CHAPTER **6**

Lines and Angles

EXERCISE 6A

1. Since the sum of two complementary angles is 90°, therefore,

(*i*) Complement of 35° = 90° – 35° = **55°**

(*ii*) Complement of 90° = 90° – 90° = **0°**

(*iii*) Complement of 87° = 90° – 87° = **3°**

(*iv*) Complement of 26° = 90° – 26° = **64°**

2. Since the sum of two supplementary angles is 180°, therefore,

(*i*) Supplement of 135° = 180° – 135° = **45°**

(*ii*) Supplement of 90° = 180° – 90° = **90°**

(*iii*) Supplement of 32° = 180° – 32° = **148°**

(*iv*) Supplement of 63° = 180° – 63° = **117°**

3. (*i*) Let the required angle be x.

Angle = Its complement [Given]

$\therefore\ x = (90 - x)°$

$\Rightarrow\ 2x = 90°$

$\Rightarrow\ x = \mathbf{45°}$

(*ii*) Let the required angle be x.

Angle = Its supplement [Given]

$\therefore\ x = (180 - x)°$

$\Rightarrow\ 2x = 180°$

$\Rightarrow\ x = \mathbf{90°}$

(*iii*) Let the required angle be x.

Angle = its complement + 24° [Given]

$\therefore\ x = (90 - x) + 24°$

$\Rightarrow\ x = (114 - x)°$

$\Rightarrow\ 2x = 114°$

$x = \frac{114}{2} = \mathbf{57°}$

(*iv*) Let the required angle be x.

Angle = Its supplement – 30°

$\therefore\ x = 180° - x - 30°$

$= 150° - x$

$\Rightarrow\ 2x = 150°$

$\Rightarrow\ x = \frac{150°}{2} = \mathbf{75°}$

(*v*) Let the required angle be x.

Angle = $\frac{1}{2}$ (its complement) [Given]

$\therefore\ x = \frac{1}{2}(90° - x)°$

$\Rightarrow\ 2x = (90 - x)°$

$\Rightarrow\ 3x = 90°$

$\Rightarrow\ x = 30°$

(*vi*) Let the required angle be x.

Angle = $\frac{1}{8}$ (its complement) [Given]

$\therefore\ x = \frac{1}{8}(90° - x)$

$\Rightarrow\ 8x = 90° - x$

$\Rightarrow\ 8x + x = 90°$

$\Rightarrow\ 9x = 90°$

$\Rightarrow\ x = \mathbf{10°}$

(*vii*) Let the required angle be x.

Angle = $\frac{1}{5}$ (its supplement) [Given]

$\therefore\ x = \frac{1}{5}(180° - x)$

$\Rightarrow\ 5x = 180° - x$

$\Rightarrow\ 6x = 180°$

$\Rightarrow\ x = \mathbf{30°}$

(*viii*) Let the required angle be x.

Angle = 4 × its supplement [Given]

$\therefore\ x = 4(180° - x)$

$\Rightarrow\ \frac{1}{4}x = 180° - x$

$\Rightarrow\ \frac{1}{4}x + x = 180°$

$\Rightarrow\ \frac{5}{4}x = 180°$

$\Rightarrow\ x = 180° \times \frac{4}{5}$

$\Rightarrow\ x = \mathbf{144°}$

(*ix*) Let the required angle be x.

Angle = $\frac{1}{4}$ (its supplement) + 10° [Given]

$\therefore\ x = \frac{1}{4}(90° - x) + 10°$

$\Rightarrow\ 4x = 90° - x + 40°$

$\Rightarrow\ 4x + x = 90° + 40°$

$\Rightarrow\ 5x = 130°$

$\Rightarrow\ x = \frac{130°}{5}$

$\Rightarrow\ x = \mathbf{26°}$

(*x*) Let the required angle be x.

Angle = $\frac{4}{5}$ (its supplement) + 36° [Given]

$\therefore\ x = \frac{4}{5}(180° - x) + 36°$

$\Rightarrow\ 5x = 4(180° - x) + 180°$

$\Rightarrow\ 5x = 720° - 4x + 180°$

$\Rightarrow\ 5x + 4x = 900°$

$\Rightarrow\ 9x = 900°$

$\Rightarrow\ x = \frac{900°}{9} = \mathbf{100°}$

4. (*i*) The given angles are complementary.

$\therefore\ (4x + 4)° + (6x - 4)° = 90°$

$\Rightarrow\ 10x + 4 - 4 = 90°$

$\Rightarrow\ 10x = 90°$

$\Rightarrow\ \boldsymbol{x = 9}$

(*ii*) The given angles are supplementary.

$\therefore \quad (5x + 6)° + (13x + 30)° = 180°$

$\Rightarrow \quad 18x = 180 - 36 = 144$

$\Rightarrow \quad x = \frac{144°}{18} = 8$

$\Rightarrow \quad x = 8$

Measures of the angles are $(5 \times 8 + 6)°$ and $(13 \times 8 + 30)°$ or **46° and 134°.**

5. Let the required angle be x.

$\therefore \quad \frac{1}{5}$(Supplement of x) = complement of x

$\Rightarrow \quad \frac{1}{5}(180° - x) = (90° - x)$

$\Rightarrow \quad 180° - x = 5(90° - x)$

$\Rightarrow \quad 180° - x = 450° - 5x$

$\Rightarrow \quad -x + 5x = 450° - 180°$

$\Rightarrow \quad 4x = 270°$

$\Rightarrow \quad x = \frac{270°}{4} = 67.5°$

Thus, the required angle is **67.5°.**

6. Let the required angle be x.

Then, Supplement of x = 4 (Complement of x)

$\therefore \quad (180° - x) = 4(90° - x)$

$\Rightarrow \quad 180° - x = 360° - 4x$

$\Rightarrow \quad -x + 4x = 360° - 180°$

$\Rightarrow \quad 3x = 180°$

$\Rightarrow \quad x = \frac{180°}{3} = \mathbf{60°}$

7. Let the required angle be x.

Then, Supplement of $x = \frac{1}{2}(x)$

$\therefore \quad (180° - x) = \frac{1}{2}(x)$

$\Rightarrow \quad 2(180° - x) = x$

$\Rightarrow \quad 360° - 2x = x$

$\Rightarrow \quad 360° = x + 2x$

$\Rightarrow \quad 3x = 180°$

$\Rightarrow \quad x = \frac{180°}{3} = 60°$

Thus, the required angle is **60°.**

Supplement of 60° = 180° – 60° = **120°**

8. Let the required angle be x.

Then,

6(Complement of x) = 2 (Supplement of x) – 12°

$\therefore \quad 6(90° - x) = 2(180° - x) - 12°$

$\Rightarrow \quad 540° - 6x = 360° - 2x - 12°$

$\Rightarrow \quad -6x + 2x = 360° - 540° - 12°$

$\Rightarrow \quad -4x = -192$

$\Rightarrow \quad x = \frac{-192}{-4} = \mathbf{48°}$

9. Let one of the angles be x.

$\therefore$ Complementary of $x = (90° - x)$

Since ratio of the given complementary angles is 2 : 3,

$\therefore \quad x : (90° - x) = 2 : 3$

$\Rightarrow \quad \frac{x}{90° - x} = \frac{2}{3}$

$\Rightarrow \quad 3(x) = 2(90° - x)$

$\Rightarrow \quad 3x = 180° - 2x$

$\Rightarrow \quad 3x + 2x = 180°$

$\Rightarrow \quad x = \frac{180°}{5} = 36°$

Complement of $x = 90° - x°$

$= 90° - 36° = 54°$

Thus, the required angles are **36° and 54°.**

10. Let one of the angles be x.

Then, its supplementary angle is $(180° - x)$.

Now, $\quad x : (180° - x) = 7 : 2$

or $\quad \frac{x}{180° - x} = \frac{7}{2}$

$\Rightarrow \quad 2x = 7(180° - x)$

$\Rightarrow \quad 2x = 1260° - 7x$

$\Rightarrow \quad 2x + 7x = 1260°$

$\Rightarrow \quad 9x = 1260°$

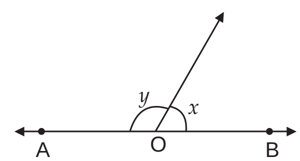
$\Rightarrow \quad x = \frac{1260°}{9} = 140°$

Supplement of $x = 180° - x = 180° - 140° = 40°$

Thus, the required angles are **140° and 40°.**

EXERCISE 6B

1. (*i*) OA and OB are opposite rays.



$\therefore \quad x + y = 180° \quad$ [Linear pair]

$63° + y = 180°$

or $\quad y = 180° - 63° = 117°$

Hence, $\boldsymbol{y} = \mathbf{117°}$.

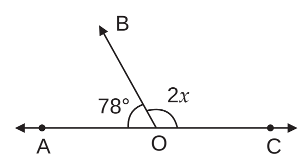
(*ii*) $\quad x + y = 180° \quad$ [Linear pair]

$x + 122° = 180°$

or $\quad x = 180° - 122° = 58°$

Hence, $\boldsymbol{x} = \mathbf{58°}$.

2. AOC is a straight line.



$78° + 2x = 180° \quad$ [Linear pair]

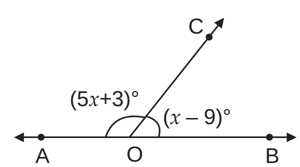
$\Rightarrow \quad 2x = 180° - 78°$

$\Rightarrow \quad 2x = 102°$

$\therefore \quad x = \frac{102°}{2} = 51°$

Hence, $\boldsymbol{x} = \mathbf{51°}$.

3. AOB is a straight line.



$\therefore \quad (5x + 3)°$ and $(x - 9)°$ form a linear pair.

$\Rightarrow \quad (5x + 3) + (x - 9) = 180°$

$\Rightarrow \quad 6x - 6 = 180°$

$\Rightarrow \quad 6x = 180° + 6 = 186°$

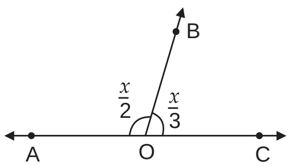
$\Rightarrow \quad x = \frac{186°}{6} = 31°$

Now, $(5x + 3)° = (5 \times 31 + 3)°$
$= 158°$

and $(x - 9)° = (31 - 9)°$
$= 22°$

Hence, $x = 31$ and measures of the angles are **158°, 22°.**

4. $\frac{x}{2}$ and $\frac{x}{3}$ from a linear pair.



$\therefore \quad \frac{x}{2} + \frac{x}{3} = 180°$

$\Rightarrow \quad \frac{3x + 2x}{6} = 180°$

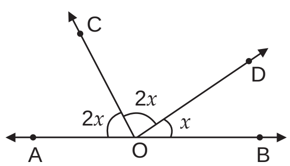
$\Rightarrow \quad 5x = 6 \times 180°$

$\Rightarrow \quad x = \frac{6x \times 180°}{5}$

$\Rightarrow \quad x = 6 \times 36° = 216°$

Hence, $x = 216°$.

5. (*i*) ΔAOB is a straight line.



$\therefore$ Sum of all angles on one side of AB as O is 180°

$\Rightarrow \quad 2x + 2x + x = 180°$

$\Rightarrow \quad 5x = 180°$

$\therefore \quad x = \frac{180°}{5} = 36°$

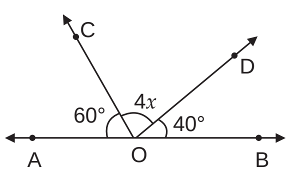
Hence, $x = 36°$.

(*ii*) $60° + 4x + 40° = 180°$

$\Rightarrow \quad 4x + 100° = 180°$

$\Rightarrow \quad 4x = 180° - 100° = 80°$

$\Rightarrow \quad x = \frac{80°}{4} = 20°$



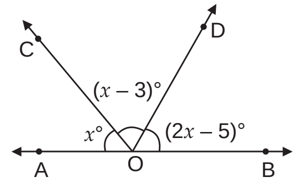
Hence, $x = 20°$.

(*iii*) $x° + (x - 3)° + (2x - 5)° = 180°$

$\Rightarrow \quad 4x - 8 = 180$

$\Rightarrow \quad 4x = 180 + 8 = 188$

$\Rightarrow \quad x = \frac{188}{4} = 47$



Hence, $x = 47$.

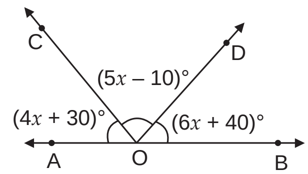
(*iv*) $(4x + 30)° + (5x - 10)° + (6x + 40)° = 180°$

$\Rightarrow \quad 15x + 30 + 40 - 10 = 180°$

$\Rightarrow \quad 15x + 60 = 180°$

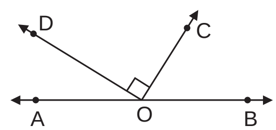
$\Rightarrow \quad 15x = 180° - 60 = 120$

$\Rightarrow \quad x = \frac{120}{15} = 8$



Hence, $x = 8$.

6. OD ⊥ OC.



$\Rightarrow \quad \angle DOC = 90° \quad \ldots (1)$

$\because \quad \angle AOD + \angle DOC + \angle COB = 180°$

[Sum of all the angles on the same side of a line at a given point is 180°] … (2)

$\therefore$ From (1) and (2), we have

$\angle AOD + 90° + \angle COB = 180°$

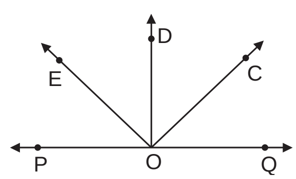
Or $\quad \angle AOD + \angle COB = 180° - 90° = 90°$

Hence, $\angle AOD + \angle COB =$ **90°**.

