles where transversal BD meets BX at B and DY at D.

Hence, **BX $\parallel$ DY**.

(*ii*) In ΔXOB and ΔYOD, we have

$\angle XBO = \angle ODY$ [From (3)]

$\angle XOB = \angle YOD$ [Vert. opp. $\angle$s]

and $XB = YD$

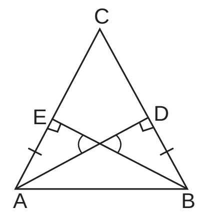
$\therefore$ $\Delta XOD \cong \Delta YOD$ [By AAS congruence]

$\Rightarrow$ $OX = OY$ and $OB = OD$ (CPCT)

$\Rightarrow$ O is the mid-point of XY and BD.

Thus, **XY and BD bisect each other**.

9.



In right ΔBDA and right ΔAEB, we have

$BD = AE$ [Given]

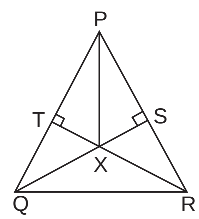
$BA = AB$ [Common]

$\therefore$ $\Delta BDA \cong \Delta AEB$ [By RHS congruence]

$\Rightarrow$ **AD = BE** [CPCT]

10. In ΔPQR, $PQ = PR$

$\Rightarrow$ $\angle PQR = \angle PRQ$ … (1)



In ΔQTR and ΔRSQ, we have

$\angle QTR = \angle RSQ$ [Each = 90°]

$\angle TQR = \angle SRQ$

[$\because$ From (1) $\angle PQR = \angle PRQ$ $\Rightarrow \angle TQR = \angle SRQ$]

$QR = RQ$ [Common]

$\therefore$ $\Delta QTR \cong \Delta RSQ$ [By AAS congruency]

$\Rightarrow$ $TQ = SR$ [CPCT] … (2)

Also $PQ = PR$ [Given]… (3)

Subtracting (2) From (3), we have

$PQ - TQ = PR - SR$

$\Rightarrow$ $PT = PS$

Now, in ΔPTX and ΔPSX, we have

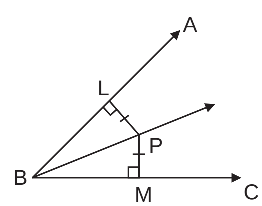
$\angle PTX = \angle PSX$

$PT = PS$

and $PX = PX$

$\therefore$ **$\Delta PTX \cong \Delta PSX$** [By RHS congruency]

11.



In rt. ΔPLB and rt. ΔPMB, we have

$PL = PM$ [Given]

$\angle PLM = \angle PMB$ [Each = 90°]

$PB = PB$ [Common]

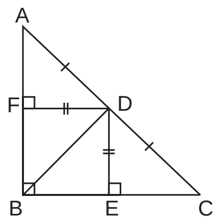
$\therefore$ $\Delta PLB \cong \Delta PMB$ [RHS congruency]

$\Rightarrow$ $\angle PBL = \angle PBM$

$\Rightarrow$ PB is the bisector of $\angle LBM$ i.e. $\angle ABC$

$\Rightarrow$ **P lies on the bisector $\angle ABC$**.

12.



In rt. ΔDEC and rt. ΔDFA, we have

$DE = DF$ [Given]

$DC = DA$

[D is the mid-point of AC]

$\Rightarrow$ $\Delta DEC \cong \Delta DFA$ [By RHS congruency]

$\therefore$ $\angle DCE = \angle DAF$ [CPCT]

$\therefore$ $\angle ACB = \angle CAB$ [CPCT]

$\Rightarrow$ $AB = BC$

[sides opp. to equal angles]

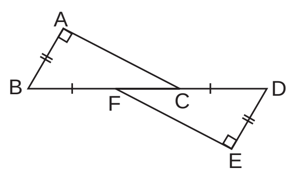
Thus, **ΔABC is an isosceles triangle**.

13. $BF = DC$ [Given]

$\therefore$ $BF + FC = DC + FC$

[Adding FC on both sides.]

$\Rightarrow$ $BC = DF$ … (1)



Now, in ΔBAC and ΔDEF, we have

$BC = DF$ [From (1)]

$AB = DE$ [Given]

$\therefore$ $\Delta BAC \cong \Delta DEF$ [RHS congruency]

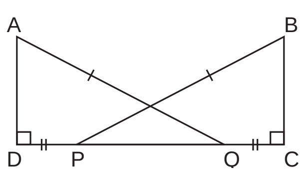
$\Rightarrow$ **$AC = EF$** [CPCT]

14. $DP = CQ$ [Given]

Adding PQ on both sides,

$DP + PQ = CQ + PQ$

$\Rightarrow$ $DQ = CP$ … (1)



Now, in rt. ΔADQ and rt. ΔBCP, we have

$AQ = BP$ [Given]

$DQ = CP$ [From (1)]

$\therefore$ $\Delta ADQ \cong \Delta BCP$

[Using RHS congruency]

$\Rightarrow$ **$\angle DAQ = \angle CBP$** [CPCT]

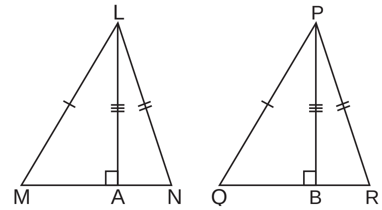
15. $\because$ $LA \perp MN$

$\Rightarrow$ $\angle LAM = 90°$

and $\angle LAN = 90°$

Similarly, $\angle PBQ = 90°$

and $\angle PBR = 90°$



In rt. ΔLMA and rt. ΔPQB, we have

$LA = PB$ [Given]

$LM = PQ$ [Given]

$\therefore$ $\Delta LMA \cong \Delta PQB$

[RHS congruency] … (1)

$\Rightarrow$ $AM = BQ$ [CPCT] … (2)

Similarly, rt. $\Delta LAN \cong$ rt. ΔPBR [RHS congruency]

$\Rightarrow$ $AN = BR$ [CPCT] … (3)

Adding (2) and (3), we have

$AM + AN = BQ + BR$

$\Rightarrow$ $MN = QR$ … (4)

In rt. ΔLMN and rt. ΔPQR, we have

$LM = PQ$ [Given]

$MN = QR$ [From (4)]

$LN = PR$ [Given]

$\Rightarrow$ $\Delta LMN \cong \Delta PQR$

[By SSS congruence] … (5)

From (1) and (5), we have

$\Delta LMA \cong \Delta PQB$ and **$\Delta LMN \cong \Delta PQR$**

16. In ΔAOB and ΔAOD, we have

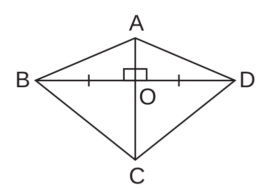
$AO = AO$ [Common]

$\angle AOB = \angle AOD$ [Each = 90°]

$BO = DO$ [Given]

$\therefore$ $AOB \cong \Delta AOD$ [SAS congruency]

$\Rightarrow$ $AB = AD$ [CPCT]… (1)



In ΔBOC and ΔDOC, we have

$CO = CO$ [Common]

$\angle BOC = \angle DOC$ [Each = 90°]

$OB = OD$ [Given]

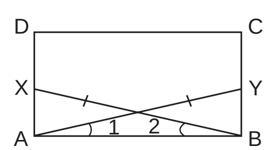
$\therefore$ $\Delta BOC \cong \Delta DOC$ [By SAS congruency]

$\Rightarrow$ $BC = CD$ [CPCT] … (2)

From (1) and (2), we have

$AB = AD$ and **$BC = CD$**

17. ABCD is a rectangle.



$\therefore$ $\angle DAB = 90°$ and $\angle CBA = 90°$

$\Rightarrow$ $\angle DAB = \angle CBA$ … (1)

In rt. ΔYBA and rt. ΔXAB, we have

$BA = AB$ [Common]

$AY = BX$ [Given]

$\therefore$ $\Delta YBA \cong \Delta XAB$ [By RHS congruency]

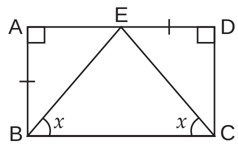
$BY = AX$ [CPCT] … (1)

and $\angle BAY = \angle ABX$ [CPCT] … (2)

From (1) and (2), we have

$BY = AX$ and $\angle BAY = \angle ABX$

18.



In rt. ΔBAE and rt. ΔEDC, we have

$BE = BC$

[In ΔEBC sides opp. to equal angles]

$BA = ED$ [Given]

$\therefore$ $\Delta BAE \cong \Delta EDC$ [By RHS congruency]

$\angle AEB + \angle BEC + \angle CED = 180°$ [Straight angle]

$\Rightarrow$ $(\angle AEB + \angle CED) + \angle BEC = 180°$

$\Rightarrow$ $90° + \angle BEC = 180°$

[$\angle AEB$ and $\angle CED$ are complementary angles $\Rightarrow \angle AEB + \angle CED = 90°$]

$\Rightarrow$ $\angle BEC = 180° - 90° = 90°$

In ΔBEC, we have

$x + x + 90° = 180°$ [Sum of $\angle$s of a Δ]

or, $2x = 180° - 90°$

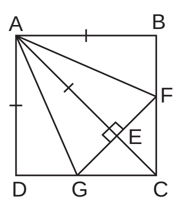
$= 90°$

$\therefore$ $x = \dfrac{90°}{2} = 45°$

Thus, $x = 45°$.

Hence, **$\Delta BAE \cong \Delta EDC$** and **$x = 45°$**.

19.



In rt. ΔABF and rt. ΔAEF, we have

$AB = AE$ [Given]

$AF = AF$ [Common]

$\therefore$ $\Delta ABF \cong \Delta AEF$ [By RHS congruency]

$\therefore$ $\angle BAF = \angle EAF = x$ (Say)

[CPCT] … (1)

Similarly, $\Delta ADG \cong \Delta AEG$ [By RHS congruency]

$\therefore$ $\angle DAG = \angle EAG = y$ (Say)

[CPCT] … (2)

Now, we have

$(\angle BAF + \angle EAF + \angle DAG + \angle EAG) = \angle BAD$

[An Angle of square ABCD]

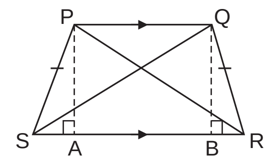
$\Rightarrow$ $x + x + y + y = 90°$

$\Rightarrow$ $2x + 2y = 90°$

$x + y = \dfrac{90°}{2} = 45°$

Thus, **FAG = $\frac{1}{2}$ (right angle)**.

20. Draw $PA \perp SR$ and $QB \perp SR$.



In rt. ΔPAS and rt. ΔQBR, we have

$PA = QB$

[Perpendicular between $\parallel$ lines are equal]

$PS = QR$ [Given]

$\therefore$ $\Delta PAS \cong \Delta QDR$ [By RHS congruence]

$\therefore$ $\angle PSA = \angle QRB$ [CPCT]

$\Rightarrow$ $\angle PSR = \angle QRS$ … (1)

Now, in ΔPSR and ΔQRS, we have

$PS = QR$ [Given]

$\angle PSR = \angle QRS$ [From (1)]

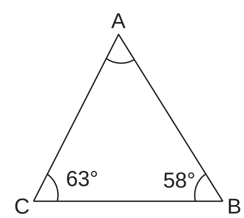
$SR = RS$ [Common]

$\therefore$ $\Delta PSR \cong \Delta QRS$ [By SAS congruency]

$\Rightarrow$ **$PR = QS$** [CPCT]

EXERCISE 7D

1.



In ΔABC, we have $\angle B = 58°$ and $\angle C = 63°$

$\therefore$ $\angle A = 180° - (\angle B + \angle C)$

[Sum of $\angle$s of a Δ]

$= 180° - (58° + 63°)$

$= 180° - 121°$

$= 59°$

$\angle C > \angle A > \angle B$

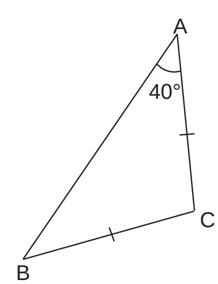
Greatest angle $= \angle C$

$\Rightarrow$ Greatest side $= AB$

[$\because$ Greater angle has longer side opp. to it]

Thus, the greatest side is **AB**.

2.



In ΔABC, $BC = CA$

$\Rightarrow$ $\angle B = \angle A$

$\therefore$ $\angle A = 40°$

$\Rightarrow \quad \angle B = 40°$

$\therefore \quad \angle C = 180° - (40° + 40°)$

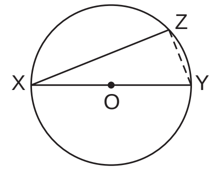
$= 100°$ [Sum of $\angle$s of a Δ]

Since $\angle C > \angle A$,

$\therefore$ **AB > BC**

[Greater angle has longer side opp. to it.]

3. XOY is diameter of the circle.



If we join YZ then,

$\angle XZY = 90°$

[Angle in a semi circle] … (1)

$\angle ZXY + \angle ZYX = 90°$

[Sum of $\angle$s of a Δ is 180°]

$\Rightarrow$ $\angle ZYX$ is an acute angle. … (2)

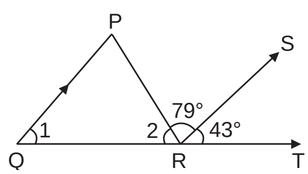
From (1) and (2), we get

$\angle XZY > \angle ZYX$

$\therefore$ **XY > XZ**

[greater angle has longer side opp. to it]

4. PQ ∥ RS and QT is a transversal.



$\therefore \quad \angle 1 = 43°$ [Corr. angles]

Also, $\angle 2 + \angle 79° + 43° = 180°$ [St. angle]

$\Rightarrow \quad \angle 2 = 180° - 79° - 43° = 58°$

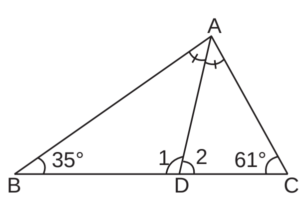
In ΔPQR, $\angle 2 > \angle 1$

$\Rightarrow$ [side opp. to $\angle 2$] > [side opp. to $\angle 1$]

$\Rightarrow$ **PQ > PR**

Thus, **PQ is greater**.

5.



In ΔABC, $\angle BAC = 180° - (35° + 61°)$

[Sum of $\angle$s of a Δ]

$= 180° - 96°$

$= 84°$

$\angle CAD = \angle BAD$

$= \dfrac{\angle BAC}{2}$

$= \dfrac{84°}{2} = 42°$

[$\because$ AD is the bisector of $\angle BAC$]

In ΔABD, $\angle 1 = 180° - (35° + 42°) = 103°$

[Sum of $\angle$s of a Δ]

$\angle ADB > \angle BAD > \angle ABD$

$\Rightarrow \quad AB > BD > AD$ … (1)

In ΔADC,

$\angle 2 = 180° - (42° + 61°) = 77°$

[Sum of $\angle$s of a Δ]

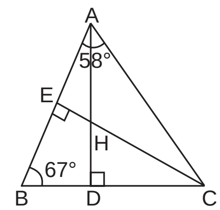
$\Rightarrow \quad \angle ADC > \angle ACD > \angle CAD$

$\Rightarrow \quad AC > AD > DC$ … (2)

From (1) and (2), we get

BD > AD > CD

6.



In ΔABC, we have

$\angle ACB = 180° - (58° + 67°)$

$= 55°$ [Sum of $\angle$s of a Δ]

$\Rightarrow \quad \angle ACD = 55°$

In ΔADC, we have

$\angle DAC = 180° - (90° + 55°) = 35°$

[Sum of $\angle$s of a Δ]

$\Rightarrow \quad \angle HAC = 35°$

$\therefore \quad \angle ACE = 180° - (90° + 58°)$

$= 32°$ [Sum of $\angle$s of a Δ]

$\Rightarrow \quad \angle ACH = 32°$

Now, in ΔAHC

$\Rightarrow \quad \angle HAC (= 35°) > \angle ACH (= 32°)$

$\Rightarrow \quad HC > AH$

[Greater angle has longer side opp. to it] … (1)

$\angle HCD = \angle ACD - \angle ACH$

$= 55° - 32°$

$= 23°$

$\angle DHC = 180° - (90° + 23°)$

$= 67°$ [Sum of $\angle$s of a Δ]

In ΔDHC, we have

$\angle HCD (= 23°) < \angle DHC (= 67°)$

$\Rightarrow \quad DH < DC$ … (2)

[Smaller angle has shorter side opp. to it]

From (1) and (2), we have

HC > AH and **DH < DC**

7. In the figure, we have ΔABC in which

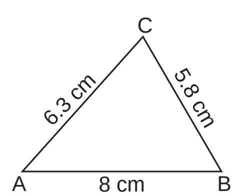
AB = 8 cm

BC = 5.8 cm

CA = 6.3 cm

The greatest side = 8cm

= AB



Angle opp. to AB is $\angle C$.

$\Rightarrow$ **$\angle C$ is the greatest angle of ΔABC**

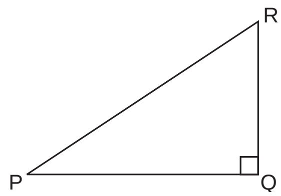
The smallest side = 5.8 cm

= BC

Angle opp. to BC is $\angle A$

$\Rightarrow$ **$\angle A$ is the smallest angle of ΔABC.**

8. $\because$ In a Δ, there can be only one right angle.



$\therefore$ $\angle Q$ is the greatest angle.

$\Rightarrow$ PR is the longest side.

$\because$ The sides PQ : QR = 3 : 2

$\therefore$ $PQ > QR$

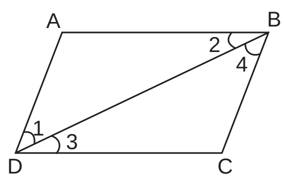
$\Rightarrow$ $\angle R > \angle P$

[Longer side has greater angle opp. to it]

$\because$ $\angle Q > \angle R > \angle P$

$\Rightarrow$ **$\angle P$ is the least angle.**

9.



In ΔABD, $AB > AD$ [Given]

$\Rightarrow$ $\angle 1 > \angle 2$ … (1)

$\because$ $\angle 2 = \angle 3$

[Alt. angles, AB ∥ DC]… (2)

$\therefore$ From (1) and (2),

$\angle 1 > \angle 3$

$\Rightarrow$ **$\angle ADB > \angle BDC$**

10. $\therefore$ $\angle PQD + \angle PQR = 180°$ [Linear pair]

$\therefore$ $118° + \angle PQR = 180°$

$\Rightarrow$ $\angle PQR = 180° - 118$

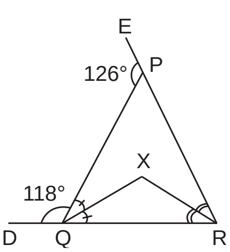
$= 62°$

$\angle XQR = \dfrac{\angle PQR}{2}$

$= \dfrac{62^0}{2}$

$= 31°$

[$\because$ XQ is bisector of $\angle PQR$]



Now, ext. $\angle EPQ = \angle PQR + \angle PRQ$

$\Rightarrow$ $\angle PRQ = \angle EPQ - \angle PQR$

$\Rightarrow$ $\angle PRQ = 126° - 62° = 64°$

$\Rightarrow$ $\angle XRQ = \dfrac{64^0}{2} = 32°$

[$\because$ XR bisects $\angle PRQ$]

Now, in ΔXQR, we have

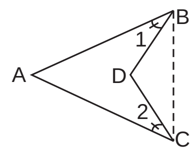
$\angle XRQ\ (= 32°) > \angle XQR\ (= 31°)$

$\Rightarrow$ **XQ > XR**

[Greater angle has longer side opp. to it]

Thus, **XQ** is greater.

11. Join BC.



In ΔABC, $AB > AC$ [Given]

$\therefore$ $\angle ACB > \angle ABC$

[Longer side has greater angle opp. to it] … (1)

$\therefore$ $\angle 2 = \angle 1$ [Given] … (2)

Subtracting (2) From (1), we have

$\angle ACB - \angle 2 > \angle ABC - \angle 1$

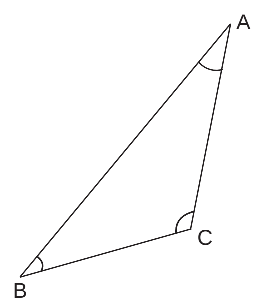
$\Rightarrow$ $\angle DCB > \angle DBC$

$\Rightarrow$ **DB > DC**

[Greater angle has longer side opp. to it.]

Thus, **DB** is greater.

12. Let ΔABC be a triangle is which the side AB is greatest. We know that longer side has greater angle opp. to it.



In ΔABC, we have

$AB > BC$ [Given]

$\therefore$ $\angle C > \angle A$ … (1)

Also, $AB > AC$

$\therefore$ $\angle C > \angle B$ … (2)

Adding (1) and (2), we get

$\angle C + \angle C > \angle A + \angle B$

$\Rightarrow$ $2\angle C > \angle A + \angle B$ … (3)

Adding $\angle C$ to both sides of (3), we get

$(2\angle C) + \angle C > (\angle A + \angle B) + \angle C$

$\Rightarrow$ $3\angle C > \angle A + \angle B + \angle C$

$\Rightarrow$ $3\angle C > 180°$ [Sum of $\angle$s of a Δ]

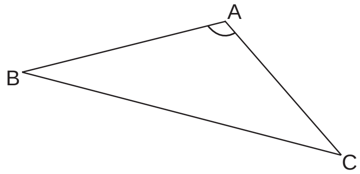
or, $\angle C > \dfrac{180°}{3}$

or, $\angle C > 60°$

or, $\angle C > \dfrac{2}{3}$ of a rt. $\angle$(= 60°)

Hence, the angle opposite to greatest side of a triangle is greater than two-third of a right angle i.e. 60°.

13. We have ΔABC in which $\angle BAC$ is an obtuse angle.
In a Δ, obtuse angle is the greatest angle.



$\therefore$ $\angle A > \angle B$ and $\angle A > \angle C$

$\Rightarrow$ $BC > AC$ and $BC > AB$

[Greater angle has longer side opp. to it.]

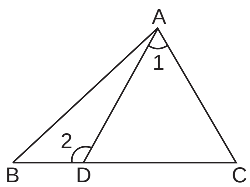
$\therefore$ BC, the side opposite to obtuse $\angle A$ is the greatest side of the obtuse ΔABC.