7. $\angle POC = \angle QOE$ [Given]

$\Rightarrow \quad \angle POE + \angle EOD + \angle COD = \angle EOD + \angle COD + \angle QOC$

$\Rightarrow \quad \angle POE = \angle QOC$



Since the sum of all the angles on the same side of a line at a given point is 180°,

$\therefore \quad \angle POE + (\angle EOD + \angle COD + \angle QOC) = 180°$

$\Rightarrow \quad \angle POE + \angle QOE = 180°$

$\Rightarrow \quad \angle POE + 135° = 180°$

[$\because$ ∠QOE = 135°, given]

$\Rightarrow \quad \angle POE = 180° - 135° = 45° \quad \ldots (1)$

Now, $\quad \angle POC = 135°$ [Given]

$\Rightarrow \quad \angle POE + \angle EOD + \angle DOC = 135°$

$\Rightarrow \quad 45° + 2\angle DOC = 135°$

[$\because$ ∠EOD = ∠DOC and using (1)]

$\Rightarrow \quad 2\angle DOC = 135° - 45°$
$= 90°$

$\Rightarrow \quad \angle DOC = \frac{90°}{2} = 45°$

$\angle EOC = \angle EOD + \angle DOC = 2\angle DOC$

$[\because \angle EOD = \angle DOC, \text{ given}]$

$\Rightarrow \quad \angle EOC = 2 \times 45° = 90°$

Hence, **∠POE = 45°, ∠EOC = 90°, ∠DOC = 45°.**

8. If AOC becomes a straight line, then sum of all angles one side of AC as O is 180°.

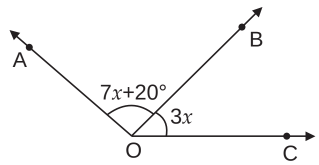
(*i*) $7x + 20° + 3x = 180°$

$[\because$ AOC is taken to be a st. line$]$

$7x + 3x = 180° - 20° = 160°$

$10x = 160°$

or $x = \dfrac{160°}{10} = 16°$



Hence, ***x* = 16°.**

(*ii*) When AOC is a straight line, then

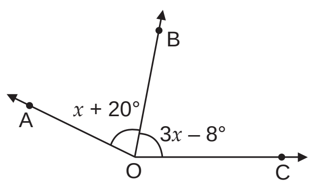
$x + 20° + 3x - 8° = 180°$

$\Rightarrow \quad x + 3x = 180° - 20° + 8°$

$= 168°$

$\Rightarrow \quad 4x = 168°$

or $x = \dfrac{168°}{4} = 42°$



Hence, ***x* = 42°.**

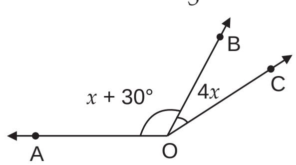
(*iii*) When AOC is a straight line, then

$x + 30° + 4x = 180°$

$\Rightarrow \quad 4x + x = 180° - 130° = 150°$

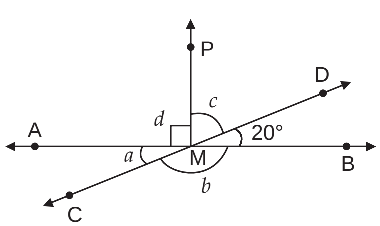
$\Rightarrow \quad 5x = 150°$

$\Rightarrow \quad x = \dfrac{150°}{5} = 30°$



Hence, ***x* = 30°.**

9. AB and CD intersect each other.



$\therefore \quad \angle AMC = \angle BMD = 20°$

[Vertically opposite angles]

$\therefore \quad a = 20°$

$PM \perp AB = \angle AMP = 90°$

$\Rightarrow \quad d = 90°$

$PM \perp AB$

$\Rightarrow \quad \angle BMP = 90°$

$\Rightarrow \quad 20° + c = 90°$

$\Rightarrow \quad c = 70°$

Also, $a + b + c + d + 20° = 360°$ [∠s about a point]

$\therefore \quad 20° + b + 70° + 90° + 20° = 360°$

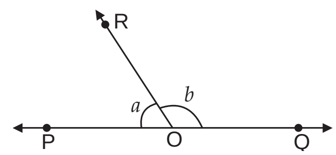
$\Rightarrow \quad 200° + b = 360°$

$\Rightarrow \quad b = 360° - 200° = 160°$

Hence, ***a* = 20°, *b* = 160°, *c* = 70° and *d* = 90°.**

10. ∠PQR and ∠QOR form a linear pair. [Given]

$\therefore \quad a + b = 180°$



(*i*) Now, $a : b = 2 : 3$

Let $a = 2x$ and $b = 3x$.

$\therefore \quad 2x + 3x = 180°$

$\Rightarrow \quad 5x = 180°$

$\Rightarrow \quad x = \dfrac{180°}{5} = 36°$

Thus, $a = 2 \times 36° = 72°$

and $b = 3 \times 36° = 108°$

Hence, ***a* = 72°, *b* = 108°.**

(*ii*) $b - a = 50°$ … (1)

$a + b = 180°$ (Linear pair) … (2)

Adding (1) and (2), we get

$2b = 50° + 180° = 230°$

$\Rightarrow \quad b = 115°$

Substituting $b = 115°$ in equation (1) we get

$115° - a = 50°$

$\Rightarrow \quad a = 65°$

Hence, ***a* = 65°, *b* = 115°.**

(*iii*) $a + b = 180°$ [Linear pair] … (1)

$2a - b = -30°$ $[\because 2a = b - 30°]$ … (2)

Adding (1) and (2), we get

$3a = 150°$

$\Rightarrow \quad a = 50°$

Substituting $a = 50°$ in equation (1), we get

Now, $50° + b = 180°$

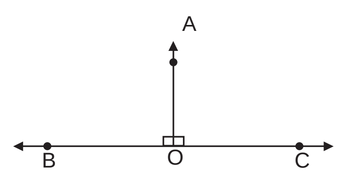
$\Rightarrow \quad b = 180° - 50°$

$\Rightarrow \quad b = 130°$

Hence, ***a* = 50°, *b* = 130°.**

11. $\angle AOC = 90°$ and $\angle AOB = 90°$

$\angle AOC + \angle AOB = 90° + 90° = 180°$



Sum of all angles at O and on the same side of BOC is 180°.

$\therefore$ BOC is a straight line.

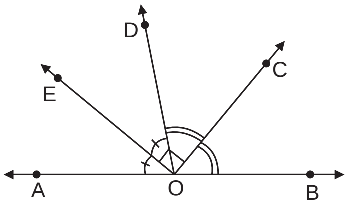
12. OE ⊥ OC and ∠EOC = 90°.

⇒ ∠EOD + ∠DOC = ∠EOC = 90° … (1)

∵ OE is bisector of AOB

∴ ∠AOE = ∠EOD … (2)

Similarly, ∠BOC = ∠DOC … (3)



From (1), (2) and (3), we get

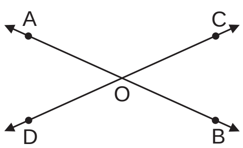
∠AOE + ∠BOC = 90°

Now, (∠AOE + ∠BOC) + ∠EOC = 90° + 90° = 180°

⇒ Sum of all angles at O and on the same side of AOB is 180°

∴ AOB i.e. (AB) is a straight line.

13. Let ∠AOD = $3x$ and ∠BOD = $5x$.



Since ∠AOD and ∠BOD make a linear pair,

∴ ∠AOD + ∠BOD = 180°

⇒ $3x + 5x = 180°$

⇒ $8x = 180°$

⇒ $x = \frac{180°}{8} = \frac{45°}{2}$

∴ $\angle AOD = 3 \times \frac{45°}{2} = \frac{135°}{2} = 67.5°$

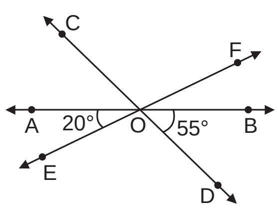
∴ $\angle BOD = 5 \times \frac{45}{2} = \frac{225}{2} = 112.5°$

∠BOC = ∠AOD = 67.5° [V. opp. ∠s]

and ∠AOC = ∠BOD = 112.5° [V. opp. ∠s]

Hence, **∠AOD = 67.5°, ∠BOD = 112.5°, ∠BOC = 67.5°, ∠AOC = 112.5°.**

14. AB and EF intersect at O.



∴ ∠AOE = ∠BOF = 20° (Vertically opposite angles)

AB and CD intersect as O.

∴ ∠AOC = ∠BOD = 55°

Since the sum of all the angles on the same side of a straight line at a given point is 180°,

∴ ∠AOC + ∠COF + ∠BOF = 180°

⇒ 55° + ∠COF + 20° = 180°

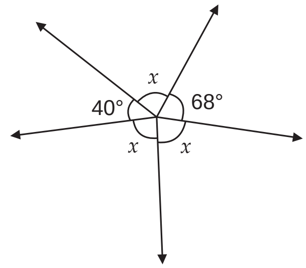
⇒ ∠COF = 180° – 55° – 20°

= 105°

⇒ ∠DOE = ∠COF = 105° [V. opp. ∠s]

Hence, **∠AOC = 55°, ∠COF = 105°, ∠DOE = 105°, ∠BOF = 20°.**

15. (*i*) Since sum of all angles about a point is 360°, therefore,



⇒ $40° + x + 68° + x + x = 360°$

⇒ $3x + 108° = 360°$

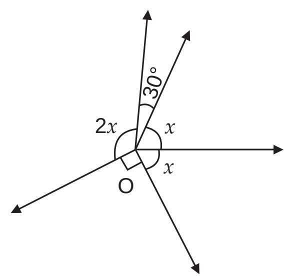
⇒ $3x = 360° - 108°$

⇒ $3x = 252°$

⇒ $x = \frac{252°}{3} = 84°$

Hence, $\boldsymbol{x = 84°}$.

(*ii*)



$2x + 30° + x + x + 90° = 360°$

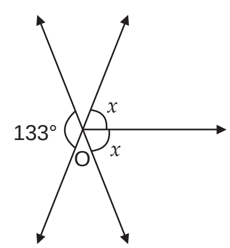
⇒ $4x + 120° = 360°$

⇒ $4x = 360° - 120° = 240°$

⇒ $x = \frac{240°}{4} = 60°$

Hence, $\boldsymbol{x = 60°}$.

(*iii*)



$133° + (180 - 133)° + (180 - 133)° + x + x = 360°$

⇒ $133° + 47° + 47° + 2x = 360°$

⇒ $227° + 2x = 360°$

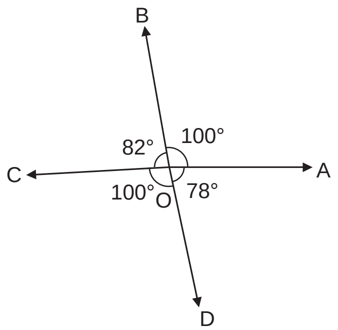
⇒ $2x = 360° - 227° = 133°$

⇒ $x = \frac{133°}{2}$

$= 66.5°$

Hence, $\boldsymbol{x = 66.5°}$.

16. For a straight line, sum of all angles on the same side of the line at a point must be equal to 180°.



$\because \quad \angle AOB + \angle BOC = 100° + 82° = 182° \neq 180°$

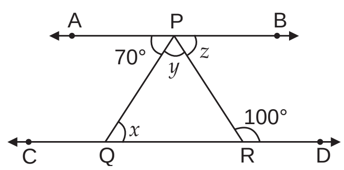
So, AOC cannot be a straight line.

Also, $\angle BOD = 100° + 78° = 178° \neq 180°$

So, BOD cannot be a straight line.

EXERCISE 6C

1. (*i*) AB ∥ CD and PR is a transversal.



$\therefore \quad 100° + z = 180°$ [Cointerior angles]

$\therefore \quad z = 180° - 100° = 80°$

Also, AB ∥ CD and PQ is a transversal then

$x = 70°$ [Alt. angles]

Since the sum of all the angles on the same side of a line at a point is 180°,

$\therefore \quad 70° + y + z = 180°$

$\Rightarrow \quad 70° + y + 80° = 180°$

$\Rightarrow \quad y + 150° = 180°$

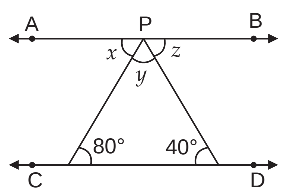
or $\quad y = 180° - 150° = 30°$

Thus, $\mathbf{x = 70°, y = 30°, z = 80°}$.

(*ii*) AB ∥ CD

$\therefore \quad x = 80°$ [Interior alt. angles]

$\therefore \quad z = 40°$



Since AB is a straight line,

$\therefore \quad x + y + z = 180°$

$\Rightarrow \quad 80° + y + 40° = 180°$

$\Rightarrow \quad y + 120° = 180°$

$\Rightarrow \quad y = 180° - 120° = 60°$

Thus, $\mathbf{x = 80°, y = 60°, z = 40°}$.

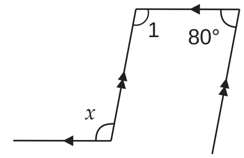
2. $\angle 1 + 80° = 180°$ [Cointerior angles]

$\therefore \quad \angle 1 = 180° - 80° = 100°$ … (1)

Also, $\quad \angle 1 = \angle x$ [Alt. angles] … (2)

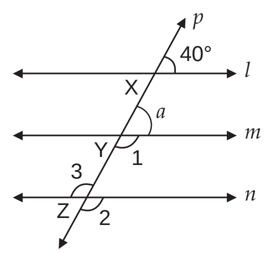
From (1) and (2),

$x = 100°$



Hence, $\mathbf{x = 100°}$.

3.



$l \parallel m$

$\Rightarrow \quad a = 40°$ [Corresponding angles]

But $\quad \angle 1 + a = 180°$ [Linear pair]

$\Rightarrow \quad \angle 1 = 180° - a$

$\Rightarrow \quad \angle 1 = 180° - 40° = 140°$

$m \parallel n$

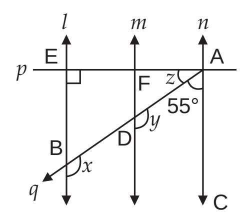
$\Rightarrow \quad \angle 1 = \angle 3$ [Alternate angles]

$\therefore \quad \angle 3 = 140°$

$\therefore \quad \angle 2 = \angle 3 = 140°$ [V. opp ∠s]

Thus, $\mathbf{\angle 1 = \angle 2 = \angle 3 = 140°}$.

4. $l \parallel n$ and P is a transversal.



$\angle EAC + \angle AEB = 180°$

$\angle EAC + 90° = 180°$

$\Rightarrow \quad \angle EAC = 180° - 90° = 90°$

But $\quad EAC = z + 55°$

$\Rightarrow \quad z + 55° = 90°$

or $\quad z = 90° - 55° = 35°$

$m \parallel n$ and AB is a transversal.

$\therefore \quad y + 55° = 180°$ [Cointerior angles]

$\Rightarrow \quad y = 180° - 55°$

$= 125°$

$l \parallel m$ and AB is a transversal.

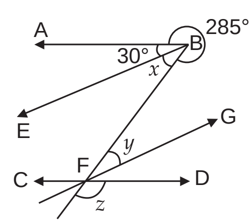
$\Rightarrow \quad x = y$ [Corr. Angles]

$\therefore \quad x = y = 125°$

Thus, $\mathbf{x = 125°, y = 125°, z = 35°}$.

5. $x + 30° + 285° = 360°$ [Angles about a point]

$\Rightarrow \quad x = 360° - 30° - 285° = 45°$



BE ∥ GF and BF is a transversal.

$\therefore \quad x = y$ [Alt. angles]

$\Rightarrow \quad y = 45°$ [$\because x = 45°$]

AB ∥ CD and BF is a transversal.

$\therefore$ ∠BFC and ∠ABF are cointerior angles.

$\therefore \quad \angle BFC + \angle ABF = 180°$

$\Rightarrow \quad \angle BFC + (30° + 45°) = 180°$

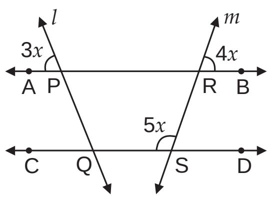
$\Rightarrow \quad \angle BFC = 180° - 30° - 45° = 105°$

But, $\angle BFC = z$ [Ver. opp. ∠s]

$\therefore \quad z = 105°$

Thus, $\boldsymbol{x = 45°, y = 45°, z = 105°}$.

6. AB and m intersects at R.



$\therefore \quad \angle PRS = 4x$ [Ver. opp. ∠s]

AB ∥ CD and m is a transversal.

$\therefore \quad 4x + 5x = 180°$ [Cointerior angles]

$\Rightarrow \quad 9x = 180°$

$\Rightarrow \quad x = \frac{180°}{9} = 20°$

AB ∥ CD and PQ is a transversal.