In an obtuse angled triangle, the side opposite to the obtuse angle is the greatest.

14. We have, $AB > AC$

$\Rightarrow$ $\angle C > \angle B$

[Longer side has greater angle opp. to it] … (1)



In ΔADC, Ext. $\angle ADB = \angle 1 + \angle C$

$\therefore$ $\angle ADB > \angle 1$

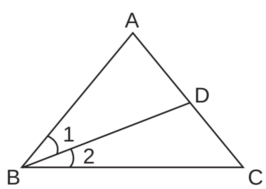
and $\angle ADB > \angle C$ … (2)

From (1) and (2),

$\angle ADB > \angle B$

$\Rightarrow$ **AB > AD**

15. ABC is an equilateral triangle.



$\therefore$ $\angle A = \angle B = \angle C$

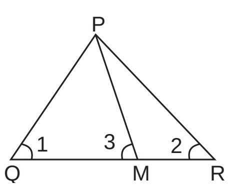
Now, $\angle B = \angle 1 + \angle 2$

$\Rightarrow$ $\angle A > \angle 1 + \angle 2$

$\Rightarrow$ **BD > AD**

[$\because$ In ΔABD, side opp. to greater angle is greater.]

16. (*i*)



$\therefore$ $PQ = PR$ [Given]

$\therefore$ $\angle 1 = \angle 2$

[Angles opp. to equal sides are equal] … (1)

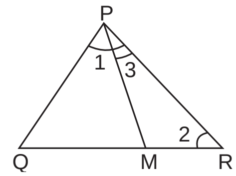
Ext. $\angle 3 > \angle 2$

$\Rightarrow$ $\angle 3 > \angle 1$ [From (1)]

$\therefore$ **PQ > PM**

[Greater angle has longer side opp. to it]

(*ii*)



$\angle 1 = \angle 2$ [$\therefore$ PQ = QR]

Since $\angle 1 > \angle 3$

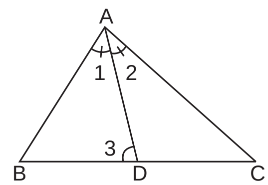
$\Rightarrow$ $\angle 2 > \angle 3$

$\therefore$ **PM > MR**

[Greater angle has longer side opp. to it]

17. In ΔABC, AD is bisector of $\angle A$.

$\therefore$ $\angle 1 = \angle 2$ … (1)



(*i*) Consider ΔADC whose side CD is produced to B.

Then, Ext. $\angle ADB > \angle 2$

[$\because$ In a Δ, ext. angle is greater than each of the int. opp. angles.]

$\therefore$ $\angle 3 > \angle 2$

$\Rightarrow$ $\angle 3 > \angle 1$ [From (1)]

$\therefore$ **AB > BD**

[Side opp. to greater angle is longer.]

(*ii*) Similarly,

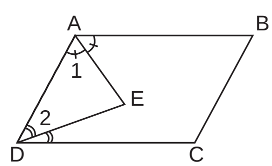
Ext. $\angle ADC > \angle 1$

$\Rightarrow$ $\angle ADC > \angle 2$

$\Rightarrow$ **AC > DC**

[Side opp. to greater angle is longer]

18. ABCD is $\parallel$gm.



$\Rightarrow$ $\angle A + \angle D = 180°$ [Coint. $\angle$s AB $\parallel$ DC]

$\because$ $\angle A$ is obtuse.

$\therefore$ $\angle A > \angle D$

$\frac{1}{2}\angle A > \frac{1}{2}\angle D$

$\Rightarrow$ $\angle 1 > \angle 2$

In ΔAED,

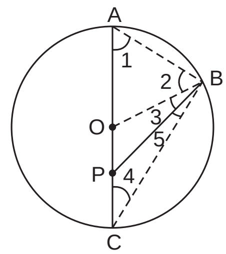
$\because$ $\angle 1 > \angle 2$

$\therefore$ $DE > AE$

[$\because$ In a Δ, side opp. to greater angle is greater]

Thus, **DE > AE**.

19. (*i*) Join AB and OB.



In AOB, $OA = OB$

$\therefore$ $\angle 1 = \angle 2$

[Angles opp. equal side of a Δ] … (1)

$(\angle 2 + \angle 3) > \angle 2$

$\therefore$ $(\angle 2 + \angle 3) > \angle 1$ [Using (1)]

$\Rightarrow$ $\angle ABP > \angle 1$

Now, in ΔABP,

$\therefore$ $\angle ABP > \angle 1$

$\Rightarrow$ $PA > PB$

[$\because$ In a Δ, side opp. to a greater angle is longer.]

or, **PA > PB**

(*ii*) Join BC.

$\therefore$ O is centre of the circle.

$\therefore$ $OB = OC$ [Radii of the same circle]

$\therefore$ $\angle OBC = \angle OCB$

or, $(\angle 3 + \angle 5) = \angle 4$

or $\angle 5 < \angle 4$

In ΔPBC, $\angle 5 < \angle 4$

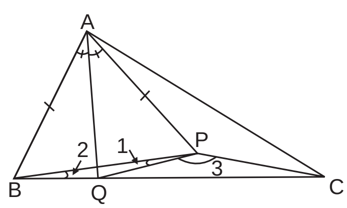
$\Rightarrow$ $PC < PB$

[Smaller angle has smaller side opp. to it.]

$\Rightarrow$ **PC < PB**

20. In ΔAPB,

AQ is bisector of $\angle PAB$.



$\therefore$ $\angle BAQ = \angle PAQ$ … (1)

Now, in ΔABQ and ΔAPQ

$AQ = AQ$ [Common]

$\angle BAQ = \angle PAQ$ [From (1)]

$AB = AP$ [Given]

$\therefore$ $\Delta ABQ \cong \Delta APQ$

[Using SAS congruency]

$\Rightarrow$ $BQ = PQ$ [CPCT]

Now, in ΔBQP, $BQ = PQ$

$\Rightarrow$ $\angle 1 = \angle 2$ … (2)

$\angle 1 + \angle 3 > \angle 1$

$\Rightarrow$ $\angle BPC > \angle 2$ [Using (2)]

$\angle BPC > \angle PBC$ … (3)

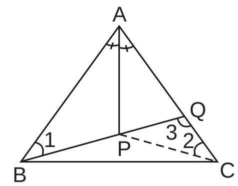
In ΔBPC, we have

$\angle BPC > \angle PBC$ [From (3)]

BC > CP

[Greater angle has longer side opp. to it]

21. Join PC.



In ΔABP and ΔACP,

$AB = AC$ [Given]

$\angle BAP = \angle PAC$

[$\therefore$ AP is bisector of $\angle A$]

$AP = AP$ [Common]

$\Rightarrow$ $\Delta ABP \cong \Delta ACP$ [By SAS congruence]

$\therefore$ $\angle 1 = \angle 2$ [CPCT]

and $BP = PC$ [CPCT]

In ΔABQ, Ext. $\angle 3 > \angle 2$

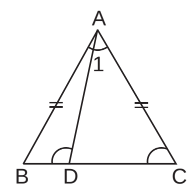
$\Rightarrow$ $PC > PQ$

[Greater $\angle$ has longer side opp. to it.]

$\Rightarrow$ $PB > PQ$ [$\because$ $PB = PC$]

Thus, **BP is greater than PQ**.

22. In ΔABC, $AB = AC$



$\Rightarrow$ $\angle B = \angle C$ … (1)

Ext. $\angle ADB > \angle C$

[$\because$ Ext. angle is greater than each of the int. opp. angle.]

But $\angle B = \angle C$ [From (1)]

$\because$ $\angle ADB > \angle B$

Now, in ΔABD,

$\therefore$ $\angle ADB > \angle B$

$\Rightarrow$ $AB > AD$

[Greater angle has longer side opp. to it.] … (2)

Similarly, in ΔADC,

$\angle ADC > \angle C$

$\Rightarrow$ $AC > AD$ … (3)

From (2) and (3), we conclude that AD is less than AB as well as AC.

Hence, **the straight line segment joining the vertex of an isosceles triangle to any point in the base is less than either of the equal sides of the triangle.**

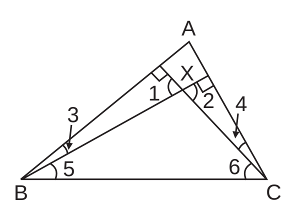
23. In ΔABC,

$AB > AC$

$\Rightarrow$ $\angle ACB > \angle ABC$

[Greater side has greater angle opp. to it]

$\Rightarrow$ $(\angle 4 + \angle 6) > (\angle 3 + \angle 5)$ … (1)



In ΔBXP and ΔCXQ,

$\angle P = \angle Q$ [Each. 90°]

$\angle 1 = \angle 2$ [Vert. Opp. $\angle$s]

$\Rightarrow$ remaining $\angle 3$ = remaining $\angle 4$ … (2)

From (1) and (2),

$(\angle 4 + \angle 6) > \angle 3 + \angle 5)$

$\Rightarrow$ $(\angle 3 + \angle 6) > (\angle 3 + \angle 5)$

$\Rightarrow$ $\angle 6 > \angle 5$

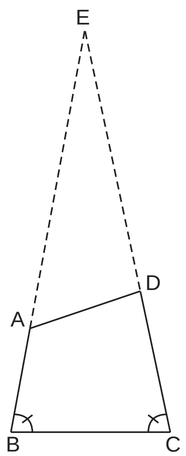
In ΔXBC,

$\because$ $\angle 6 > \angle 5$

$\therefore$ **XB > XC**

[Greater angle has longer side opp. to it]

24. Produce CD and BA to meet at E.



$\angle C = \angle B$ [Given]

$\therefore$ $BE = CE$ [Sides opp. equal angles]

$AE + AB = ED + CD$

But $CD > AB$ [Given]

$\therefore$ $AE > ED$

$\Rightarrow$ $\angle EDA > \angle EAD$

[Greater side has greater angles opp. to it]

$\Rightarrow$ $180° - \angle EDA < 180° - \angle EAD$

$\Rightarrow$ $\angle ADC < \angle BAD$

$\Rightarrow$ $\mathbf{\angle BAD > \angle ADC}$

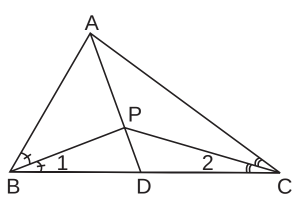
25. $AC > AB$ [Given]

$\therefore$ $\angle ABC > \angle ACB$

$\Rightarrow$ $\frac{1}{2}\angle ABC > \frac{1}{2} ACB$ … (1)

$\Rightarrow$ $\angle 1 > \angle 2$

[$\because$ BP and CP are bisectors of $\angle B$ and $\angle C$ respectively] … (2)



In ΔBPC,

$\because$ $\angle 1 > \angle 2$ [From (2)]

$\therefore$ **PC > PB**

[Side opp. to greater angle in longer]

26. (*i*) **No.**

$AB = 2$ cm

$BC = 3.5$ cm

$CA = 6.5$ cm

$\because$ $AB + BC = (2 + 3.5)$ cm

$= 5.5$ cm

and $5.5 < 6.5$

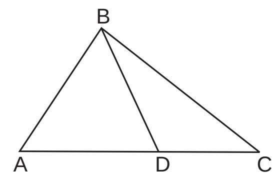
$\Rightarrow$ $(AB + BC) < CA$ … (1)

We know that a triangle can be drawn only when the sum of any two sides is greater that the third side.