$\therefore \quad \angle PQC = 3x$ [Corresponding angles]

$= 3 \times 20 = 60°$

AB ∥ CD and m is a transversal.

$\therefore \quad \angle RSD = 4x$ [Corr. Angles]

$= 4 \times 20 = 80°$

Thus, $\boldsymbol{x = 20°, \angle PQC = 60°, \angle RSD = 80°}$.

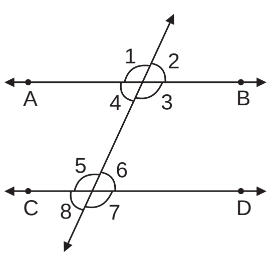
7. (*i*) We have, $\angle 3 = \angle 1$ [Ver. opp. ∠s]

$\therefore \quad \angle 3 = (3x - 10)°$

$\Rightarrow \quad 3x - 10 = 5x - 30$

$\Rightarrow \quad -2x = -20$

$\Rightarrow \quad x = 10$



Now, $\angle 1 = (3x - 10)°$

$= (3 \times 10 - 10°) = 20°$

$\angle 7 = (5x - 30)°$

$= (5 \times 10 - 30)° = 20°$

Thus, $\boldsymbol{\angle 1 = \angle 7 = 20°}$.

(*ii*) If $\angle 4 : \angle 7 = 4 : 5$

Let $\angle 4 = 4x$ and $\angle 7 = 5x$.

$\angle 3 = \angle 7$ [Corr. ∠s, AB ∥ CD]

$\therefore \quad \angle 3 = 5x$

$\angle 3 + \angle 4 = 180°$ [Linear pair]

$5x + 4x = 180°$

$\Rightarrow \quad 9x = 180°$

or $\quad x = 20°$

$\therefore \quad \angle 3 = \angle 7 = 5x = 5 \times 20° = 100°$

$\angle 4 = 4x = 4 \times 20° = 80°$

Thus, $\boldsymbol{\angle 4 = 80°, \angle 7 = 100°}$.

(*iii*) We have,

Complement of ∠6 = Supplement ∠3

$(90° - \angle 6) = (180° - \angle 3)$

or $\quad \angle 3 - \angle 6 = 180° - 90°$

$\angle 3 - \angle 6 = 90°$ … (1)

$\angle 3 + \angle 6 = 180°$

[Cointerior angles, AB ∥ CD] … (2)

Adding (1) and (2),

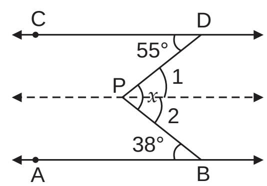
$2\angle 3 = 90° + 180° = 270°$

$\Rightarrow \quad \angle 3 = 270° \div 2 = 135°$

and $\quad \angle 6 = 180° - 135° = 45°$

Thus, $\boldsymbol{\angle 3 = 135°, \angle 6 = 45°}$.

8. (*i*) Through P, draw l ∥ CD or AB.



$\Rightarrow \quad CD \parallel l \parallel AB$

$\therefore \quad \angle 1 = 55°$ [Alt. angles] … (1)

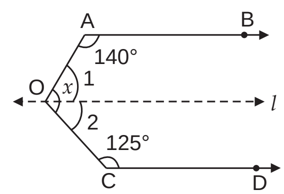
$\therefore \quad \angle 2 = 38°$ [Alt. angles] … (2)

Adding equation (1) and equation (2), we get

$\angle 1 + \angle 2 = 55° + 38° = 93°$

Hence, $\boldsymbol{x = 93°}$.

(*ii*) Though O, draw l ∥ AB and CD.



$\therefore \quad \angle 1 + 140° = 180°$ [Coint. ∠s, AB ∥ l]

$\Rightarrow \quad \angle 1 = 180° - 140°$

$\Rightarrow \quad \angle 1 = 40°$ … (1)

Similarly, $\angle 2 = 180° - 125° = 55°$

$\Rightarrow \quad \angle 2 = 55°$ … (2)

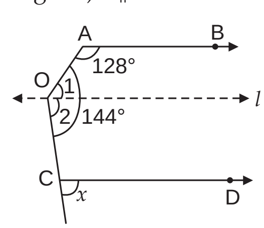
Adding equation (1) and equation (2), we get

$\angle 1 + \angle 2 = 40° + 55°$

$\Rightarrow \quad x = 95°$

Hence, $\boldsymbol{x = 95°}$.

(*iii*) Draw (through O) l ∥ AB and CD.



$AB \parallel l$

$\Rightarrow \quad \angle 1 + 128° = 180°$ [Coint. ∠s]

$\Rightarrow \quad \angle 1 = 180° - 128° = 52°$ … (1)

But $\angle 1 + \angle 2 = 144°$

$\Rightarrow$ $52° + \angle 2 = 144°$

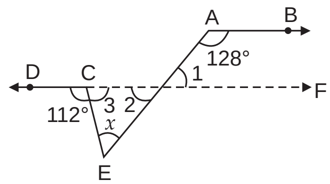
$\Rightarrow$ $\angle 2 = 144° - 52° = 92°$

But $\angle x = \angle 2$ [Corr. $\angle$s, $l \parallel CD$]

$\Rightarrow$ $x = 92°$

Hence, $\boldsymbol{x = 92°}$.

(*iv*) Extend DC to F.



Now, AB $\parallel$ DCF

$\therefore$ $\angle 1 + 128° = 180°$ [Cointerior angles]

$\Rightarrow$ $\angle 1 = 180° - 128° = 52°$

$\Rightarrow$ $\angle 2 = \angle 1$ [V. opp $\angle$s]

$\Rightarrow$ $\angle 2 = 52°$

$\angle 3 + 112° = 180°$ [Linear pair]

$\Rightarrow$ $\angle 3 = 180° - 112° = 68°$

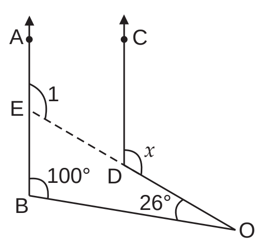
Now, $x + \angle 2 + \angle 3 = 180°$ [sum of $\angle$s of a Δ]

$\Rightarrow$ $x + 52° + 68° = 180°$

$\Rightarrow$ $x = 180° - 52° - 68° = 60°$

Hence, $\boldsymbol{x = 60°}$.

(*v*) Extend OD to meet AB at E.



Ext $\angle 1 = 100° + 26°$

[Ext. $\angle$ = sum of int. opp $\angle$s]

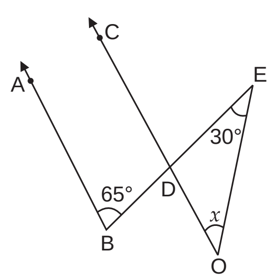
$\Rightarrow$ $\angle 1 = 126°$

$x = \angle 1$ [Corr. $\angle$s, AB $\parallel$ CD]

$x = 126°$

Hence, $\boldsymbol{x = 126°}$.

(*vi*)



$\angle CDE = \angle ABD$ [Corr. $\angle$s, AB $\parallel$ CDO]

$\therefore$ $\angle CDE = 65°$

Ext $\angle CDE = x + 30°$

[Ext $\angle$ = sum of int. opp$\angle$s]

$\Rightarrow$ $65° = x + 30°$

$\Rightarrow$ $x = 65° - 30° = 35°$

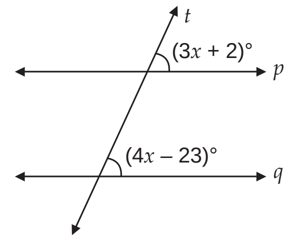
Hence, $\boldsymbol{x = 35°}$.

9. (*i*) p and q will be parallel if

$(4x - 23)° = (3x + 2)°$ [Corr. angles]

$\Rightarrow$ $4x - 3x = 2 + 23$

$\Rightarrow$ $\boldsymbol{x = 25}$



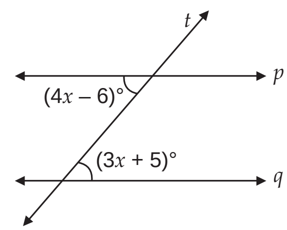
Hence, $\boldsymbol{x = 25}$.

(*ii*) p and q will be parallel if

$(4x - 6)° = (3x + 5)°$ [Alt. angles]

$\Rightarrow$ $4x - 3x = 5 + 6$

$\Rightarrow$ $x = 11$



Hence, $\boldsymbol{x = 11}$.

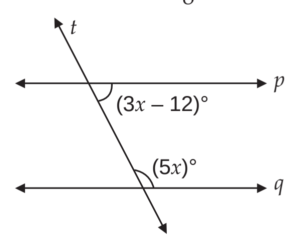
(*iii*) p and q will be parallel if

$(3x - 12)° + (5x)° = 180°$ [Cointerior angles]

$\Rightarrow$ $3x + 5x = 180 + 12 = 192$

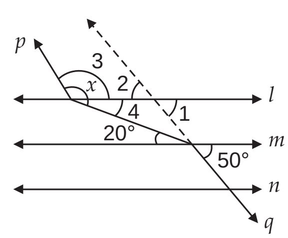
$\Rightarrow$ $8x = 192$

$\Rightarrow$ $x = \frac{192}{8} = 24$



Hence, $\boldsymbol{x = 24}$.

10. Extending q to intersect l, we get



$\angle 1 = 50°$ [Corr. $\angle$s, $l \parallel m$]

$\angle 2 = \angle 1 = 50°$ [V. opp. $\angle$s]

$\angle 2 + \angle 3 = 180°$ [Cointerior angles]

$\Rightarrow$ $50° + \angle 3 = 180°$ [Using (2)]

$\Rightarrow$ $\angle 3 = 180° - 50° = 130°$ … (1)

Also, $\angle 4 = 20°$ [Alt. $\angle$s, $l \parallel m$] … (2)

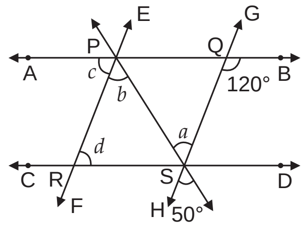
Adding (1) and (2), we get

$$\angle 3 + \angle 4 = 130° + 20° = 150°$$

$\Rightarrow$ $x = 150°$

Hence, $\boldsymbol{x = 150°}$.

11.



$\angle a = 50°$ [Ver. opp. ∠s]

$\angle b = \angle a = 50°$ [Alt. ∠s, EF ∥ GH]

$\angle PQS + 120° = 180°$ [Linear pair]

$\angle PQS = 180° - 120° = 60°$

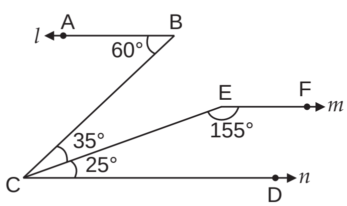
$\Rightarrow$ $\angle PQS = \angle c = 60°$ (Corr. angles, EF ∥ GH)

$\angle d = \angle c = 60°$ [Alt. ∠s, AB ∥ CD]

Thus, $\boldsymbol{\angle a = 50°, \angle b = 50°, \angle c = 60°, \angle d = 60°}$.

12. (*i*) 25° and 155° are cointerior angles.

and $25° + 155° = 180°$



$\therefore$ $m \parallel n$ … (1)

Also, $60° = 35° + 25°$

They are pair of alternate angles.

$\therefore$ $l \parallel n$ … (2)

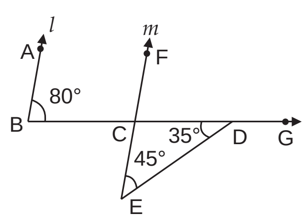
From (1) and (2),

$\boldsymbol{l \parallel m}$

(*ii*) Considering ΔCDE, we get

Ext $\angle FCG = 45° + 25° = 80°$ [Ext ∠ = sum of int. opp. ∠s]

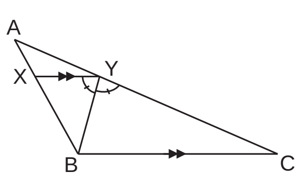
$\therefore$ $\angle ABC = FCG = 80°$



But they are corresponding angles.

$\Rightarrow$ $l \parallel m$

13. XY ∥ BC and BY is a transversal.



$\therefore$ $\angle XYB = \angle CBY$ [Alt. angles] … (1)

$\because$ BY bisects ∠XYC.

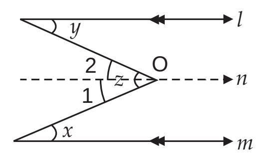
$\therefore$ $\angle XYB = \angle CYB$ … (2)

From (1) and (2), we get

$\angle CBY = \angle CYB$

Hence, **∠CBY = ∠CYB.**

14. Draw $n \parallel l$ or m though O.



$\because$ $n \parallel l$

$\therefore$ $\angle 2 = y$ [Alt. ∠s] … (1)

Also, $m \parallel n$.

$\therefore$ $\angle 1 = x$ [Alt. ∠s] … (2)

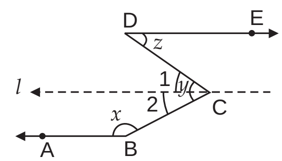
Adding (1) and (2),

$\angle 2 + \angle 1 = \angle y + \angle x$

$\Rightarrow$ $z = x + y$

Hence, $\boldsymbol{x + y = z}$.

15. Though C, draw $l \parallel$ DE or AB.



$\Rightarrow$ $\angle 1 = z$ [Alt. angles, $l \parallel$ DE] … (1)

and $\angle 2 + x = 180°$ [Cointerior angles, $l \parallel$ AB]

$\Rightarrow$ $\angle 2 = 180° - x$ … (2)

Adding (2) and (1), we get

$\angle 2 + \angle 1 = 180° - x + z$

$\Rightarrow$ $y = 180° - x + z$

$\Rightarrow$ $x + z = 180° + z$

Hence, $\boldsymbol{x + z = 180° + z}$.

EXERCISE 6D

1. Let the third angle of the triangle be $x°$.

$\therefore$ $41° + 48° + x° = 180°$ [sum of ∠s of a Δ]

$\Rightarrow$ $x = 180° - 41° - 48°$

$\Rightarrow$ $x = 91°$

The required third angle is **91°.**

2. Let the smaller acute angle of the given right triangle be x.

Then, the other acute angle = $2x$

$\therefore$ $x + 2x + 90° = 180°$ [sum of ∠s of a Δ]

$\therefore$ $x + 2x = 90°$

$\Rightarrow$ $3x = 90°$

$\Rightarrow$ $x = 30°$

$\therefore$ $2x = 2 \times 30 = 60°$

Thus, the acute angles are **30°** and **60°.**

3. Let the angles be $5x°$, $6x°$ and $7x°$.

$\therefore$ $5x + 6x + 7x = 180°$

$\Rightarrow$ $18x = 180°$

$\Rightarrow$ $x = 10°$

$\therefore$ The angles are $5 \times x° = 5 \times 10° = 50°$

$\therefore$ $6 \times x° = 6 \times 10° = 60°$

$\therefore$ $7 \times x° = 7 \times 10° = 70°$

Thus the required angles are **50°, 60°, 70°.**

4. The angles are: $(x + 10)^\circ$, $(x + 40)^\circ$ and $(2x - 30)^\circ$.

$\therefore \quad x + 10 + x + 40 + 2x - 30 = 180$ [sum of $\angle$s of a Δ]

$\Rightarrow \quad (x + x + 2x) + 10 + 40 - 30 = 180$

$\Rightarrow \quad 4x + 20 = 180$

$\Rightarrow \quad 4x = 180 - 20 = 160$

$\Rightarrow \quad x = \dfrac{160}{4} = 40$

The angles are

$(x + 10)^\circ = (40 + 10)^\circ = 50^\circ$

$(x + 40)^\circ = (40 + 40)^\circ = 80^\circ$

and $(2x - 30)^\circ = (2 \times 40 - 30)^\circ = 50^\circ$

As the two angles are equal,

$\therefore$ the special name of the triangle is isosceles triangle.

Hence, $\boldsymbol{x = 40}$ and the triangle is **isosceles** triangle.

5. Let each of the equal angles be x.

$\Rightarrow$ Then, m (vertex angles) $= x + 30^\circ$

$\therefore \quad x + x + x + 30^\circ = 180^\circ$ [sum of $\angle$s of a Δ]

$\Rightarrow \quad 3x = 180^\circ - 30^\circ$

$= 150^\circ$

$\Rightarrow \quad x = 50^\circ$

$\therefore$ Angles are $(50 + 30)^\circ, 50^\circ, 50^\circ$ i.e., $80^\circ, 50^\circ, 50^\circ$

Thus, the measures of three angles are **80°, 50°, 50°.**

6. One angle $= 65^\circ$

Let the other two angles be $(x - 20)^\circ$ and x°.

$\therefore \quad 65 + (x - 20) + x = 180^\circ$

$\Rightarrow \quad 2x + 45 = 180$

$\Rightarrow \quad 2x = 180 - 45 = 135$

$\Rightarrow \quad x = \dfrac{135}{2} = 67.5$

$\therefore$ The other two angles are $(67.5^\circ - 20)^\circ$ and 67.5° i.e. 47.5° and 67.5°

Hence, the other two angles are **47.5°, 67.5°.**

7. Let one of the angle be x.

$\therefore \quad$ Other angle $= (80^\circ - x)$

Difference of the two angles $= 20^\circ$

$\therefore \quad (80^\circ - x) - x = 20^\circ$

$\Rightarrow \quad 80^\circ - 2x = 20^\circ$

$\Rightarrow \quad 2x = 80^\circ - 20^\circ = 60^\circ$

$\Rightarrow \quad x = \dfrac{60^\circ}{2} = 30^\circ$

$\therefore$ The two angles are: 30°, $(80 - x)^\circ$ i.e. 30° and 50°

Thus, the third angle $= 180^\circ - (30 + 50)^\circ = 100^\circ$ [Sum of $\angle s$ of a Δ]

Hence, the angles of the Δ are **30°, 50°, 100°.**

8. $\angle A - \angle B = 15^\circ$

$\Rightarrow \quad \angle A = 15^\circ + \angle B \quad \ldots (1)$

$\angle B - \angle C = 30^\circ$

$\Rightarrow \quad \angle C = \angle B - 30^\circ \quad \ldots (2)$

and $\quad \angle A + \angle B + \angle C = 180^\circ$ [Sum of $\angle$s of a Δ]

$\therefore \quad (15^\circ + \angle B) + \angle B + (\angle B - 30^\circ) = 180^\circ$

$\Rightarrow \quad 3\angle B - 15^\circ = 180^\circ$

$\Rightarrow \quad 3\angle B = 180^\circ + 15^\circ = 195^\circ$

$\Rightarrow \quad \angle B = \dfrac{195^\circ}{3} = 65^\circ$

$\angle A = 15^\circ + \angle B$

$= 15^\circ + 65^\circ$

$= 80^\circ$

$\angle C = \angle B - 30^\circ$

$= 65^\circ - 30^\circ$

$= 35^\circ$

Thus, the angles of the triangle are: **35°, 65°, 80°.**

9. $\angle A + \angle B = 122^\circ$

$\Rightarrow \quad \angle A = 122^\circ - \angle B \quad \ldots (1)$

$\angle B + \angle C = 111^\circ$

$\Rightarrow \quad \angle C = 111^\circ - \angle B \quad \ldots (2)$

Now, $\quad \angle A + \angle B + \angle C = 180^\circ$ [Sum of $\angle$s of a Δ]

$\Rightarrow \quad 122^\circ - \angle B + \angle B - 111^\circ - \angle B = 180^\circ$

$\Rightarrow \quad \angle B + 233^\circ = 180^\circ$

$\Rightarrow \quad \angle B = 233^\circ - 180^\circ$

$\Rightarrow \quad \angle B = 53^\circ$

From (2),

$\angle C = 111^\circ - 53^\circ = 58^\circ$

Thus, $\angle$**B = 53°**, $\angle$**C = 58°.**

10. Each angle of the Δ < (sum of the other two $\angle$ of the Δ)

i.e. $\angle A < (\angle B + \angle C)$

$\Rightarrow \quad (\angle A + \angle A) < (\angle A + \angle B + \angle C)$

$\Rightarrow \quad 2\angle A < (180^\circ)$

$\Rightarrow \quad 2\angle A < (2 \times 90^\circ)$

$\Rightarrow \quad \angle A < 90^\circ$

$\Rightarrow \quad \angle A$ is an acute angle.

Similarly, $\quad \angle B < \angle A + \angle C$

$\Rightarrow \quad \angle B < 90^\circ$

i.e. $\angle B$ is an acute angle.

and $\quad \angle C < \angle A + \angle B$

$\Rightarrow \quad \angle C < 90^\circ$

i.e. $\angle C$ is an acute angle.

$\therefore$ Each angle of the Δ is acute.

$\therefore$ The given Δ is an acute Δ.

11. [One angle of the triangle] > [sum of other two angles]

Let $\quad \angle A > \angle B + \angle C$

$\Rightarrow \quad (\angle A + \angle A) > (\angle B + \angle C) + \angle A$

$\Rightarrow \quad 2\angle A > \angle A + \angle B + \angle C$

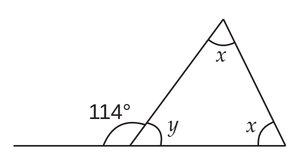
$\Rightarrow \quad 2\angle A > 180^\circ$

$\Rightarrow \quad \angle A > 90^\circ$

$\Rightarrow \quad \angle A$ is an obtuse angle.

$\Rightarrow \quad \Delta ABC$ is an obtuse angle.

12. (*i*)



$x + x = 144^\circ$

[Ext. $\angle$ = sum of int. opp. $\angle$s]

$\Rightarrow \quad 2x = 144^\circ$

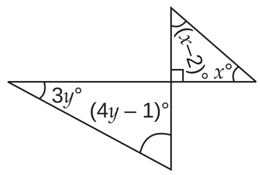
$\Rightarrow \quad x = \dfrac{114^\circ}{2} = 57^\circ$

Also, $\quad 144^\circ + y = 180^\circ$ [Linear pair]

$\Rightarrow \quad y = 180^\circ - 144^\circ = 66^\circ$

Thus, $\boldsymbol{x = 57^\circ, y = 66^\circ}$.

(*ii*)



$$(x - 2)^\circ + x^\circ + 90^\circ = 180^\circ \quad \text{[Sum of } \angle\text{s of a } \Delta]$$

$\Rightarrow \quad (x - 2) + x = 90$

$\Rightarrow \quad 2x = 90 + 2 = 92$

$\Rightarrow \quad x = \dfrac{92}{2} = 46$

Also, $(3y)^\circ + (4y - 1)^\circ + 90^\circ = 180^\circ$

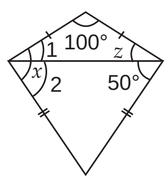
Also, $(3y) + (4y - 1) = 90$

$\Rightarrow \quad 7y = 90 + 1 = 91$

$\Rightarrow \quad y = \dfrac{91}{7} = 13$

Thus, $\mathbf{x = 46, y = 13}$.

(*iii*)



$\angle 1 + \angle 2 + 100^\circ = 180^\circ \quad$ [Sum of $\angle$s of a Δ]

$\Rightarrow \quad 2\angle 1 = 180^\circ$

$\quad$ [$\because \angle 1 = \angle 2$, $\angle$s opp. equal sides of a Δ]

$\Rightarrow \quad \angle 1 = 40^\circ \quad \ldots (1)$

Also, $\quad \angle 2 = 50^\circ$

$\quad$ [$\angle$s opp. equal sides of a Δ] … (2)

Adding (1) and (2), we get

$\angle 1 + \angle 2 = 40^\circ + 50^\circ$

$\Rightarrow \quad x = 90^\circ$

Hence, $\mathbf{x = 90^\circ}$.

13. (*i*) $\angle BAC + \angle ACD = 180^\circ \quad$ [Coint $\angle$s, AB $\parallel$ CD]

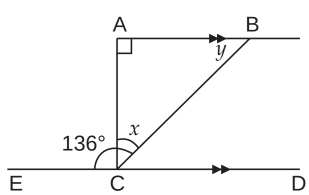
$\Rightarrow \quad 90^\circ + \angle ACD = 180^\circ$

$\Rightarrow \quad \angle ACD = 90^\circ$

$\Rightarrow \quad AC \perp CD$

$x + 90^\circ = 136^\circ$

$\Rightarrow \quad x = 136^\circ - 90^\circ = 46^\circ$



Also, $\quad x + y + 90^\circ = 180^\circ \quad$ [Sum of $\angle$s of a Δ]

$x + y = 90^\circ$

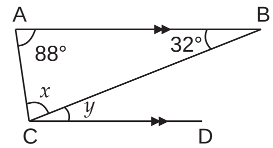
$\Rightarrow \quad 46^\circ + y = 90^\circ$

$\Rightarrow \quad y = 90^\circ - 46^\circ$

$y = 44^\circ$

Thus, $\mathbf{x = 46^\circ, y = 44^\circ}$.

(*ii*) AB $\parallel$ CD and BC is transversal.



$\therefore \quad y = 32^\circ \quad$ [Alt. angles]

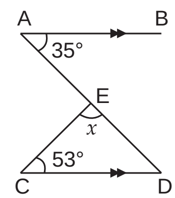
In ΔABC,

$x = 180^\circ - (88 + 32)^\circ \quad$ [Sum of $\angle$s of a Δ]

$= 180^\circ - 120^\circ = 60^\circ$

Thus, $\mathbf{x = 60^\circ, y = 32^\circ}$.

(*iii*) AB $\parallel$ CD and AD is a transversal.



$\therefore \quad \angle D = 35^\circ \quad$ [Alt. angles]

$53^\circ + 35^\circ + x^\circ = 180^\circ \quad$ [Sum of angles of a Δ]

$\Rightarrow \quad x = 180^\circ - 53^\circ - 35^\circ = 92^\circ$

Thus, $\mathbf{x = 92^\circ}$.

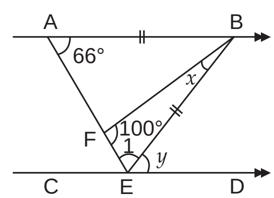
(*iv*) $\quad \angle 1 = 66^\circ$

$\quad$ [$\angle$s opp. equal sides of ΔABE] … (1)

$66^\circ + \angle 1 + y = 180^\circ \quad$ [coint $\angle$s, AB $\parallel$ CD]

$\Rightarrow \quad 66^\circ + 66^\circ + y = 180^\circ \quad$ [Using (1)]

$\Rightarrow \quad y = 180^\circ - 132^\circ = 48^\circ$



In ΔBEF,

$x + \angle 1 + 100^\circ = 180^\circ \quad$ (sum of $\angle$s of a Δ)

$x = 180^\circ - 100^\circ - \angle 1$

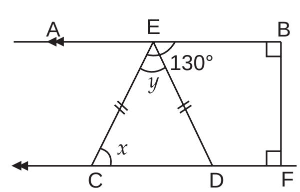
$= 180^\circ - 66^\circ$

$= 14^\circ$

Thus, $\mathbf{x = 14^\circ, y = 48^\circ}$.

(*v*) $\quad 130^\circ + x = 180^\circ \quad$ [Coint $\angle$s, AB $\parallel$ CD]

$\Rightarrow \quad x = 50^\circ$



In ΔEDC, $\quad EC = ED$

$\angle D = \angle x$

$\therefore \quad \angle x + \angle y + \angle D = 180^\circ \quad$ [Sum of $\angle$s of a Δ]

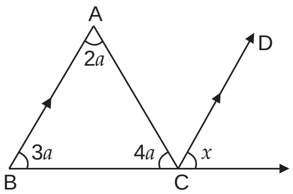
$\Rightarrow \quad 50^\circ + y + 50^\circ = 180^\circ$

$\Rightarrow \quad y + 100° = 180°$

$\Rightarrow \quad y = 180° - 100° = 80°$

Thus, $\boldsymbol{x = 50°, y = 80°}$

(*vi*)



In ΔABC,

$2a + 3a + 4a = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow \quad 9a = 180°$

$\Rightarrow \quad a = \frac{180°}{9} = 20°$

Now, AB $\parallel$ CD and BC is a transversal.

$\therefore \quad x = 3a$ [corr. angles]

$\Rightarrow \quad x = 3 \times 20° = 60°$

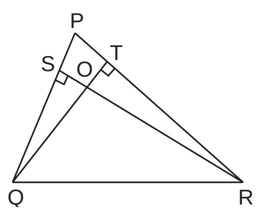
Thus, $\boldsymbol{x = 60°}$.

14. In rt. ΔRSQ,

$\angle SRQ + \angle Q + 90° = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow \quad \angle SRQ = 180° - 90° - \angle Q$

$\Rightarrow \quad \angle SRQ = 90° - \angle Q$ ... (1)



Similarly, in rt ΔRTQ,

$\Rightarrow \quad \angle TQR = 90° - \angle R$ ... (2)

Now, in a ΔROQ,

$\angle ORQ + \angle OQR + \angle QOR = 180°$ [Sum of $\angle$s of a Δ]

$\angle SRQ + \angle TQR + \angle QOR = 180°$

$90° - \angle Q + 90° - \angle R + \angle QOR = 180°$ [Using (1) and (2)]

$\angle QOR = \angle Q + \angle R$

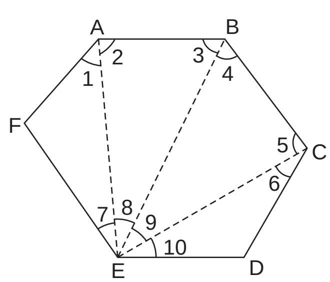
$\angle QOR = 180° - \angle P$

[$\angle P + \angle Q + \angle R = 180° \Rightarrow \angle Q + \angle R = 180° - \angle P$]

Thus, $\boldsymbol{\angle QOR = 180° - \angle P}$.

15. Let ABCDEF is a hexagon.

Join AE, BE and CE such that 4 Δs are formed.



In ΔAEF, sum of its angles = 180°.