From (1),

The sum of two sides is less than the third side.

Thus, we cannot draw the triangle.



(*ii*) In ABD, we have

$(BA + AD) > BD$ … (1)

[$\because$ Sum of any two sides of a Δ is greater than the third side]

Adding DC on both sides,

$(BA + AD) + DC > (BD + DC)$

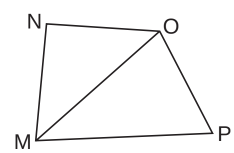
$\Rightarrow$ $BA + (AD + DC) > (BD + DC)$

$\Rightarrow$ **BA + AC > BD + DC**

27. In MNO,

$MO < (MN + ON)$

[$\because$ Sum of two sides in a Δ is greater than the third side]



Similarly, in ΔMOP,

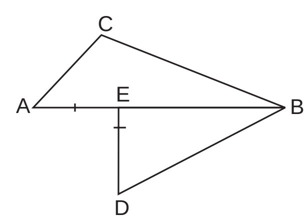
$MO < (OP + MP)$

Adding (1) and (2),

$(MO + (MO) < (MN + ON + OP + MP)$

or **2MO < (MN + NO + OP + PM)**

28. We have $DE = EA$



In ΔABC, we have

$AC + BC > AB$

[Sum of two sides of Δ is greater than the third side]

$\Rightarrow$ $AC + BC > EA + EB$

$\Rightarrow$ $AC + BC > DE + EB$

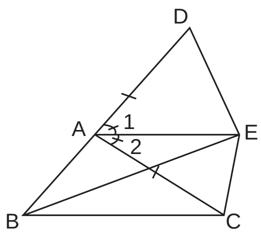
[$\because$ EA = DE, given] … (1)

Now, in ΔEDB, we have

$$DE + EB > BD$$

$\Rightarrow$ $\mathbf{AC + BC > BD}$ [Using (1)]

29.



In ADE and ΔACE, $AD = AC$ [Given]

$\angle 1 = \angle 2$ [AE is bisector of $\angle A$]

$AE = AE$ [common]

$\therefore$ $\Delta ADE \cong \Delta ACE$ [SAS congruency]

$\Rightarrow$ $ED = EC$ [CPCT] … (1)

Now, in ΔBED, we have

$$EB + ED > BD$$

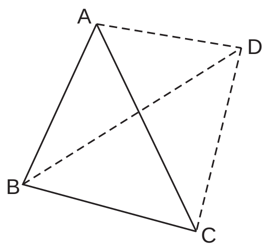
[Sum of two sides of a Δ > third side]

$\Rightarrow$ $EB + EC > BA + AD$ [Using (1)]

$\Rightarrow$ $\mathbf{EB + EC > AB + AC}$

[$\because$ AD = AC given]

30. Join OA, OB and OC.



In ΔAOB, $AB < (OA + BO)$

[$\because$ Sum of two sides in a Δ is more than the third side]

Similarly, we have

$BC < (OB + OC)$ … (2)

and $AC < (OA + OC)$ … (3)

Adding (1), (2) and (3),

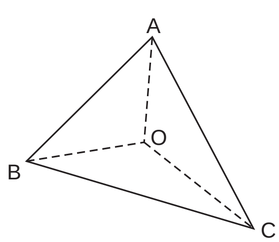
$AB + BC + AC < [OA + OB) + (OB + OC) + (OA + OC)]$

$\Rightarrow$ $AB + BC + AC < [2OA + 2OB + 2OC]$

$\Rightarrow$ $AB + BC + AC < 2\,[OA + OA + OC]$

$\Rightarrow$ $\mathbf{\frac{1}{2}[AB + BC + AC] < [OA + OB + OC]}$

31. Join, AO, BO and CO.



In ΔAOB, we have

$(OA + OB) > AB$ … (1)

[Sum of two sides in a Δ is greater than the third side]

Similarly, in ΔBOC,

$(OB + OC) > BC$ … (2)

and in ΔCOA,

$(OC + OA) > AC$ … (3)

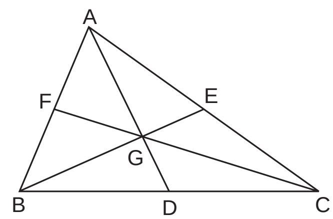
Adding (1), (2) and (3), we have

$(OA + OB) + (OB + OC) + (OC + OA) > (AB + BC + CA)$

$\Rightarrow$ $2[OA + OB + OC] > (AB + BC + CA)$

$\Rightarrow$ $\mathbf{[OA + OB + OC] > \frac{1}{2}[AB + BC + CA]}$

32. Let AD, BE and CF be the medians of ΔABC and let them intersect at G.



G, the centroid of ΔABC, divides each median in the ratio 2 : 1.

$\therefore$ $AG : GD = 2 : 1$

$\Rightarrow$ $\frac{AG}{GO} = \frac{2}{1}$

$\Rightarrow$ $\frac{GD}{AG} = \frac{1}{2}$

$\Rightarrow$ $\frac{GD}{AG} + 1 = \frac{1}{2} + 1$

$\Rightarrow$ $\frac{GD + AG}{AG} = \frac{3}{2}$

$\Rightarrow$ $\frac{AD}{AG} = \frac{3}{2}$

$\Rightarrow$ $AG = \frac{2}{3}AD$ … (1)

Similarly, $BG = \frac{2}{3}BE$ and $CG = \frac{2}{3}CF$

Now, in ΔAGB, we have

$$AG + BG > AB$$

$\Rightarrow$ $\frac{2}{3}AD + \frac{2}{3}BE > AB$ [Using (1) and (2)] … (3)

Similarly, by considering ΔBGC and ΔCGA, we have

$\frac{2}{3}BE + \frac{2}{3}CF > BC$ … (4)

and $\frac{2}{3}CF + \frac{2}{3}AD > AC$ … (5)

Adding (3), (4) and (5), we have

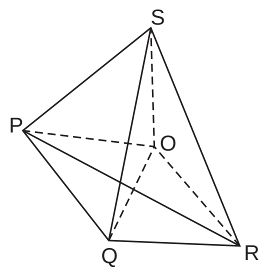
$$2\left[\frac{2}{3}AD + \frac{2}{3}BE + \frac{2}{3}CF\right] > AB + BC + CA$$

$\Rightarrow$ $\frac{4}{3}(AD + BE + CF) > AB + BC + CA$

$\Rightarrow$ $4(AD + BE + CF) > (AB + BC + CA)$

$\Rightarrow$ 4 (Sum of the medians) > 3 (perimeter of ΔABC)

33. Let PQRS be a quadrilateral whose diagonals PQ and QS are joined. O is any point inside quadrilateral PQRS, not lying on PR or QS.
Join OP, OQ, OR and OS.



Now, in ΔPOR,

$(OP + OR) > PR$

[Sum of two sides of a Δ > third side] … (1)

Similarly, in ΔSOR,

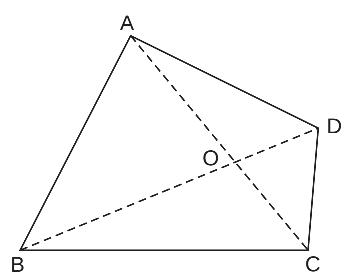
$(OS + OQ) > SQ$ … (2)

Adding (1) and (2), we have

$(OP + OR) + (OS + OQ) > (PR + SQ)$

⇒ **$(OP + OQ + OR + OS) > (PR + QS)$**

34. Let ABCD be a quadrilateral whose diagonals AC and BD intersect at O.



In ΔABO, $(AO + BO) > AB$ … (1)

[∵ Sum of two sides is greater than the third side]

Similarly, in ΔBOC,

$(BO + CO) > BC$ … (2)

and in ΔCOD,

$(CO + OD) > CD$ … (3)

Also in ΔAOD,

$(OD + AO) > AD$ … (4)

Adding (1), (2), (3) and (4), we have

$[(AO + BO) + (BO + CO) + (CO + DO) + (DO + AO)]$
$> AB + BC + CD + AD$

⇒ $[(AO + CO) + (AO + CO) + (BO + DO) + (BO + DO)$
> [Perimeter of the quad. ABCD]

⇒ $[AC + AC) + (BD + BD)]$
> [Perimeter of the quad. ABCD]

⇒ $(2AC + 2BD) >$ [Perimeter the quad. ABCD]

⇒ $2(AC + BD) >$ [Perimeter of the quad. ABCD]

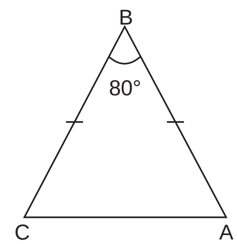
⇒ $AC + BD > \frac{1}{2}$ [Perimeter of quad. ABCD]

Hence, **the sum of the diagonals of a quadrilateral is greater than half its perimeter.**

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (*c*) **50°**



∵ $BC = AB$

∴ $\angle A = \angle C$ … (1)

$\angle A + \angle C = 180° - 80° = 100$

[Sum of ∠s of a Δ] … (2)

⇒ $\angle A = \angle C = \frac{100^o}{2} = 50°$

[Using (1) and (2)]

2. (*b*) **25°**

Angles are supplementary.

∴ $(30 - a) + (125 + 2a) = 180°$

⇒ $a = 180 - 155$

⇒ $a = 25°$

3. (*c*) **An exterior angle of a triangle is always greater than opposite interior angle.**

Sum of int. opp. angles = Exterior ∠

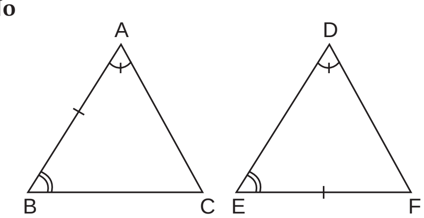
⇒ Each of int. opp. angle is smaller than the exterior angle.

4. (*b*) **SSA**

In two congruent Δs, angles included by two corresponding sides must be equal.

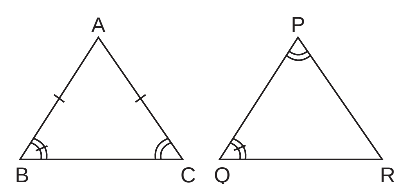
∴ It should be SAS instead of SSA.

5. (*b*) **No**



The triangle are not congruent because side AB of ΔABC does not correspond to side DE of ΔDEF.

6. (*a*) **Isosceles but not necessary congruent.**



Two equal angles of one triangle are equal to two equal angles of the other.

⇒ Triangles are isosceles

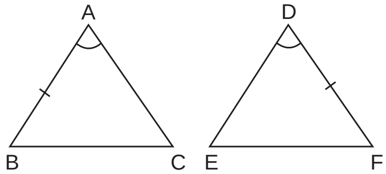
But for congruency the information is not sufficient.

7. (*c*) **3.4 cm**

In a triangle, the difference of two sides must be smaller than the third side.

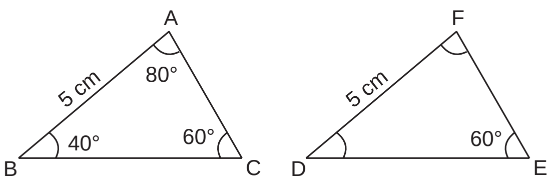
Here, (7.0 – 3.5) cm > 3.4 cm

8. (*d*) **AC = DE**



The two triangles ABC and DEF will be congruent by SAS axiom if AC = DE because ∠A is included between the sides AB and AC and ∠D is included between the sides DF and DE.