$\therefore \quad \angle F + \angle 1 + \angle 7 = 180°$ ... (1)

Similarly, in ΔABE,

$\angle 2 + \angle 3 + \angle 8 = 180°$ ... (2)

In ΔBCE,

$\angle 4 + \angle 5 + \angle 9 = 180°$ ... (3)

In ΔCDE,

$\angle 6 + \angle D + \angle 10 = 180°$ ... (4)

Adding (1), (2), (3) and (4), we get

$(\angle F + \angle 1 + \angle 7) + (\angle 2 + \angle 3 + \angle 8) + (\angle 4 + \angle 5 + \angle 9) + (\angle 6 + \angle D + \angle 10)$

$= 180° + 180° + 180° + 180°$

$\Rightarrow \quad \angle F + (\angle 1 + \angle 2) + (\angle 3 + \angle 4) + (\angle 5 + \angle 6) + \angle D + (\angle 7 + \angle 8 + \angle 9 + \angle 10)$

$= 720°$

$\Rightarrow \quad \angle F + \angle A + \angle B + \angle C + \angle D + \angle E = 720°$

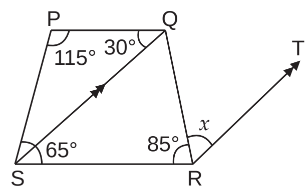
$\Rightarrow$ Sum of angles of hexagon ABCDEF = 720°

Thus, sum of angles of hexagon is 720°.

16. In ΔPQS,

$\angle PSQ = 180° - (115° + 30°) = 35°$ [sum of $\angle$s of a Δ]

$\therefore \quad \angle QSR = 65° - 35° = 30°$



$\because$ SQ $\parallel$ RT and SR is a transversal

$\therefore \quad \angle QSR + \angle SRT = 180°$ [Cointerior angles]

$\Rightarrow \quad 30° + \angle SRT = 180°$

$\angle SRT = 180° - 30° = 150°$

$\Rightarrow \quad 85° + x = 150°$

$\Rightarrow \quad x = 150° - 85° = 65°$

Thus, $\boldsymbol{x = 65°}$.

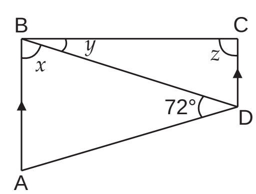
Now, $\angle P + \angle S = 115° + 65° = 180°$

But they are cointerior angles.

$\Rightarrow \quad$ SR $\parallel$ PQ

$\therefore$ PQRS is a **trapezium.**

17. AB $\parallel$ CD and BC is a transversal.



$\therefore \quad (x + y) + z = 180°$ [Cointerior $\angle$s]

$\Rightarrow \quad \left(\frac{4}{3}y + y\right) + z = 180°$

$\Rightarrow \quad \frac{7}{3}y + z = 180°$

$\Rightarrow \quad \frac{7}{3} \times \frac{3}{8}z + z = 180°$ [$\because y = \frac{3}{8}z$, given]

$\Rightarrow \quad \frac{7}{8}z + z = 180°$

$\Rightarrow \quad \frac{15}{8}z = 180°$

$\Rightarrow \quad z = 96°$

$\Rightarrow \quad \angle BCD = 96°$

Since $x = \frac{4}{3}y$

$= \frac{4}{3} \times \frac{3}{8}z$

$= \frac{z}{2}$

$= \frac{96°}{2}$

$= 48°$

and $y = \frac{3}{8}z$

$= \frac{3}{8} \times 96°$

$= 3 \times 12°$

$= 36°$

Now, $\angle ABC = x + y = 48° + 36° = 84°$

In ΔABD,

$x + 72° + \angle BAD = 180°$ [Sum of $\angle$s of a Δ]

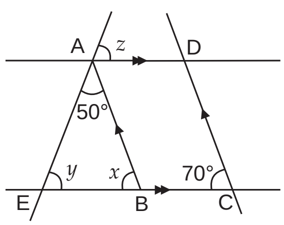
$48° + 72° + \angle BAD = 180°$

$\Rightarrow$ $\angle BAD = 180° - 48° - 72° = 60°$

Thus, **$\angle BCD = 96°$, $\angle ABC = 84°$ and $\angle BAD = 60°$.**

18. ABCD is a $\parallel$gm.

$\Rightarrow$ AB $\parallel$ CD and BC is a transversal.



$\therefore$ $x = 70°$ [Corr. angles]

In ΔABE, $x + y + 50° = 180°$ [Sum of $\angle$s of ΔABE]

$\Rightarrow$ $70° + y + 50° = 180°$

or $y = 180° - 70° - 50° = 60°$

Now, AD $\parallel$ EC and AE is a transversal.

$\therefore$ $z = y = 60°$ [Corr. $\angle$s]

Thus, **$x = 70°$, $y = 60°$, $x = 60°$.**

19. In ΔABC,

$\because$ $\angle B = 45°$

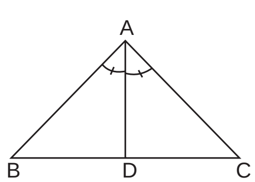
and $\angle C = 55°$

$\therefore$ $\angle BAC = 180° - (\angle B + \angle C)$

[Sum of $\angle$s of a Δ]

$= 180° - (45° + 55°)$

$= 80°$



But AD is the bisector of $\angle BAC$.

$\therefore$ $\angle BAD = \frac{80°}{2} = 40°$

and $\angle CAD = \frac{80°}{2} = 40°$

Ext. $\angle ADB = \angle CAD + \angle C$

[Ext. $\angle$ = sum of int. opp. $\angle$s]

$= 40° + 55°$

$= 95°$

Ext. $\angle ADC = \angle BAD + \angle B$

[Ext. $\angle$ = sum of int. opp. $\angle$s]

$= 40° + 45°$

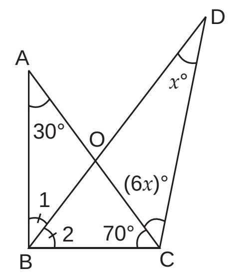
$= 85°$

Thus, **$\angle ADB = 95°$, $\angle ADC = 85°$.**

20. In ΔABC, $\angle B = 180° - (20° + 70°)$

[Sum of $\angle$s of a Δ]

$= 180° - 100° = 80°$



BD is bisector of $\angle B$.

$\therefore$ $\angle 1 = 40°$ and $\angle 2 = 40°$

Now, in ΔBCD,

$\angle BCD + \angle D + \angle DBC = 180°$ [Sum of angles of a Δ]

$\Rightarrow [70° + (6x)°] + [x°] + \angle 2 = 180°$

$\Rightarrow$ $70° + 6x° + x° + 40° = 180°$

$\Rightarrow$ $(7x) = 180 - 70 - 40 = 70$

$\Rightarrow$ $x = \frac{70}{7} = 10$

Thus, **$x = 10$.**

21. In ΔXYZ,

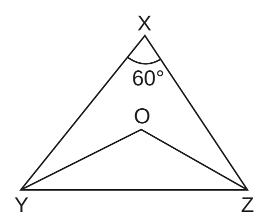
$\angle X + \angle XYZ + \angle XZY = 180°$

[$\angle$s sum property of a Δ]

$\Rightarrow$ $60° + 54° + \angle XZY = 180°$

$\Rightarrow$ $\angle XZY = 180° - [60° + 54°]$

$= 66°$ ... (1)



ZO and YO are the bisectors of $\angle XZY$ and $\angle XYZ$ respectively.

$\therefore$ $\angle OZY = \frac{1}{2}(\angle XZY)$

$= \frac{1}{2} \times 66°$

$= 33°$ [Using (1)] ... (2)

$\angle OYZ = \frac{1}{2}(\angle XYZ)$

$= \frac{1}{2} \times 54°$

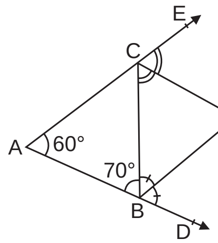
$= 27°$

Now, $\angle YOZ = 180° - (\angle OZY + OYZ)$
[Sum of ∠s of a Δ]
$= 180° - (33° + 27°)$ [Using 2]
$= 120°$

Thus, **∠OZY = 33°, ∠YOZ = 120°.**

22. In ΔABC,

$\angle ACB = 180° - [60° + 70°]$
[Sum of ∠s of a Δ]
$= 180° - 130°$
$= 50°$



Now, $\angle ACB + \angle BCE = 180°$ [Linear pair]
$\Rightarrow$ $50° + \angle BCE = 180°$
$\Rightarrow$ $\angle BCE = 130°$

Since CO is bisector of ∠BCE,

$\therefore$ $\angle BCO = \dfrac{130°}{2} = 65°$ … (1)

Now, $\angle CBD + 70° = 180°$ [Linear pair]
$\Rightarrow$ $\angle CBD = 180° - 70°$ [Linear pair]
$= 110°$

$\Rightarrow$ $\angle CBO = \dfrac{110°}{2} = 55°$

[BO is bisector of ∠CBD] … (2)

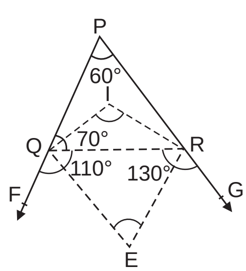
Now, in ΔBOC, we have

$\angle BCO + \angle CBO + \angle COB = 180°$
$\Rightarrow$ $65° + 55° + \angle BOC = 180°$ [Using (1) and (2)]
$\Rightarrow$ $\angle BOC = 180° - (65° + 55°)$
$= 180° - 120°$
$= 60°$

Thus, **∠BOC = 60°.**

23. (*i*) In ΔPQR, ∠P = 60°, ∠Q = 70°

$\therefore$ $\angle R = 180° - [\angle P + \angle Q]$
[Sum of ∠s of a Δ]
$= 180° - [60 + 70]$
$= 50°$



QI is bisectors ∠Q.

$\therefore$ $\angle IQR = \dfrac{1}{2}\angle Q$

$= \dfrac{1}{2}(70°)$

$= 35°$ … (1)

$\angle RQF + 70° = 180°$ [Linear pair]
$\Rightarrow$ $\angle RQF = 180° - 70° = 110°$

$\therefore$ $\angle EQR = \dfrac{1}{2}\angle RQF$

$= \dfrac{1}{2}(110°)$

$= 55°$ … (2)

[∵ QE is the bisector of ∠RQF]

Adding (1) and (2), we get

$\angle IQR + \angle EQR = 35° + 55° = 90°$

Thus, **∠IQR + ∠EQR = 90°.**

(*ii*) In ΔQIR, $\angle QIR = 180° - \left[\dfrac{1}{2}(70°) + \dfrac{1}{2}(50)°\right]$

[Sum of ∠s of a Δ and QI and RI are bisectors of ∠Q and ∠R respectively]

$= 180° - [35° + 25°]$
$= 180° - [60°]$
$= 120°$

In ΔQER, $\angle QER = \left[180° - \left\{\dfrac{1}{2}(100°) + \dfrac{1}{2}(130°)\right\}\right]$

[Sum of ∠s of a Δ and QE and RE are bisectors of ∠RQF and ∠QRG respectively]

$= 180 - (55° + 65°)]$
$= 180° - 120°$
$= 60°$

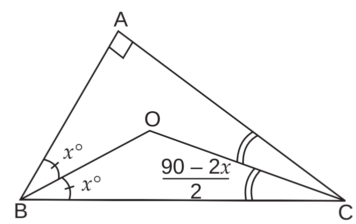
Now, $\angle QIR + \angle QER = 120° + 60° = 180°$

Thus, **∠QIR + ∠QER = 180°.**

24. In rt. ΔABC, ∠A = 90°.

Let the acute angle $\angle B = 2x°$.

BO is the bisector of ∠B.



$\Rightarrow$ $\angle OB = x°$ … (1)

$\angle A + \angle B + \angle C = 180°$ [Sum of ∠s of a Δ]
$\Rightarrow$ $90° + 2x + \angle C = 180°$
$\angle C = 180° - 90° - 2x$
$= 90° - 2x$

$\angle OCB = \dfrac{90^o - 2x}{2}$

[∵ CO is the bisector ∠C] … (2)

Now, in ΔBOC, we have

$\angle BOC + \angle OBC + \angle OCB = 180°$

[Sum of the angles of a Δ]

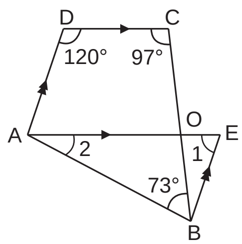
$\angle BOC + x + \dfrac{90° - 2x}{2} = 180°$ [Using (1) and (2)]

$\Rightarrow$ $\angle BOC = 180° - x - \dfrac{90° - 2x}{2}$

$= 180° - x - (45° - x)$
$= 180° - 45° - x + x$
$= 135°$

Thus the angle between the bisectors of two acute angles of a rt. Δ is 135°.

25. AD ∥ BE and AB is a traversal.



$\therefore \quad \angle DAE = \angle 1 \quad$ [Alt. angles] … (1)

DC ∥ AE and AD is a transversal.

$\angle DAE + 120° = 180° \quad$ [Cointerior angles]

$\Rightarrow \quad \angle DAE = 180° - 120°$

$= 60° \quad$ … (2)

From (1) and (2),

$\angle 1 = 60°$

ABCD is a quadrilateral.

$\therefore \quad \angle BAD = 360° - [120° + 97° + 73°]$

[Sum of ∠s of quadrilateral is 360°]

$\Rightarrow \quad \angle 2 = \angle BAD - \angle DAE$

$= 70° - 60° \quad$ [Using (3) and (2)]

$= 10°$

In ΔABE,

$\angle ABE = 180° - [10° + 60°]$

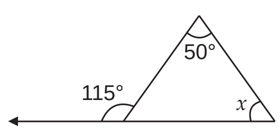
$= 110°$

[Sum of ∠s of a Δ]

⇒Angles of ΔABE are: **10°, 110°, 60°.**

EXERCISE 6E

1. (*i*) Ext. ∠ = 115°



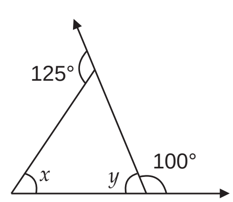
$\Rightarrow \quad x + 50° = 115°$

[Sum of int. opp ∠s = Ext. ∠]

$\Rightarrow \quad x = 115° - 50° = 65°$

Hence, $\boldsymbol{x = 65°}$.

(*ii*) $y + 100° = 180° \quad$ [Linear pair]



$\Rightarrow \quad y = 180° - 100° = 80°$

$x + y = 125°$

[Sum of int. opp. ∠s = Exterior ∠]

$x + 80° = 125°$

$\Rightarrow \quad x = 125° - 80° = 45°$

Thus, $\boldsymbol{x = 45°}$.

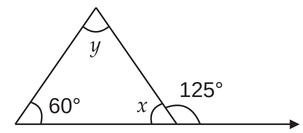
(*iii*) $x + 125° = 180° \quad$ [Linear pair]

$x = 180° - 125° = 55°$

$60° + y = 125°$

[Sum of int. opp. ∠s = Ext. ∠]

$\Rightarrow \quad y = 125° - 60° = 65°$



Thus, $\boldsymbol{x = 55°, y = 65°}$.

(*iv*) $y + y = 116°$

[Sum of int. opp ∠s = Ext. ∠]

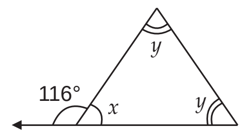
$\Rightarrow \quad 2y = 116°$

$\Rightarrow \quad y = 58°$

$x + 116° = 180° \quad$ [Linear pair]

$\Rightarrow \quad x = 180° - 116°$

$= 64°$



Thus, $\boldsymbol{x = 64°, y = 58°}$.

(*v*) $x + 130° = 180° \quad$ [Linear pair]

$\therefore \quad x = 180° - 130°$

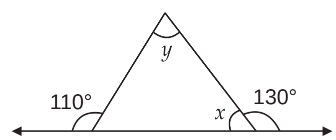
$= 50° \quad$ … (1)

$\therefore \quad x + y = 110°$

[Sum of int. opp. ∠s = Ext. ∠]

$\Rightarrow \quad 50° + y = 110° \quad$ [Using (1)]

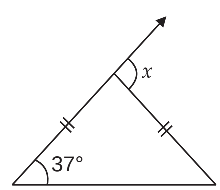
$\Rightarrow \quad y = 110° - 50° = 60°$



Thus, $\boldsymbol{x = 50°, y = 60°}$.

(*vi*) Ext. ∠ = x

The two opp. (interior) angles are equal.



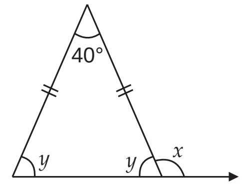
∵ ∠s opp. equal sides of a Δ are equal.

$\therefore \quad 37° + 37° = x$

[Sum of int. opp. ∠s = Ext. ∠]

Hence, $\boldsymbol{x = 74°}$.

(*vii*) Let each base ∠ of isosceles Δ be y.



Then, $40° + y + y = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow 2y = 140°$

$\Rightarrow y = 70°$

$x + y = 180°$ [Linear pair]

$\Rightarrow x + 70° = 180°$

Hence, $\boldsymbol{x = 110°}$.

(*viii*) $\angle 1 = 180° - (90° + 50)$

$= 40°$ [Sum of ∠s of a Δ]

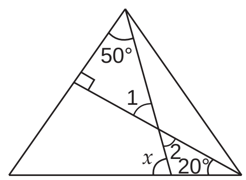
$\because \angle 1 = \angle 2$ [Ver. opp. angles]

$\Rightarrow \angle 2 = 40°$

Ext. $x = \angle 2 + 20°$

[Ext. ∠ = sum of int. opp. ∠s a Δ]

$\Rightarrow x = 40° + 20° = 60°$



Thus, $\boldsymbol{x = 60°}$.

(*ix*) $\angle 1 + 110° = 180°$ [Linear Pair]

$\Rightarrow \angle 1 = 180° - 110°$

$= 70°$

Similarly, $\angle 2 = 180° - 165°$

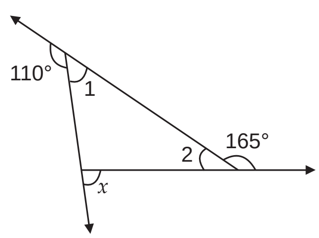
$= 15°$

Ext. $x = \angle 1 + \angle 2$

[Ext. ∠ = sum of int. opp. ∠s]

$\Rightarrow x = 70° + 15°$

$= 85°$



Thus $\boldsymbol{x = 85°}$.

2. (*i*) $x° + (x + 10)° = 130°$

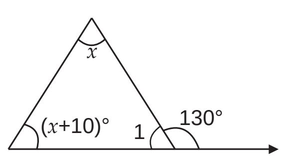
[Ext. ∠ = sum of int. opp. ∠s]

$\Rightarrow 2x = 120°$

$\Rightarrow x = 60°$

$\angle 1 + 130° = 180°$ [Linear pair]

$\Rightarrow \angle 1 = 50°$

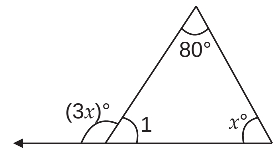


Angles of the triangle are $x° = 60°$, $(x + 10)°$ $= (60 + 10)° = 70°$ and $50°$.

Hence, angle of the triangle are **60°, 70°, 50°.**

(*ii*) $\angle 1 + 3x° = 180°$ [Linear Pair]

$\angle 1 = 180° - (3x°)$ … (1)



Now, we have

$\angle 1 + 80° + x° = 180°$ [Sum of ∠s of a Δ]

$\therefore 180° - (3x)° + 80° + x° = 180°$ [Using (1)]

$\Rightarrow -2x = -80$

$\Rightarrow x = \dfrac{-80}{-2} = 40$

$\therefore \angle 1 = 180° - (3x)°$

$= 180° - (3 \times 40°)$

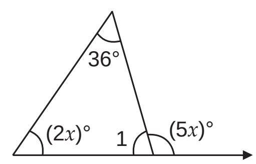
$= 180° - 120° = 60°$

$x° = 40°$

$\therefore$ The angles of the triangle are **40°, 80°, 60°.**

(*iii*) $\angle 1 + 5x = 180°$ [Linear Pair]

$\Rightarrow \angle 1 = 180° - (5x)°$



Now,

$36° + (2x)° + \angle 1 = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow 36 + (24) + 180 - (5x) = 180$

$\Rightarrow -3x = 180° - 36° - 180°$

$\Rightarrow -3x = -36°$

or $x = 12°$

$\therefore 2x = 3 \times 12$

$= 24°$

Now, $\angle 1 = 180 - (5x)°$

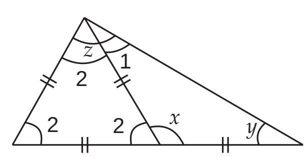
$= 180° - (5 \times 12)°$

$= 180° - 60°$

$= 120°$

Thus, the angles of the triangle are **36°, 24°, 120°.**

3. (*i*)



$\angle 1 = y$

[∠s opp. equal sides of a Δ] … (1)

$x = \angle 2 + \angle 2$

[Ext. ∠ = Sum of int. opp. ∠s]

$\Rightarrow x = 2\angle 2 = 2 \times 60°$

[∵ ∠2 = 60°, angle of can equilateral Δ]

$\Rightarrow x = 120°$ … (2)

Also, $\angle 2 = \angle 1 + y$

[Ext. ∠ = sum of int. opp. ∠s]

$\Rightarrow \angle 2 = y + y$ [Using (1)]

$\Rightarrow \quad 60° = 2y$

[$\because \angle 2 = 60°$ angle of an equilateral Δ]

$\Rightarrow \quad y = 30°$

$z = \angle 1 + \angle 2$

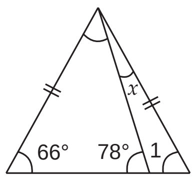
$= y + \angle 2$ [Using (1)]

$= 30° + 60°$

$= 90°$

Hence, $\boldsymbol{x = 120°, y = 30°, z = 90°}$.

(*ii*)



$\angle 1 = 66°$ [$\angle$s opp. equal sides]

$x + \angle 1 = 78°$

$\Rightarrow \quad x + 66° = 78°$

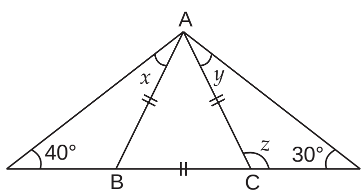
[Ext. $\angle$ = sum of int. opp. $\angle$s]

$\Rightarrow \quad x = 78° - 66°$

$= 12°$

Hence, $\boldsymbol{x = 12°}$.

(*iii*)



ABC is an equilateral triangle.

$\therefore \quad \angle A = \angle B = \angle C = 60°$

$\therefore \quad 30° + y = 60°$

[Sum of int. opp. $\angle$s = Ext. $\angle$]

$\Rightarrow \quad y = 60° - 30°$

$= 30°$

and $\quad 40° + x = 60°$

[Sum of int. opp. $\angle$s = Ext. $\angle$]

$x = 60° - 40° = 20°$

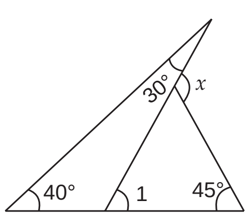
$z + 60° = 180°$ [Linear Pair]

$\Rightarrow \quad z = 180° - 60°$

$= 120°$

Thus, $\boldsymbol{x = 20°, y = 30°, z = 120°}$.

(*iv*)



Ext. $\angle 1 = 30° + 40° = 70°$

[Ext. $\angle$ = Sum of int. opp. $\angle$s] … (1)

Ext. $\angle x = \angle 1 + 45°$

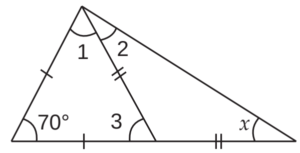
[Ext. $\angle$= sum of int. opp. $\angle$s]

$= 70° + 45°$

$= 115°$ [Using (1)]

Thus, $\boldsymbol{x = 115°}$.

(*v*)



$\angle 1 = \angle 3$

[$\angle$s opp. equal sides of a Δ]… (1)

$\angle 1 + \angle 3 + 70° = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow \quad 2\angle 1 + 70° = 180°$ [Using (1)]

$\Rightarrow \quad \angle 1 = \dfrac{180° - 70°}{2}$

$= 55°$ … (2)

$\angle 2 = x$

[$\angle$s opp. to equal sides] … (3)

Now, $70° + (\angle 1 + \angle 2) + x = 180°$

[sum of angles of a Δ]

$\Rightarrow \quad 70° + 55° + x + x = 180°$ [Using (2) and (3)]

$\Rightarrow \quad 2x = 180° - 70° - 55°$

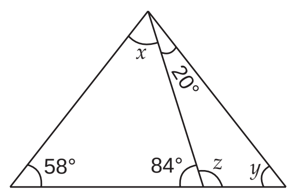
$= 55°$

$\Rightarrow \quad x = \dfrac{55^o}{2}$

$= 27.5°$

Thus $\boldsymbol{x = 27.5°}$.

(*vi*)



$x + 58° + 84° = 180°$ [Sum of angles of a Δ]

$\Rightarrow \quad x = 180° - 58° - 84°$

$= 38°$

Also, $\quad 84° + z = 180°$ [Linear pair]

$\Rightarrow \quad z = 180° - 84° = 96°$

$20° + y = 84°$

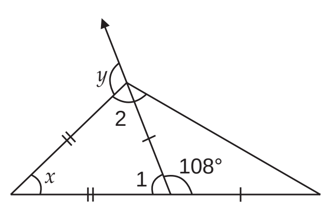
[Sum of int. opp. $\angle$s = Ext. $\angle$]

$\Rightarrow \quad y = 84° - 20°$

$= 64°$

Thus, $\boldsymbol{x = 38°, y = 64°, z = 96°}$

(*vii*)



$\angle 1 + 108° = 180°$ [Linear Pair]

$\Rightarrow \quad \angle 1 = 180° - 108° = 72°$

$\angle 2 = \angle 1 = 72°$

[$\angle$s opp. to equal sides of a Δ]

$\therefore \quad x + \angle 1 + \angle 2 = 180°$

$\Rightarrow \quad x + 72° + 72° = 180°$

$\Rightarrow \quad x = 180° - 144° = 36°$

$y = x + \angle 1$

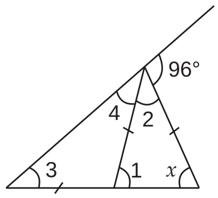
[Ext. $\angle$ = sum of int. opp. $\angle$s]

$= 36° + 72°$

$= 108°$

Thus $\boldsymbol{x = 36°}$ and $\boldsymbol{y = 108°}$.

(*viii*)



$\angle 1 = x$ [$\angle$s opp. equal sides of a Δ]... (1)

$\therefore \quad \angle 2 = 180° - \angle 1 - x$ [Sum of $\angle$s of a Δ]

$= (180° - 2x)$ [Using (1)] ... (2)

$\angle 1 = \angle 3 + \angle 4$ [Ext. $\angle$ = sum of int. opp. $\angle$s]

$\Rightarrow \quad x = 2\angle 4$ [$\because$ $\angle 3 = \angle 4$ and using (1)]

$\angle 4 = \dfrac{x}{2}$ ... (3)

Now, $\angle 4 + \angle 2 + 96° = 180°$

[Sum of all $\angle$s on the same side of a line at a point is 180°]

$\Rightarrow \quad \left(\dfrac{x}{2}\right)^{\circ} + (180° - 2x) + 96° = 180°$

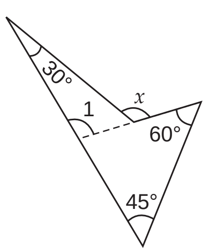
[Using (2) and (3)]

$\Rightarrow \quad -\dfrac{3}{2}x = 180° - 96° - 180° = -96°$

$\Rightarrow \quad x = -96° \times \left(-\dfrac{2}{3}\right) = 64°$

Hence, $\boldsymbol{x = 64°}$.

(*ix*)



Ext. $\angle 1 = 45° + 60°$

$= 105°$

[Ext. $\angle$ = sum of int. opp. $\angle$s ... (1)]

Ext. $\angle x = \angle 1 + \angle 30°$

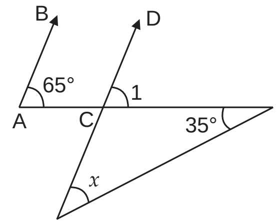
[Ext. $\angle$ = sum of int. opp. $\angle$s]

$= 105° + 30°$ [Using (1)]

$= 135°$

Thus, $\boldsymbol{x = 135°}$.

4. (*i*) AB $\parallel$ CD and AC is transversal.



$\therefore \quad \angle 1 = 65°$ [corr. angle]

Now, $\quad x + 35° = \angle 1$

[Sum of int. opp. $\angle$s = ext. $\angle$]

$\Rightarrow \quad x + 35° = 65°$ [Using (1)

$\Rightarrow \quad x = 65° - 35°$

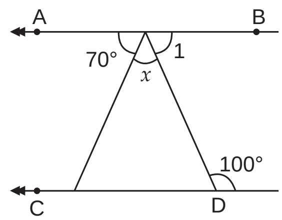
$= 30°$

Thus, $\boldsymbol{x = 30°}$.

(*ii*) $\quad 70° + x = 100°$ [Alt. $\angle$s, AB $\parallel$ CD]

$\Rightarrow \quad x = 100° - 70°$

$= 30°$



Hence, $\boldsymbol{x = 30°}$.

(*iii*) $\quad x + x + 80° = 180°$ [Coint. $\angle$s, AB $\parallel$ CD]

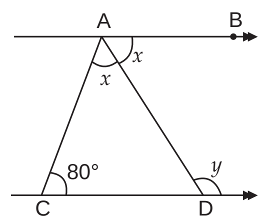
$\Rightarrow \quad 2x = 100°$

$\Rightarrow \quad x = 50°$

$x + y = 180°$ [Coint $\angle$s, AB $\parallel$ CD]

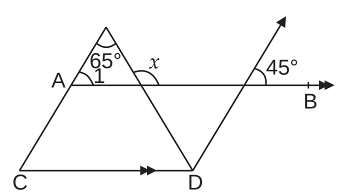
$50° + y = 180°$

$\Rightarrow \quad y = 130°$



Hence, $\boldsymbol{x = 50°, y = 130°}$.