9. (*c*) **DF = 5 cm, ∠E = 60°**



$\because$ ΔABC ≅ ΔFDE

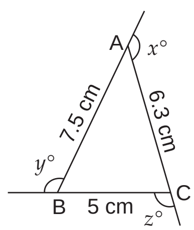
$\therefore$ AB = FD, ∠C = ∠E

In ΔABC, ∠A = 80°, ∠B = 40°,

$\therefore$ ∠C = 180° – (80° + 40°)

= 60° = ∠E

10. (*d*) ***z* < *y* < *x***



In ΔABC,

Greater side, AB = 7.5

and Smaller side, BC = 5 cm

$\therefore$ Greatest interior ∠ of ΔABC is ∠C and smallest interior angle is ∠A.

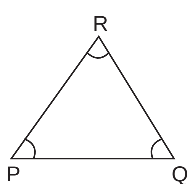
⇒ Supplementary angle of ∠A is greatest and that of ∠C is least.

⇒ x° is greatest and z° is smallest

$\therefore$ Required ascending order is z° < y° < x°

⇒ $z < y < x$

11. (*b*) **PQ > PR**

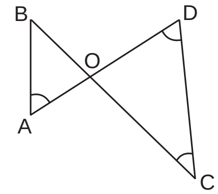


In a triangle, side opp. to a greater angle is greater.

$\therefore$ ∠R > ∠Q

⇒ PQ > PR

12. (*c*) **AD < BC**



∠B < ∠A

⇒ OA < OB … (1)

∠D > ∠C

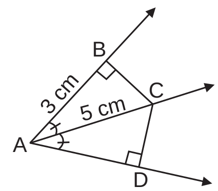
⇒ OD < OC … (2)

Adding (1) and (2),

(OA + OD) < (BO + CO)

or AD < BC

13. (*a*) **4 cm**



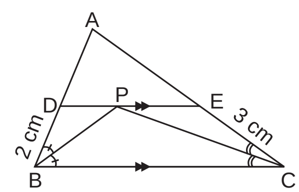
Triangles ABC and ADC are congruent.

⇒ AB = AD = 3 cm

$\therefore$In rt. ΔADC, $CD^2 = 5^2 - 3^2 = 4^2$

⇒ CD = 4 cm

14. (*c*) **5 cm**



BC ∥ DE

⇒ ∠DPB = ∠CBP [Alt. angles]

= ∠DBP

[$\because$ BP is bisector of ∠B]

⇒ ΔBDP is an isosceles triangle.

⇒ DB = DP = 2 cm

Similarly, ΔCEP is an isosceles triangle.

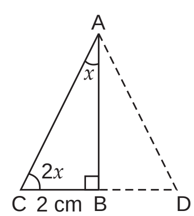
⇒ CE = EP = 3 cm

$\therefore$ DE = PD + DE

= (2 + 3) cm

= 5 cm

15. (*b*) **4 cm**



In rt. ΔABD , ∠BAD = x

⇒ ∠CAD = $2x$

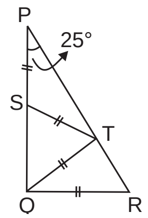
Also, ∠CDA = $2x$

$\therefore$ ADC is an equilateral triangle.

$\therefore$ CD = CB + BD
= 2 cm + 2 cm
= 4 cm

$\Rightarrow$ AC = CD
= 4 cm

16. (*c*) **75°**



Ext. $\angle$s = 25° + 25°
= 50°

$\Rightarrow$ $\angle$STQ = 180° – (50° + 50°)
= 80°

$\because$ $\angle$PTS + $\angle$STQ + $\angle$QTR = 180°

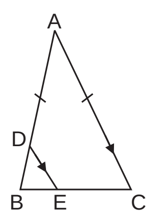
$\therefore$ 25° + 80° + $\angle$QTR = 180°

$\Rightarrow$ $\angle$QTR = 180° – 25° – 80°
= 75°

In ΔQTR, QT = QR

$\Rightarrow$ $\angle$R = $\angle$QTR = 75°

17. (*d*) **1.5 cm**



AB = AC

$\Rightarrow$ $\angle$B = $\angle$C … (1)

DE ∥ AC and BC is a transversal.

$\therefore$ $\angle$C = $\angle$DEB [Corr. angles] … (2)

From (1) and (2),

$\angle$DEB = $\angle$B

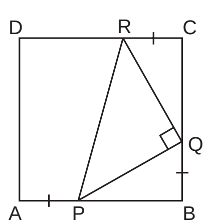
$\Rightarrow$ ΔBDE is an isosceles triangle with $\angle$B = $\angle$E.

$\therefore$ BD = DE [Sides opp. equal angles]

But BD = 1.5 cm

$\Rightarrow$ DE = 1.5 cm

18. (*c*) **45°**



ΔPBQ $\cong$ ΔQCR [By SAS congruencey]

$\Rightarrow$ PQ = QR

$\therefore$ $\angle$PRQ = $\angle$RPQ

[Angles opp. equal sides of ΔQRP] … (1)

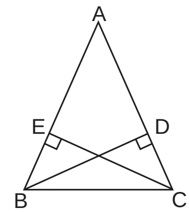
But $\angle$PRQ + $\angle$RPQ = 180° – 90°
= 90° [Sum of $\angle$s of a Δ] … (2)

$\Rightarrow$ $\angle$PRQ = $\angle$RPQ = 45°

[Using (1) and (2)]

Thus, $\angle$RPQ = 45°.

19. (*c*) **5 cm**



ΔBEC $\cong$ ΔCDB [By RHS congruence]

$\Rightarrow$ $\angle$EBC = $\angle$DCB [CPCT]

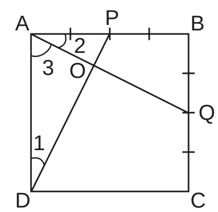
$\Rightarrow$ $\angle$B = $\angle$C

$\Rightarrow$ AC = AB [Sides opp. equals]

But AB = 5 cm.

$\therefore$ AC = 5 cm

20. (*b*) **90°**



In ΔAPD and ΔBQA,

AP = BQ

[Each = $\frac{1}{2}$ of the side of square ABCD]]

$\angle$PAD = $\angle$QBA [Each 90°]

AD = BA

[Each side of square ABCD]

$\Rightarrow$ ΔAPD $\cong$ ΔBQA [By SAS congruence]

$\Rightarrow$ $\angle$1 = $\angle$2 [CPCT] … (1)

$\because$ $\angle$2 + $\angle$3 = 90°

[Each angle of a square = 90°]

$\therefore$ $\angle$1 + $\angle$3 = 90° [Using (1)] … (2)

In ΔAOD,

$\angle$1 + $\angle$3 = 90° [From (2)

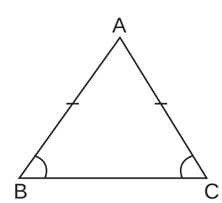
$\Rightarrow$ $\angle$AOD = 90° [Sum of $\angle$s of a Δ]

$\angle$AOD = $\angle$POQ [Vert. opp. angles]

$\Rightarrow$ $\angle$POQ = 90°

SHORT ANSWER QUESTIONS

1.



In ΔABC, AB = AC

$\Rightarrow$ $\angle$B = $\angle$C … (1)

$\angle$A + $\angle$B + $\angle$C = 180° [Sum of the $\angle$s of a Δ]

$\Rightarrow \quad \angle A + 2\angle B = 180°$ [Using (1)]

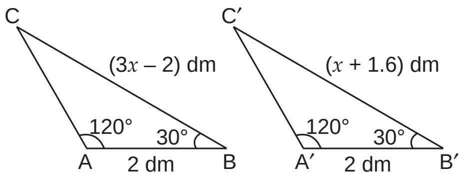
$\Rightarrow \quad \angle A + 2\left(\frac{2}{5}\angle A\right) = 180°$

$\Rightarrow \quad \frac{9}{5}\angle A = 180°$

$\Rightarrow \quad \angle A = \frac{5}{9} \times 180^0$

$\Rightarrow \quad$ **$\angle A = 100°$**

2.



In ΔABC and $\Delta A'B'C'$, we have

$\angle A = \angle A' = 120°$

$AB = A'B' = 2$ dm and $\angle B = \angle B' = 30°$

$\therefore \quad \angle ABC \cong \Delta A'B'C'$

[By ASA congruencey]

$\therefore \quad BC = B'C'$ [CPCT]

$\Rightarrow \quad (3x - 2)$ dm $= (x + 1.6)$ dm

$\Rightarrow \quad 3x - x = 1.6 + 2$

$= 3.6$ dm

$\Rightarrow \quad 2x = 3.6$ dm

$\Rightarrow \quad x = \frac{3.6}{2} = 1.8$ dm

Now, $B'C' = x + 1.6$ dm $= (1.8 + 1.6)$ dm $= 3.4$ dm

Thus, **$B'C' = 3.4$ dm.**

3. $BC = CD$ [Given]

$\Rightarrow \quad \angle B = \angle 1$

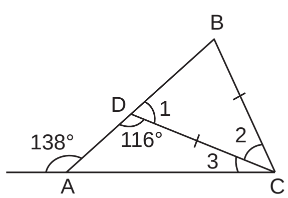
[$\angle$s opp. equal sides of a Δ]

$\angle 1 + 116° = 180°$ [Linear pairs]

$\Rightarrow \quad \angle 1 = \angle B$

$= 180° - 116°$

$= 64°$



Now, $\quad \angle 2 = 180° - (\angle 1 + \angle B)$

$= 180° - (64° + 64°)$

$= 52°$ [Sum of $\angle$s of a Δ] … (1)

$\because \quad$ Ext. $\angle 138° = 116° + \angle 3$

$\Rightarrow \quad \angle 3 = 138° - 116° = 22°$ … (2)

Adding (1) and (2),

$\angle 2 + \angle 3 = 52° + 22°$

$= 74°$

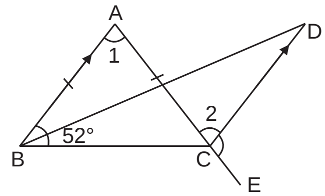
$\Rightarrow \quad$ **$\angle ACB = 74°$**

4. $\because \quad AB = AC$

$\therefore \quad \angle ACB = \angle ABC$

[$\angle$s opp. equal sides of a Δ]

$\Rightarrow \quad \angle ACB = 52°$



$\because \quad CD \parallel AB$ and AC is a transversal.

$\therefore \quad \angle 1 = \angle 2$

But, $\quad \angle 1 = 180° - \angle B - \angle ACB$

[Sum of $\angle$s of a Δ is 180°]

$= 180° - 52° - 52°$

$= 76°$

$\Rightarrow \quad \angle 2 = 76°$

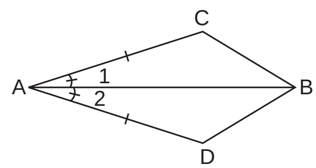
Now, $\quad \angle DCE = 180° - \angle 2$ [Linear pair]

$= 180° - 76°$

$= 104°$

Thus, $\angle DCE = 104°$

5.



In ΔACB and ADB, $AC = AD$ [Given]

$\angle 1 = \angle 2$ [Given]

$AB = AB$ [Common]

$\Rightarrow \quad \Delta ACB \cong \Delta ADB$ [SAS congruency]

$\therefore \quad BC = BD$ [CPCT]

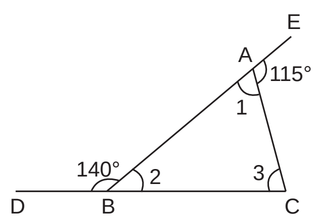
$\Rightarrow \quad BC = BD = 2.6$ cm

$\therefore \quad 2\,BD + \frac{BC}{2} = 2\,(2.6\text{ m}) + \frac{2.6\text{ m}}{2}$

$= (5.2 + 1.3)$ cm

$= 6.5$ cm

6.



$\angle 1 = 180° - 115°$

$= 65°$ [Linear pair] … (1)

$\angle 1 + \angle 3 =$ Ext. $\angle 140°$

[Ext. $\angle$ is equal to sum of int. opp. $\angle$s]

$\Rightarrow \quad \angle 3 = 140° - 65°$

$= 75°$ [Using (1)]

$\angle 2 = 180° - 140°$

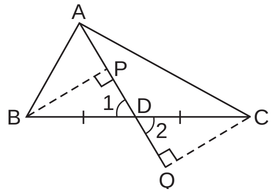
$= 40°$ [Linear pair]

$\therefore \quad \angle 2 < \angle 1 < \angle 3$

[Greater angle has greater side opposite to it and vice versa]

$\therefore \quad$ **$AC < BC < AB$**

7.



In ΔBPD and ΔCQD, $\angle P = \angle Q$ [Each = 90°]

$\angle 1 = \angle 2$ [Vert. opp. $\angle$s]

$BD = CD$ [Given]

$\therefore$ $\Delta BPD \cong \Delta CQD$ [AAS congruency]

$\Rightarrow$ $DP = DQ$ [CPCT]

$\because$ $DP = 3.5$ cm [Given]

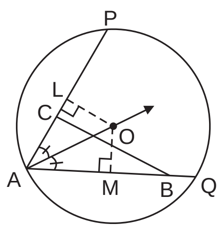
$\therefore$ $DQ = 3.5$ cm

Now, $PQ = DP + DQ$

$= 3.5 \text{ cm} + 3.5 \text{ cm}$

$=$ **7 cm**

8. O is the centre of circle and AP is a chord and $OL \perp AP$. [Given]



$\therefore$ L is mid-point of AP.

[Perpendicular from the centre of a circle to the chord bisects it]

$\Rightarrow$ $AL = \frac{1}{2} AP$

$\Rightarrow$ $AP = 2AL$ … (1)

Similarly, $AM = \frac{1}{2} AQ$

$\Rightarrow$ $AQ = 2AM$ … (2)

Now, in ΔALO and ΔAMO,

$AO = AO$ [Common]

$\angle L = \angle M$ [Each = 90°]

$\angle OAL = \angle OAM$

[AO is bisector of $\angle A$]

$\therefore$ $\Delta ALO \cong \Delta AMO$ [By SAA congruence]

$\therefore$ $AL = AM$ [CPCT]

$\because$ $AL = 3.5 \text{ cm} = AM$

$\therefore$ $AP + AQ = 2AL + 2AM$

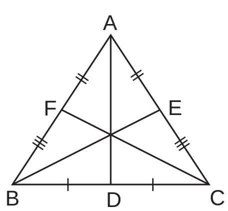
[Using (1) and (2)]

$= 2 (3.5 \text{ cm}) + 2(3.5 \text{ cm})$

$= 7 \text{ cm} + 7 \text{ cm}$

$=$ **14 cm**

9. Let ABC be an equilateral triangle and AD, BE and CF be its medians.



In ΔABD and ΔBAE,

$AB = BA$ [Common]

$\angle B = \angle A$

[Angles of an equilateral triangle]

$BD = AE$

$[\because BC = AC \Rightarrow \frac{1}{2} BC = \frac{1}{2} AC]$

So, $\Delta ABD \cong BAE$ [By SAS congruency]

$\Rightarrow$ $AD = BE$ … (1)

Similarly, $\Delta ADC \cong \Delta CFA$

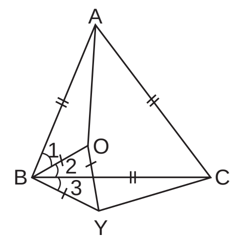
$\Rightarrow$ $AD = CF$ … (2)

From (1) and (2),

$AD = BE = CF$

Hence, **medians of an equilateral triangle are equal].**

10. ΔABC is an equilateral triangle.



$\therefore$ $\angle ABC = 60°$

$\Rightarrow$ $\angle 1 + \angle 2 = 60°$ … (1)

ΔBOY is an equilateral triangle.

$\therefore$ $\angle OBY = 60°$

$\Rightarrow$ $\angle 2 + \angle 3 = 60°$ … (2)

From (1) and (2),

$\angle 1 + \angle 2 = \angle 2 + \angle 3$

$\Rightarrow$ $\angle 1 = \angle 3$

Now, in ΔABO and ΔCBY, we have

$AB = CB$

[Sides of an equilateral triangle]

$\angle 1 = \angle 3$ [From (1)]

$BO = BY$

[Sides of an equilateral Δ]

$\therefore$ $\Delta ABO \cong \Delta CBY$ [SAS congruency]

$\Rightarrow$ $AO = CY$ [CPCT]

But $AO = 2.8$ cm [Given]

$\therefore$ **CY = 2.8 cm**

VALUE-BASED QUESTION

(*i*) In ΔPSB,

$PS = BS$

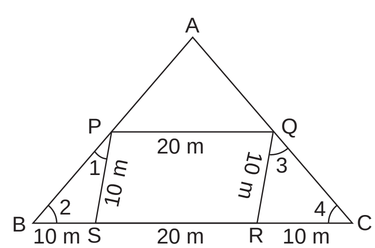
[Each is equal to 10 m, given]

$\therefore$ $\angle 1 = \angle 2 = x$ (say)

[Angles opposite to equal sides are equal] ... (1)

and Ext. $\angle PSR = x + x = 2x$

[Ext. $\angle$ = sum of int. opp. $\angle$s]



In ΔQRC,

$$QR = RC$$
[Each is equal to 10 m, given]

$\therefore$ $\angle 3 = \angle 4 = y$ (say)
[Angles opposite equal sides are equal] ... (2)

and Ext. $\angle QRS = y + y = 2y$
[Ext. $\angle$ = sum of int. opp. $\angle s$]

PS ∥ QR and transversal BSRC intersects PS at S and QR at R.

$\therefore$ $2x + 2y = 180°$ [Cointerior $\angle$s, PS ∥ QR]

$\Rightarrow$ $x + y = 90°$ ... (3)

In ΔBAC,

$\angle BAC + \angle 2 + \angle 4 = 180°$ [Sum of angles of a Δ]

$\angle BAC + x + y = 180°$ [Using (1) and (2)]

$\angle BAC + 90° = 180°$ [Using (3)]

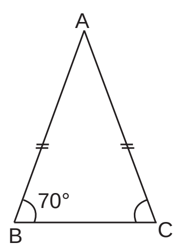
$\Rightarrow$ $\angle BAC = 180° - 90° = 90°$

Hence, ΔBAC is a right angled triangle right angled at A.

(*ii*) Empathy, caring, compassion, social responsibility and environmental protection.

UNIT TEST

1. (*i*) **40°**



$AB = AC$

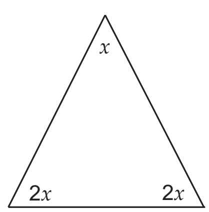
$\Rightarrow$ $\angle B = \angle C$ [$\angle$s opp equal sides]

$\angle B = 70° = \angle C$ [Sum of $\angle$s of a Δ]

$\therefore$ $\angle A = 180° - 2(70°)$

$= \mathbf{40°}$

(*ii*) **72°**



$x + 2x + 2x = 180°$ [Sum of $\angle$s a Δ]

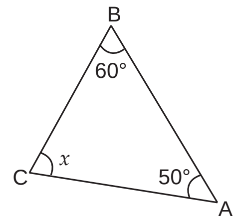
$\Rightarrow$ $x = 36°$

$\therefore$ $2x = 2 \times 36°$

$= 72°$

$\Rightarrow$ Each base angle = 72°

(*iii*) **BC < CA < AB**



$x = 180° - (60° + 50°)$

$= 70°$

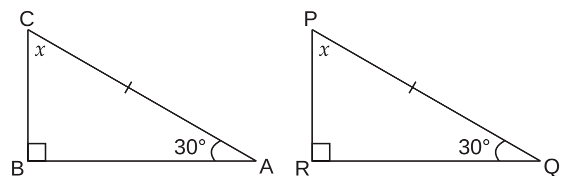
$\because$ $70° > 60° > 50°$

$\therefore$ $AB > AC > BC$
[Greater angle has greater side opp. to it]

$\Rightarrow$ Required ascending order

$\therefore$ $BC < CA < AB$

(*iv*) **QRP** $\Delta ABC \cong \Delta QRP$



In ΔABC and ΔQRP, we have

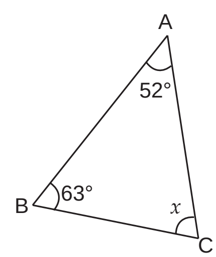
$AC = QP$ [Given]

$\angle CBA = \angle PRQ$ [Each is 90°]

$\angle CAB = \angle PQR$ [Each is 30°]

$\therefore$ $\Delta ABC \cong \mathbf{\Delta QRP}$ [By SAA congruence]

(*v*) **AB**



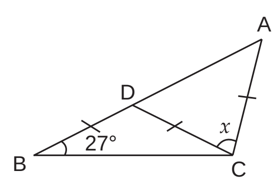
$\angle C = 180° - (52° + 63°)$

$= 65°$ [Sum of $\angle$s of a Δ]

$\angle C = 65°$ is the greatest angle.

$\therefore$ Side opp. to $\angle C$ i.e. **AB** is the greatest side.
[Greater angle has greater side opp. to it]

2. (*i*) **72°**



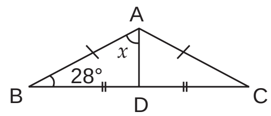
Ext. $\angle ADC = 27° + 27° = 54°$

$\angle ACD = \angle ADC = 54°$

$\therefore$ $x = 180° - (54° + 54°)$

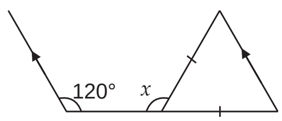
$= \mathbf{72°}$ [Angle sum property]

(*ii*) **62°**



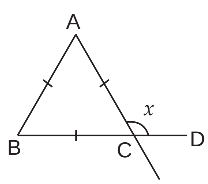
$\angle ACD = \angle ABC$
$= 28°$ [$\angle$s opp. equal sides]
$\Delta ADB \cong \Delta ADC$ [By SAS congruence]
$\Rightarrow$ $ADB = \angle ADC$ [CPCT]
But $\angle ADB + \angle ADC = 180°$
$\therefore$ $\angle ADB = 90°$
$\Rightarrow$ $x = 90° - 28° = \mathbf{62°}$

(*iii*) **120°**



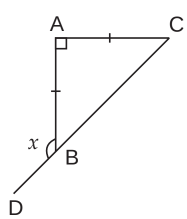
$\angle DCB + \angle ABC = 180°$ [coint $\angle$s, CD $\parallel$ BA]
$120° + \angle ABC = 180°$
$\Rightarrow$ $\angle ABC = 60°$
$\angle EAB = \angle EBA$ (= $\angle ABC$)
$= 60°$
Ext. $\angle x = \angle ABE + \angle EBA$
$= 60° + 60°$
$= \mathbf{120°}$

(*iv*) **120°**
ABC is an equilateral triangle.