(*iv*) AB $\parallel$ CD



$\Rightarrow \quad \angle 1 = 45°$ [coint. $\angle$s] ... (1)

Ext. $\angle x = \angle 1 + 65°$

[Ext $\angle$ = sum of int. opp. $\angle$s]

$= 45° + 65°$ [Using (1)]
$= 110°$

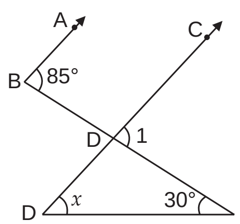
Thus $\boldsymbol{x = 110°}$.

(*v*) $\angle 1 = 85°$ [Corr. ∠s, AB ∥ CD] … (1)

$x + 30° = \angle 1$ [Ext. ∠ = sum of int. opp. ∠s]

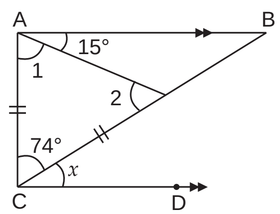
$\Rightarrow \quad x = 85° - 30°$

$\Rightarrow \quad x = 55°$



Hence, $\boldsymbol{x = 55°}$.

(*vi*)



$\angle 1 + \angle 2 + 74° = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow \quad 2\angle 1 + 74° = 180°$ [∵ ∠1 = ∠2, ∠s opp. equal sides]

$\Rightarrow \quad \angle 1 = \dfrac{180° - 74°}{2}$

$= \dfrac{106°}{2} = 53°$ … (1)

∵ AB ∥ CD

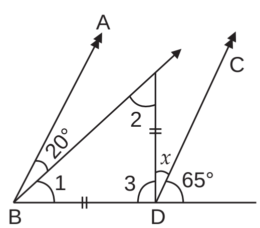
∴ $(\angle 1 + 15°) + (74 + x)° = 180°$ [coint. ∠s]

$\Rightarrow \quad (53 + 15)° + 74° + x = 180°$ [Using (1)]

$\Rightarrow \quad x = 180° - 53° - 15° - 74°$
$= 38°$

Thus, $\boldsymbol{x = 38°}$.

(*vii*) AB ∥ CD and BD is transversal.



∴ $(\angle 1 + 20°) = 65°$ [corr. angles]

$\Rightarrow \quad \angle 1 = 65° - 20° = 45°$ … (1)

$\angle 1 + \angle 2 + \angle 3 = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow \quad 2\angle 1 + \angle 3 = 180°$ [∵ ∠1 = ∠2, ∠s opp. equal sides]

$\Rightarrow \quad (2 \times 45°) + \angle 3 = 180°$ [Using (1)

$\Rightarrow \quad \angle 3 = 90°$ … (2)

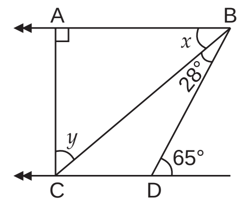
Now, $\angle 3 + x + 65° = 180°$ [Linear pair]

$\Rightarrow \quad 90° + x + 65° = 180°$ [Using (2)]

$\Rightarrow \quad x = 180° - 90° - 65°$
$= 25°$

Thus, $\boldsymbol{x = 25°}$.

(*viii*) AB ∥ CD



∴ $(x + 28°) = 65°$ [Alt. angles]

$\Rightarrow \quad x = 65° - 28°$
$= 37°$ …(1)

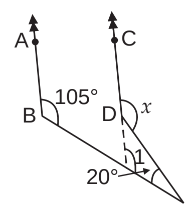
$x + y + 90° = 180°$ [Sum of ∠s of a Δ]

$\Rightarrow \quad 37° + y = 90°$ [Using (1)]

$\Rightarrow \quad y = 90° - 37°$
$= 53°$

Thus, $\boldsymbol{x = 37°, y = 53°}$.

(*ix*) AB ∥ CD



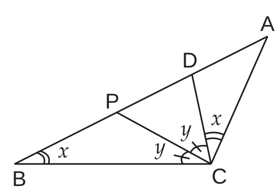
∴ $\angle 1 = 105°$ [Corresponding ∠s] … (1)

$x = \angle 1 + 20°$ [Ext. ∠ = Sum of int. opp. ∠s]

$\Rightarrow \quad x = 105° + 20°$ [Using (1)]
$= 125°$

Thus, $\boldsymbol{x = 125°}$.

5.



$\angle ABC = \angle ACD = x$ (say) [Given] … (1)

$\angle PCB = \angle PCD = y$ (say) [∵ CP bisects ∠BCD] … (2)

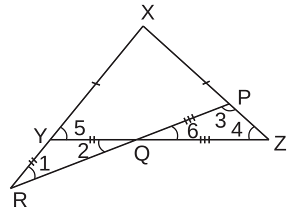
Ext. $\angle APC = \angle PBC + \angle PCB$ [Ext. ∠= sum of int. opp. ∠s]
$= \angle ABC + \angle PCB$
$= x + y$ [Using (1) and (2)]

$\angle ACP = \angle ACD + \angle PCD$
$= x + y$ [Using (1) and (2)]

From (3) and (4), we get

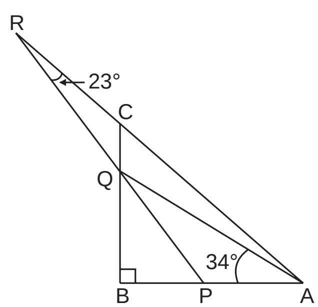
$\Rightarrow \quad \boldsymbol{\angle APC = \angle ACP}$

6. $\angle 1 = \angle 2 = x$ say [$\angle$s opp. equal sides of ΔYQR] … (1)
$\angle 3 = \angle 4 = y$ say [$\angle$s opp. equal sides of ΔPQZ] … (2)
$\angle 1 + \angle 2 =$ Ext. $\angle 5$ [Sum of int. opp. $\angle$s = Ext. $\angle$]



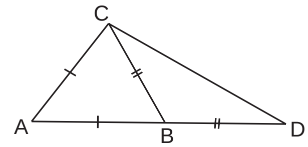
$\Rightarrow \quad x + x = \angle 5$ [Using (1)]
$\Rightarrow \quad 2x = \angle 5$ … (3)
Also, $\angle 4 = \angle 5$
[$\angle$s opp. equal sides of ΔXYZ]
$y = 2x$ [Using (2) and (3)] … (4)
Also, $\angle 6 = \angle 2$ [V. opp. $\angle$s]
$\Rightarrow \quad \angle 6 = x$ [Using (1)]
In ΔPQZ, we have
$\angle 6 + \angle 3 + \angle 4 = 180°$ [Sum of $\angle$s of a Δ]
$x + y + y = 180°$
$x + 2x + 2x = 180°$ [Using]
$\Rightarrow \quad 5x = 180°$
$\Rightarrow \quad x = 36°$
$\angle y = 2x = 2 \times 36° = 72°$,
$\angle 4 = y = 2x = 2 \times 36° = 72°$
$\angle x = 180° - 72°$[sum of $\angle$s of a Δ]
$= 36°$
Hence, the angles of ΔXYZ are **36°, 72°, 72°.**

7. We have rt. ΔABC such that $\angle B = 90°$ and BC = BA.



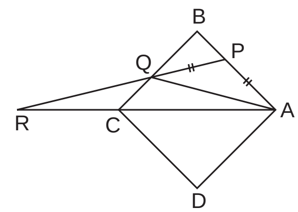
$\therefore \quad \angle C = \angle A = 45°$
i.e. $\quad \angle BCA = \angle BAC = 45°$
$\Rightarrow \quad \angle CAQ = 45° - 34°$
$= 11°$
In ΔACQ, the side CD is produced to B.
$\therefore \quad$ Ext. $\angle BQA = 45° + 11°$
$= 56°$
In ΔQRC, the side RC is produced to A.
$\therefore \quad \angle CQR =$ Ext. $\angle ACQ - \angle CRQ$
$\Rightarrow \quad \angle CQR = 45° - 23°$
$= 22°$
$\angle PQB = \angle CQR$
$= 22°$ [Vert. opp. angles]
Now, $\quad \angle PQA = \angle BQA - \angle PQB$
$= 56° - 22°$
$= 34°$ … (1)
$\angle PAQ = 34°$ [Given]
From (1) and (2),
$\angle$**PAQ** = $\angle$**PQA**

8. AC = BC



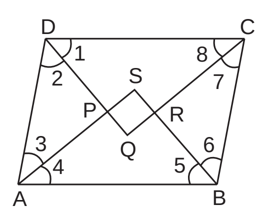
$\therefore \quad \angle ACB = \angle ABC$
[$\angle$s opp. equal sides] … (1)
$\because \quad$ BC = BD
$\therefore \quad \angle BCD = \angle CDB$
[$\angle$s opp. equal sides] … (2)
Adding (1) and (2), we get
$\angle ACB + \angle BCD = \angle ABC + \angle CDB$ … (3)
But Ext. $\angle ABC = \angle BCD + \angle BDC$
[Ext. $\angle$ = sum of int. opp. $\angle$s]
$= \angle BDC + \angle BDC$ [Using (2)]
$\Rightarrow \quad$ Ext. $\angle ABC = 2\angle BDC$ … (4)
From (3) and (4)
$(\angle ACB + \angle BCD) = 2\,\angle BDC + \angle BDC$
$\Rightarrow \quad \angle ACD = 3\angle BDC$
$\Rightarrow \quad \angle ACD = 3\angle ADC$

9. ABCD is a square and AC is its diagonal.



$\therefore \quad \angle BAC = 45°$
$\because \quad$ AP = PQ
$\therefore \quad \angle PAQ = \angle PQA = x$ (right)
$\Rightarrow \quad \angle QAR = 45° - x$
Since the side RQ of ΔARQ is produced to P.,
$\therefore \quad$ Ext. $\angle PQA = 25° + 45° - x$
$\Rightarrow \quad$ Ext. $x = 70° - x$
$\Rightarrow \quad 2x = 70°$
$\Rightarrow \quad x = 35°$
Now, $\quad \angle PAQ = 35°$
and $\quad \angle QAC = \angle QAR$
$= 45° - x$
$\Rightarrow \quad \angle QAC = 45° - 35°$
$= 10°$
Thus $\angle$**PAQ = 35°;** $\angle$**QAC = 10°.**

10. In a parallelogram, cointerior angles are supplementary.



$\therefore \quad \angle CDA + \angle DAB = 180°$
$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$
$2\angle 2 + 2\angle 3 = 180°$
[$\because$ DQ and AS bisect $\angle D$ and $\angle A$ respectively]

$\Rightarrow \qquad \angle 2 + \angle 3 = \dfrac{180°}{2} = 90°$

Ext. $\angle DPS = \angle 2 + \angle 3 = 90°$

But $\quad \angle SPQ + \angle DPS = 180°$ [Linear pair]

$\therefore \quad \angle SPQ + \angle DPS = 180°$

$\therefore \quad \angle SPQ + 90° = 180°$

$\Rightarrow \quad \angle SPQ = 90°$

Similarly, $\quad \angle SRQ = 90°$

Now, $\quad \angle BCD + \angle CDA = 180°$ [Cointerior angles]

$\Rightarrow \quad \angle 7 + \angle 8 + \angle 1 + \angle 2 = 180°$

$\Rightarrow \quad 2(\angle 8 + \angle 1) = 180°$

[$\because$ CQ and DQ bisect $\angle C$ and $\angle D$ respectively]

$\Rightarrow \quad \angle 8 + \angle 1 = \dfrac{180°}{2} = 90°$

Now, in ΔDQC, we have

$\angle DQC = 180° - (\angle 8 + \angle 1)$

[Sum of $\angle$s of a Δ is 180°]

$\Rightarrow \quad \angle PQR = 180° - 90°$

$= 90°$

Similarly, we prove that $\angle PSR = 90°$.

Thus, each angle of quadrilateral is a right angle.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (*b*) **57°**

Angle = Its complement + 24° [Given]

$x = (90° - x) + 24°$

$\Rightarrow \quad 2x = 114°$

$\Rightarrow \quad \mathbf{x = 57°}$

2. (*c*) **74°**

Angle = Its supplement – 32° [Given]

$x = (180° - x) - 32°$

$\Rightarrow \quad 2x = 148°$

$\Rightarrow \quad x = \mathbf{74°}$

3. (*c*) **72°**

Angle = 4 Its complement [Given]

$x = 4\ (90° - x)$

$\Rightarrow \quad 5x = 360°$

$\Rightarrow \quad x = \mathbf{72°}$

4. (*a*) **60°**

Supplement of $\angle$ = 4 (its complement) [Given]

$(180° - x) = 4\ (90° - x)$

$\Rightarrow \quad 180° - x = 360° - 4x$

$\Rightarrow \quad 3x = 180°$

$\Rightarrow \quad x = 60°$

5. (*d*) **36°, 54°**

Let the angles be $2x$ and $3x$.

Then, $\quad 2x + 3x = 90°$

$\Rightarrow \quad 5x = 90°$

$\Rightarrow \quad x = 18°$

$\angle$s are $2 \times 18° = 36°$

and $\quad 3 \times 18° = 54°$

6. (*a*) **70°, 20°**

$m\angle P + m\angle Q = 90°$

$\therefore (2y + 30°) + (y) = 90°$

$\Rightarrow \quad 3y = 60°$

or $\quad y = 20°$

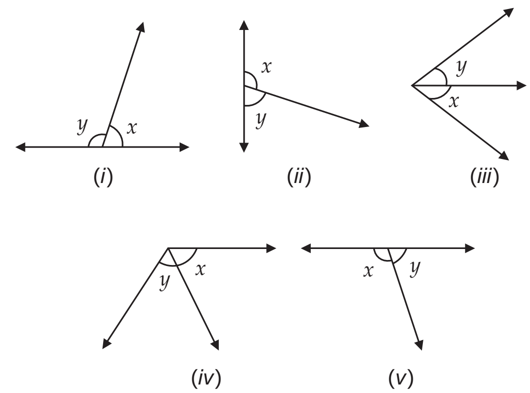
$\Rightarrow \quad m\angle P = 2y + 30°$

$= 2 \times 20° + 30°$

$= \mathbf{70°}$

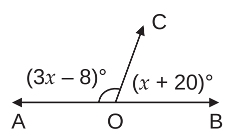
and $\quad m\angle Q = y = \mathbf{20°}$

7. (*d*) **(*i*) (*ii*) and (*v*)**



In a linear pair, the non-common arms form straight line.

8. (*d*) **42**



Opposite rays form a straight line.