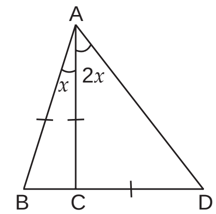
$\therefore$ $60° + x = 180°$ [Linear pair]
$\Rightarrow$ $\mathbf{x = 120°}$

(*v*) **135°**



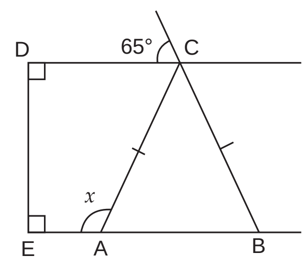
$AB = AC$
$\Rightarrow$ $\angle C = \angle B$
[$\angle$s opp equal sides] … (1)
$\angle A + \angle B + \angle C = 180°$ [Sum of $\angle$s of a Δ]
$\Rightarrow$ $90° + \angle C + \angle C = 180°$ [Using (1)]
$\Rightarrow$ $\angle C = 45°$
$x + 45° = 180°$ [Linear pair]
$\Rightarrow$ $\mathbf{x = 135°}$

(*vi*) **20°**



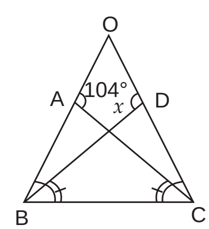
$\angle ADC = \angle CAD = 2x$
Ext. $\angle ACB = 2x + 2x = 4x$
$\angle CBA = \angle ACB = 4x$ [$\because$ AB = AC]
$\therefore$ $\angle BAC + \angle ACB + \angle CBA = 180°$
[Sum of $\angle$s of a Δ]
$\Rightarrow$ $x + 4x + 4x = 180°$
$\Rightarrow$ $9x = 180°$
$\Rightarrow$ $\mathbf{x = 20°}$

(*vii*) **115°**



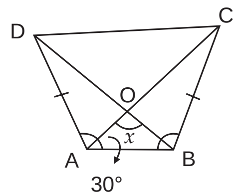
$\angle CDE + \angle DEB = 90° + 90°$
$= 180°$
But $\angle CDE$ and $\angle DEB$ are coint. $\angle$s.
$\therefore$ DC $\parallel$ EAB
$\therefore$ $\angle CBA = 65°$ [Corr. $\angle$s]
$\angle CAB = \angle CBA$
$= 65°$ [$\angle$s opp. equal sides]
$x + \angle CAB = 180°$ [Linear pair]
$\Rightarrow$ $x + 65° = 180°$
$\Rightarrow$ $\mathbf{x = 115°}$

(*viii*) **104°**
Side BA of ΔABC is produced to O.



$\therefore$ Ext. $\angle OAC = \angle ABC + \angle ACB$
[Ext. $\angle$s Sum of int. opp. $\angle$s]
$\Rightarrow$ $104° = \angle DCB + \angle DBC$
$\therefore$ $\angle ABC = \angle DBC$
and $\angle ACB = \angle DBC$
But $x = \angle DCB + \angle DBC$
$\Rightarrow$ $\mathbf{x = 104°}$

(*ix*) **120°**



In ΔDAB and ΔCBA, we have

$DA = CB$ [Given]

$\angle DAB = \angle CBA$ [Given]

$AB = BA$ [Common]

$\therefore$ $\Delta DAB \cong \Delta CBA$ [By SAS congruence]

$\Rightarrow$ $\angle ADB = \angle BCA$ [CPCT] … (1)

In ΔAOD and ΔBOC, we have

$\angle ADO = \angle BCO$ [From (1)]

$\angle AOD = \angle BOC$ [vert. opp. ∠s]

$AD = BC$ [Given]

$\therefore$ $\Delta AOD \cong \Delta BOC$ [By AAS congruencey]

$\Rightarrow$ $OA = OB$ [CPCT]

$\Rightarrow$ $\angle OBA = \angle OAB = 30°$

[∠s opp. equal sides of ΔOAB]

In ΔOAB, we have

$x + 30° + 30° = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow$ $\boldsymbol{x = 120°}$

(*x*) **150°**

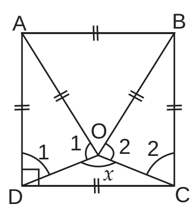
$\angle DAO = \angle DAB - \angle OAB$

$= 90° - 60°$

$= 30°$

[∵ $\angle DAB = 90°$, ∠ of a square and

$\angle BAO = 60°$, ∠ of equilateral ΔAOB]



Similarly, $\angle CBO = 30°$

Now, $\angle 1 = \dfrac{180° - 30°}{2} = 75°$

Similarly, $\angle 2 = 75°$

Angles about point O is 360°.

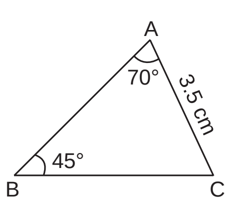
$x + \angle 1 + \angle AOB + \angle 2 = 360°$

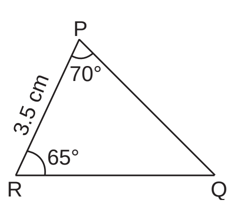
$\Rightarrow$ $x + 75° + 60° + 75° = 360°$

$\Rightarrow$ $\boldsymbol{x = 150°}$

3. (*b*) $\boldsymbol{\Delta ABC \cong \Delta PQR}$

In ΔABC, $\angle A = 70°, \angle B = 45°$





$\Rightarrow$ $\angle C = 180° - (45° + 70°)$

$= 65°$

$AC = 3.5$ cm

In ΔPQR,

$\angle P = 70°, \angle R = 65°$

$\Rightarrow$ $\angle Q = 45°$

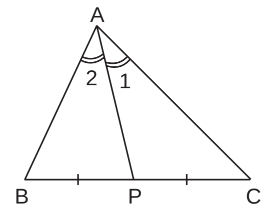
$PR = 3.5$ cm

Thus, $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$

$AC = PR$

$\therefore$ $\Delta ABC \cong \Delta PQR$

4. (*b*) **BA > BP**



Ext. $\angle APB > \angle 1$

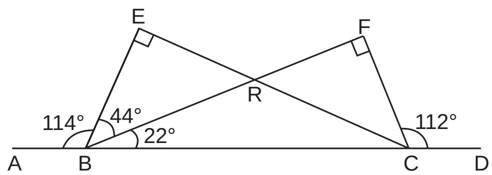
$\Rightarrow$ $\angle APB > \angle 2$ [∵ $\angle 1 = \angle 2$]

∵ Side opp. to greater angle is greater.

$\therefore$ $AB > BP$

or, **BA > BP**

5. (*d*) **EB > ER**



In ΔBER, $\angle E = 90°$,

$\angle ERB = 90° - 44°$

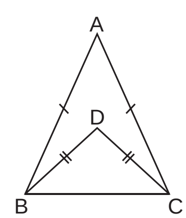
$= 46°$ [Angle sum property]

$\therefore$ $46° > 44°$

$\Rightarrow$ **EB > ER**

[Greater ∠ has greater side opp. to it]

6. (*a*) **1 : 1**



$AB = AC$

$\Rightarrow$ $\angle ACB = \angle ABC$ … (1)

$DB = DC$

$\Rightarrow$ $\angle DCB = \angle DBC$

[∠s opp. to equal sides of a Δ] … (2)

Subtract (2) from (1),

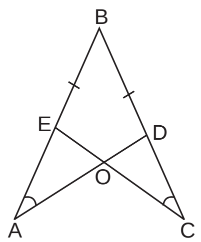
$\angle ABC - \angle DBC = \angle ACB - \angle DCB$

$\angle ABD = \angle ACD$

$\Rightarrow$ $\dfrac{\angle ABD}{\angle ACD} = \dfrac{1}{1} = 1 : 1$

$\Rightarrow$ $\boldsymbol{\angle ABD : \angle ACD = 1 : 1}$

7. (*a*) **$\Delta ADB \cong \Delta CBE$**



In ΔABD and ΔCBE,

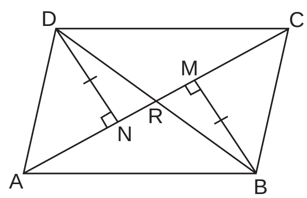
$\angle A = \angle C$ [Given]

$BA = BC$ [Given]

$\angle B = \angle B$ [Common]

$\therefore$ $\Delta ABD \cong \Delta CBE$ [By ASA congruence]

8. (*c*) **16 cm**



In ΔDNR and ΔBMR, we have

$DN = BM$ [Given]

$\angle DRN = \angle BRM$ [Vert. opp. $\angle$s]

and $\angle DNR = \angle BMR = 90°$

$\therefore$ $\Delta DNR \cong \Delta BMR$ [AAS congruency]

$\Rightarrow$ $DR = BR$ [CPCT]

$\Rightarrow$ $BR = 8$ cm

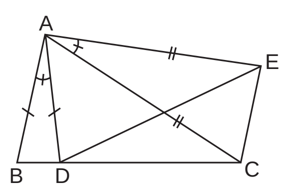
$\Rightarrow$ $DR = 8$ cm

$BD = BR + DR$

$= 8 \text{ cm} + 8 \text{ cm}$

= 16 cm

9. (*c*) **40°**



$\angle BAC = \angle BAD + \angle DAC$

$\angle DAE = \angle DAC + \angle EAC$

$\Rightarrow$ $\angle BAC = \angle DAE$

[$\because$ $\angle BAD = \angle EAC$] … (1)

In ΔABC and ΔADE, we have

$AB = AD$ [Given]

$\angle BAC = \angle DAE$ [From (1)]

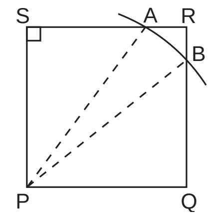
and $AC = AE$ [Given]

$\therefore$ $\Delta ABC \cong \Delta ADE$ [By SAS congruence]

$\Rightarrow$ $\angle ACB = \angle AED$

$\Rightarrow$ **$40° = \angle AED$**

10. (*b*) **1 cm**



Triangle PSA is a right angled triangle such that PA = 5 cm, SA = 3 cm.

$\therefore$ $PS = \sqrt{5^2 - 3^2} = 4$ cm

$\Rightarrow$ $PS = SR = RQ = PQ = 4$ cm [Sides of a square]

In ΔBQP and ΔASP, we have,

$BP = AP$ [radii of same arc]

$\angle BQP = \angle ASP = 90°$

$PQ = PS = 4$ cm

$\therefore$ $\Delta BQP \cong \Delta ASP$

$\Rightarrow$ $BQ = AS$ [CPCT]

$\Rightarrow$ $RQ - BR = SR - AR$

$BR = AR$ [$\because$ $RQ = SR$]

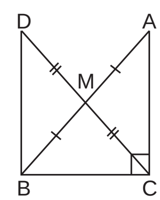
But $AR = SR - AS$

$= (4 - 3)$ cm

$= 1$ cm

$\therefore$ **BR = 1 cm**

11. (*c*) 4 cm



$\Delta BDM \cong \Delta ACM$ [By SAS congruence]

$\Rightarrow$ $BD = AC$,

$DM = MC$ … (1)

$\angle BDM = \angle ACM$

But $\angle BDM$ and $\angle ACM$ are alt. $\angle$s

$\Rightarrow$ $DB \parallel AC$

$\Rightarrow$ $\angle DBC + \angle ACB = 180°$ [Coint. $\angle$s]

$\Rightarrow$ $\angle DBC + 90° = 180°$

$\Rightarrow$ $\angle DBC = 90°$

$\Delta DBC \cong \Delta ACB$ [By SAS congruence]

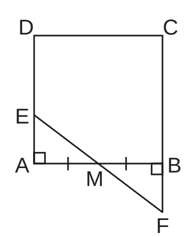
$\therefore$ $DC = AB$ [CPCT]

$\Rightarrow$ $DM + MC = AB$

$\Rightarrow$ $2\,DM = 8$ cm [Using (1)]

$\Rightarrow$ **DM = 4 cm**

12. (*c*) **6 cm**



In ΔMAE and ΔMBF, we have

$\angle MAE = \angle MBF = 90^\circ$

$AM = BM$ [M is mid-point of AB]

$\angle AME = \angle BMF$ (vert. opp. $\angle$s)

$\Rightarrow$ $\Delta MAE \cong \Delta MBF$ [ASA congruency]

$\Rightarrow$ $AE = BF = 1$ cm

$\therefore$ $CF = CB + BF$

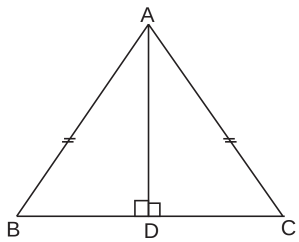
$= AD + BF$

[$\because$ CB = AD side of a square]

$= 5$ cm $+ 1$ cm

= 6 cm

13. (*i*)



In rt. ΔADB and rt. ΔADC,

$AB = AC$ [Given]

$AD = AD$ [Common]

$\therefore$ $\Delta ADB \cong \Delta ADC$

[By RHS congruency] … (1)

$\therefore$ $DB = DC$ [CPCT]

$\Rightarrow$ D is mid-point of BC.

Hence, **AD bisects BC**.

(*ii*) $\Delta ADB \cong \Delta ADC$ [From (1)]

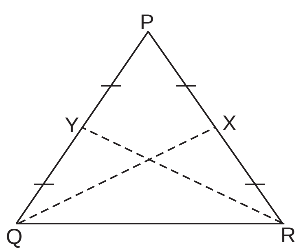
$\therefore$ $\angle BAD = \angle CAD$ [CPCT]

Hence, **AD bisects $\angle$A**.

14. In ΔPQR, $PQ = PR$ [Given]

$\Rightarrow$ $\angle PRQ = \angle PQR$

[Angles opp. to equal sides] … (1)



Now, in ΔQXR and ΔRYQ,

$XR = YQ$

[X and Y are mid-points of equal sides PR and PQ respectively]

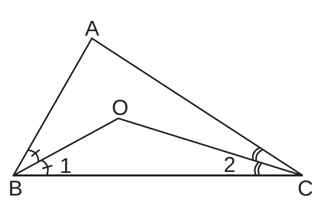
$\angle XRQ = \angle YQR$ [From (1)]

$QR = RQ$ [Common]

$\Delta QXR \cong \Delta RYQ$ [SAS congruency]

$\Rightarrow$ **QX = RY** [CPCT]

15.



In ΔABC, we have $AC > AB$

$\therefore$ $\angle ABC > \angle ACB$

[The angle opp. to a greater side is greater]

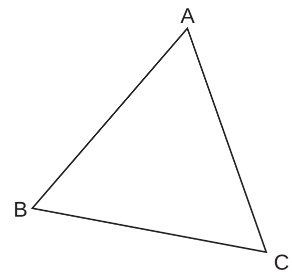
$\Rightarrow$ $\frac{1}{2}\angle ABC > \frac{1}{2}\angle ACB$

or, $\angle 1 > \angle 2$

$\Rightarrow$ **OC > OB**

[$\because$ Side opp. to greater angle is greater]

16. To prove that each angle of an equilateral triangle is 60°.
Let ABC be an equilateral triangle.



$\Rightarrow$ $AB = BC = CA$

$\because$ Angles opposite to equal sides of a Δ are equal.

$\therefore$ $AB = AC$

$\Rightarrow$ $\angle C = \angle B$ … (1)

$BC = CA$

$\Rightarrow$ $\angle A = \angle B$ … (2)

From (1) and (2), we get

$\angle A = \angle B = \angle C$ … (3)

But $\angle A + \angle B + \angle C = 180^\circ$ [Sum of $\angle$s of a Δ]

$\therefore$ $\angle A + \angle A + \angle A = 180^\circ$ [Using (3)]

$\Rightarrow$ $3\angle A = 180^\circ$

$\Rightarrow$ $\angle A = \frac{180^\circ}{3} = 60^\circ$

Thus, $\angle A = \angle B = \angle C = 60^\circ$

Now, $\angle ADE = \angle ADC + \angle CDE$

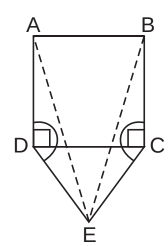
$= 90^\circ + 60^\circ$

$= 150^\circ$

[$\because$ $\angle ADC$ is $\angle$ of a square and $\angle CDE$ is an angle of an equilateral triangle]

Similarly, $\angle BCE = 150^\circ$

$\therefore$ $\angle ADE = \angle BCE = 150^\circ$ … (1)



Now, in ΔADE and ΔBCE,

$AD = BC$ [sides of a square]

$\angle ADE = \angle BCE$ [From (1)]

$DE = CE$

[Sides of an equilateral Δ]

$\therefore$ **$\Delta ADE \cong \Delta BCE$**

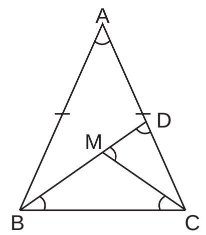
[Using SAS congruency]

17. In ΔABC, $AC = AB$ [Given]

$\Rightarrow$ $\angle ABC = \angle ACB$ [$\angle$s opp. equal sides]

$\Rightarrow \qquad \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

or $\qquad \angle 2 = \angle 1 \qquad \ldots (1)$



In ΔBMC,

$\text{Ext.}\angle DMC = \angle 1 + \angle 2$

[Ext. $\angle$ = Sum of int. opp. $\angle$s]

But $\qquad \angle 1 = \angle 2 \qquad$ [From (1)

$\therefore \qquad \angle PMC = \angle 2 + \angle 2$

$= 2\angle 2$

$= \angle ABC$

[$\because$ BM is bisector of $\angle B$]

Thus, **$\angle$PMC = $\angle$ABC**.

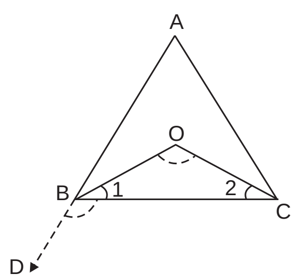
18. In ΔABC, $\qquad AC = AB$

$\Rightarrow \qquad \angle ABC = \angle ACB$

[$\because$ Angle opp. to equal sides of a Δ are equal.]

$\Rightarrow \qquad \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \qquad \angle 1 = \angle 2$



In BOC,

$\angle 1 + \angle 2 + \angle BOC = 180° \qquad$ [Sum of $\angle$s of Δ]

$\Rightarrow \qquad \angle 1 + \angle 1 + \angle BOC = 180° \qquad [\because \angle 1 = \angle 2]$

$\Rightarrow \qquad 2\angle 1 + \angle BOC = 180°$

or, $\qquad \angle BOC = 180° - 2\angle 1 \qquad \ldots (1)$

Now, $\qquad \angle ABC + \angle CBD = 180° \qquad$ [Linear pair]

$\angle CBD = 180° - \angle ABC$

$\Rightarrow \qquad \angle CBD = 180° - 2\angle 1$

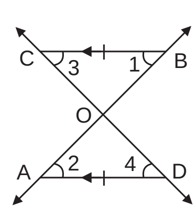
$[\because \frac{1}{2}\angle ABC = \angle 1] \ldots (2)$

From (1) and (2), we get

$\angle$CBD = $\angle$BOC

19. BC $\parallel$ DA and AB is a transversal.

$\therefore \qquad \angle 1 = \angle 2 \qquad$ [Int. alt. $\angle$s] … (1)



Similarly, $\qquad \angle 3 = \angle 4 \qquad \ldots (2)$

Now, in ΔBOC and ΔAOD

$\angle 1 = \angle 2 \qquad$ [From (1)]

$BC = AD \qquad$ [Given]

$\angle 3 = \angle 4 \qquad$ [From (2)]

$\therefore \qquad \Delta BOC \cong \Delta AOD \qquad$ [ASA congruency]

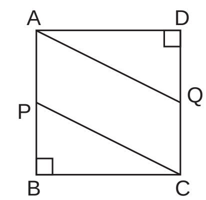
$\Rightarrow \qquad BO = AO$

$\Rightarrow$ O is mid-point of AB and CO = DO

$\Rightarrow$ O is mid-point of CD

Thus, **O is the mid-point of both AB and CD.**

20. ABCD is a square.



$AD = CB \qquad$ [Side of a square] … (1)

$CD = AB$

$\Rightarrow \qquad \frac{1}{2}CD = \frac{1}{2}AB$

$\Rightarrow \qquad DQ = BP$

[$\because$ Q and P are mid-points of CD and AB respectively]

… (2)

Now, in ΔADQ and ΔCBP,

$AD = CB \qquad$ [From (1)]

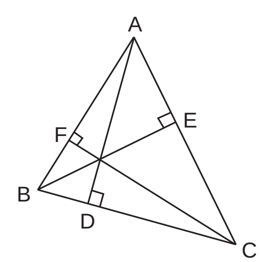
$\angle D = \angle B \qquad$ [Each = 90°]

$DQ = BP \qquad$ [From (2)]

$\therefore \qquad \Delta ADQ \cong \Delta CBP \qquad$ [SAS congruency]

$\Rightarrow \qquad$ **QA = PC** $\qquad$ [CPCT]

21. In a triangle, the perpendicular on a side from the opposite vertex is the shortest of all other line segments drawn to this side from the opp. vertex.



Now, $\qquad AD \perp BC$

$\therefore \qquad AD < AB \qquad \ldots (1)$

Again $\qquad BE \perp AC$

$\therefore \qquad BE < BC \qquad \ldots (2)$

and $\qquad CF \perp AB$

$\therefore \qquad CF < AC \qquad \ldots (3)$

Adding (1), (2) and (3), we get

$(AD + BE + CF) < (AB + BC + AC)$

Hence, **the sum of altitudes of a triangle is less than the sum of its three sides.**

Thus, $\qquad PQ + QR + RP > 2PS$