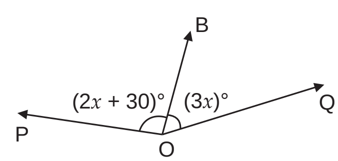
$\Rightarrow \quad (3x - 8)° + (x + 20)° = 180°$

$4x = 180 - 8 - 30$

$\Rightarrow \quad 4x = 168$

$\Rightarrow \quad x = 42$

9. (*a*) **30**



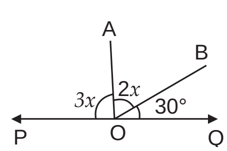
POQ will be a strainght line if

$(2x + 30)° + (3x)° = 180°$

$\Rightarrow \quad 5x = 150°$

$\Rightarrow \quad x = 30°$

10. (*b*) **30°**



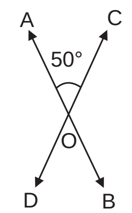
$\angle POQ$ will be a strainght line if

$3x + 2x + 30° = 180°$

$\Rightarrow \quad 5x = 150°$

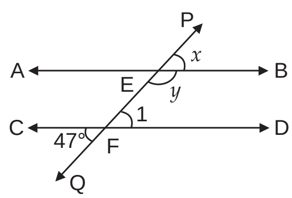
$\Rightarrow \quad \mathbf{x = 30°}$

11. (*c*) **260°**



$$\angle AOD = \angle 180° - \angle AOC$$
$$= 180° - 50°$$
$$= 130°$$
$$\angle AOD = \angle COB \quad \text{[Vert. opp. angles]}$$
$$\Rightarrow \quad \angle AOD + \angle COD = 130° + 130°$$
$$= 260°$$

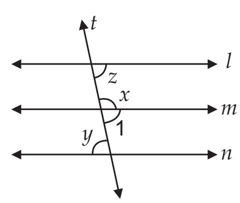
12. (*c*) **47°, 133°**



$$\angle 1 = \angle CFQ \quad \text{[Ver. opp. angles]}$$
$$\Rightarrow \quad \angle 1 = 47°$$
$$y + \angle 1 = 180° \quad \text{[Cointerior angles]}$$
$$\Rightarrow \quad y = 180° - 47° = 133°$$
$$x = \angle 1 = 47° \text{[Corr. } \angle\text{s AB} \parallel \text{CD]}$$

Hence, $\boldsymbol{x = 47°}$ and $\boldsymbol{y = 133°}$.

13. (*d*) **80°**



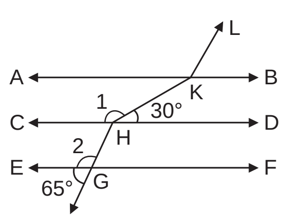
Given, $x : y = 5 : 4$

$$\Rightarrow \quad \frac{x}{y} = \frac{5}{4}$$
$$\Rightarrow \quad x = \frac{5}{4}y \quad \ldots (1)$$

Now, $\angle 1 = y$ (Alt. angles)

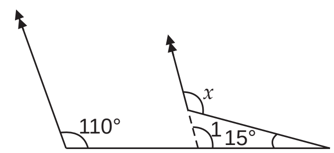
$$\Rightarrow \quad x + \angle 1 = 180°$$
$$\Rightarrow \quad x + y = 180°$$
$$\Rightarrow \quad \frac{5}{4}y + y = 180° \quad \text{[From (1)]}$$
$$\Rightarrow \quad y = 80°$$
$$\because \quad x = y \quad \text{[Alt. angles } l \parallel n\text{]}$$
$$\because \quad z = \mathbf{80°}$$

14. (*b*) **145°**



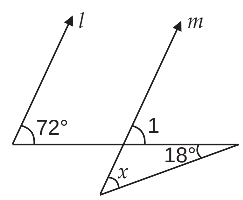
$$\angle AKH = \angle DHK = 30° \quad \text{[Alt. } \angle\text{s]}$$
$$\angle AKL = \angle 1 = \angle 2$$
$$= 180° - 65°$$
$$= 115°$$
$$\angle HKL = \angle AKL + \angle AKH$$
$$= 115° + 30°$$
$$= \mathbf{145°}$$

15. (*a*) **125°**



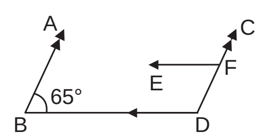
$$\angle x = \angle 1 + 15°$$
$$= 110° + 15°$$
$$= 125°$$
$$[\because \angle 1 = 110° \text{ corresponding angles}]$$

16. (*c*) **54°**



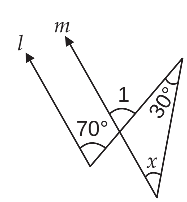
$$\angle 1 = 72° \quad \text{[Corr. } \angle\text{s]}$$
$$\text{Ext. } \angle 1 = x + 18°$$
$$\Rightarrow \quad x = 72° - 18° = 54°$$

17. (*b*) **115°**



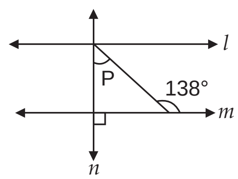
$$AB \parallel CD$$
$$\Rightarrow \quad \angle D = (180° - 65°)° = 115°$$
[Cointerior angles]
$$\angle CFE = \angle D \quad \text{(Corresponding angels)}$$
$$= 115°$$

18. (*c*) **40°**



$$\angle 1 = 70° \quad \text{[Corr. } \angle\text{s, } l \parallel m\text{]}$$
$$\therefore \quad \text{Ext. } \angle 1 = x + 30°$$
$$\Rightarrow \quad 70° = x + 30°$$
$$\Rightarrow \quad x = 40°$$

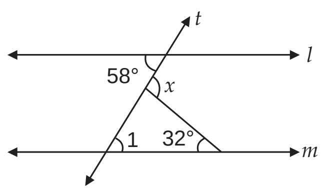
19. (*a*) **48°**



Ext. ∠138° = ∠P = 90°

⇒ ∠P = 138 − 90° = 48°

20. (*d*) **90°**



∠1 = 58° [Alt. ∠s, $l \parallel m$]

x = 32° + 58°

[Ext. ∠ = sum of int. opp. ∠s]

⇒ x = **90°**

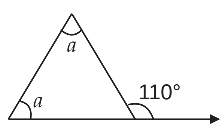
21. (*a*) **A right triangle**

Sum of two complementary angles = 90°

∴ Third angle must be = 90°

[∵ 90° + 90° = 180°]

22. (*b*) **55°**



Let each of the equal interior opposite angles be a

∴ $a + a$ = 110°

[Ext. ∠ = sum of int. opp. ∠s]

$$a = \frac{110°}{2} = 55°$$

23. (*d*) **a right triangle**

Let the ∠s of the Δ be $4x$, $5x$ and $9x$.

$4x + 5x + 9x$ = 180°

⇒ $18x$ = 180°

⇒ x = 10°

∴ Angles of triangle are 40, 50 and 90°.

⇒ It is a **right** triangle.

24. (*c*) **an obtuse angled triangle**

∵ Exterior angle is an acute angle

∴ Sum of interior opp. angles < 90°

⇒ The third angle must be > 90, so that the sum of three angles of the Δ be 180°. So, the triangle must be obtuse angled triangle.

25. (*c*) **12°**

Let the vertex angle = x

Each of the base angle = $9x$

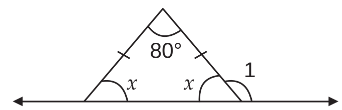
The $x + 7x + 7x$ = 180° [Sum of ∠ of a Δ]

⇒ $15x$ = 180°

⇒ x = 12°

⇒ Vertex angle = **12°**

26. (*d*) **130°**



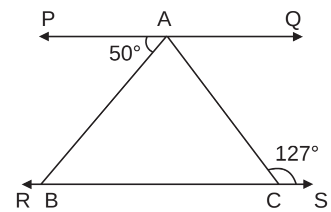
Let each base of isosceles triangle be x.

Then, $x + x$ + 80° = 180°

⇒ x = 50°

Ext. ∠1 = sum of int. opp. ∠s = 80° + 50° = 130°

27. (*b*) **77°**



PQ ∥ RS

⇒ ∠B = 50° [Alt. angles]

∠BAC + ∠B = 127°

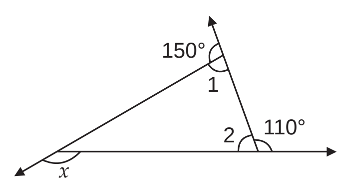
[Ext. ∠ = sum of int. opp. ∠s]

∠BAC = 127° − ∠B

= ∠127° − 50°

= 77°

28. (*a*) **100°**



∠1 = 180° − 150° = 30°

∠2 = 180° − 110° = 70°

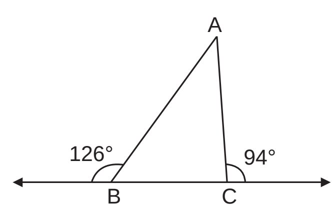
x = ∠1 + ∠2

= 30° + 70°

= 100°

[Ext. ∠x = sum of int. opp. ∠s]

29. (*c*) **40°**



∠ABC + 126° = 180° [Linear pair]

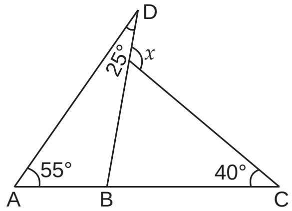
⇒ ∠ABC = 54°

∠BAC + 54° = 94°

[Ext. ∠ = sum of int. opp. ∠s]

⇒ ∠BAC = **40°**

30. (*d*) **120°**



Since Ext. ∠ = sum of int. opp. ∠s

∴ Ext. ∠DBC = 55° + 25°
= 80°

and Ext. ∠x = 40° + 80°
= 120°

31. (*b*) **152.5**

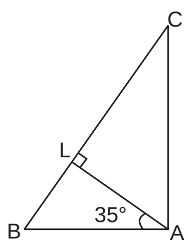
Given angle is 125°

∴ Sum of other two angles = 180° − 125° = 55°

⇒ Sum of halves of these two angles = $\frac{1}{2}$ [55°] = 27.5°

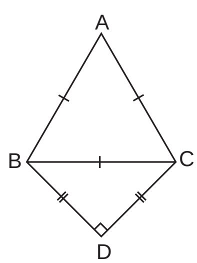
∴ Angles between the bisectors of the base angles
= 180° − 27.5°
= 152.5°

32. (*c*) **35°**



∠A = 90°
∠LAC = 90° − 35° = 55°
55° + ∠ACB = 90°
⇒ ∠ACB = 90° − 55°
= 35°

33. (*c*) **105°**



∠DBC + ∠DCB + 90° = 180°

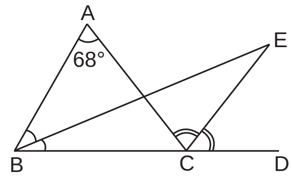
⇒ 2∠DBC = 90°
[∵ ∠DBC = ∠DCB, ∠s opp. equal sides]

⇒ ∠DCB = 45°

and ∠ABC = 60°
[Angle of an equilateral triangle]

∠ABD = ∠DBC + ∠ABC
= 45° + 60°
= **105°**

34. (*d*) **34°**



The exterior ∠ACD = ∠B + 68°

$\angle ECD = \frac{1}{2} \angle ACD$

$= \frac{\angle B}{2} + 34°$ … (1)

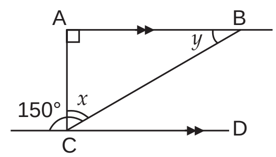
$\angle ECD = \frac{\angle B}{2} + \angle BEC$ … (2)

From (1) and (2),

$\frac{B}{2} + \angle BEC = \frac{B}{2} + 34°$

⇒ ∠BEC = **34°**

35. (*b*) **60°, 30°**



$90° + x + y = 180°$ [Sum of ∠s of a Δ]

⇒ $x + y = 90°$ … (1)

∠BCD = y [Alt. angle]

∴ $y = 180° - 150°$ [Linear pair]
$= 30°$ … (2)

From (1) and (2)

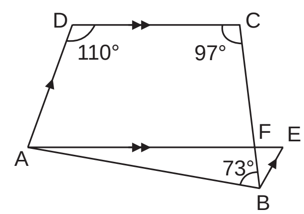
$x + y = 90°$

⇒ $x + 30° = 90°$

⇒ $x = 60°$

∴ We have x = **60°** and y = **30°**

36. (*d*) **27°**



DC ∥ AE

⇒ 110° + ∠DAE = 180° [Cointerior angles]

⇒ ∠DAE = 180° − 110°
= 70°

AD ∥ BE

⇒ ∠DAE = ∠BEA [Alt. angles]

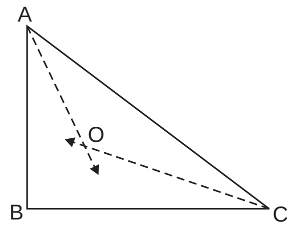
⇒ ∠BEA = 70°

∠AFC = 180° − 97°
= 83° [Coint. ∠s]

∠BFE = ∠AFC [∠s opp. to 83°]

∴ ∠EBF = 180° − (83° + 70°)
= 180° − 153°
= 27°

37. (*a*) **135°**



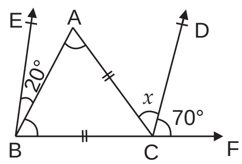
$$\angle B = 90°$$

$$\Rightarrow \quad \angle A + \angle C = 90°$$

$$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle C = 45°$$

$$\Rightarrow \quad \angle AOC = 180° - 45° = \mathbf{135°}$$

38. (*c*) **30°**



When parallel lines are intersected by a transversal

$\therefore$ Corresponding angles are equal

$$\Rightarrow \quad \angle EBC = 70°$$

$$\therefore \quad \angle ABC = 70° - 20° = 50°$$

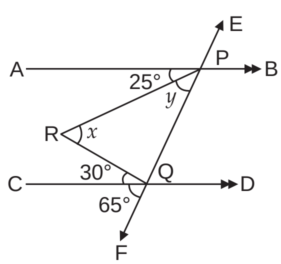
$$\angle CAB = \angle ABC = 50° \quad [\angle\text{s opp. equal sides}]$$

$$\text{Ext. } \angle ACF = \angle CAB + \angle ABC$$

$$\Rightarrow \quad 70° + x = 50° + 50° = 100° \quad [\because \angle CAB = \angle ABC]$$

$$\Rightarrow \quad x = 100° - 70° = \mathbf{30°}$$

39. (*a*) **55°, 40°**



AB ∥ CD and EF is a transversal.

$$\Rightarrow \quad 25° + y = 65° \quad [\text{Corr. } \angle\text{s.}]$$

$$\Rightarrow \quad y = 65° - 25° = 40°$$

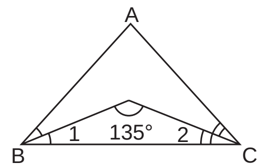
$$\text{Ext. } \angle RQF = x + y$$

$$\Rightarrow \quad 30° + 65° = x + 40°$$

$$\Rightarrow \quad x = 30° + 65° - 40° = \mathbf{55°}$$

Thus, $x = 55°$ and $y = 40°$

40. (*d*) **A right triangle.**



In $\Delta BOC = \angle 1 + \angle BOC + \angle 2 = 180°$

$$\Rightarrow \quad \angle 1 + \angle 2 = 180° - 135° = 45°$$

$$\Rightarrow \quad \angle 1 + \angle 2 = 45°$$

$$\Rightarrow \quad 2\angle 1 + 2\angle 2 = 90°$$

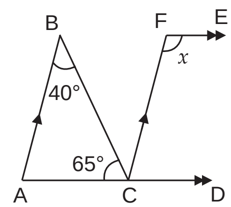
$$\Rightarrow \quad \angle B + \angle C = 90°$$

[$\because$ BO & CO are bisectors of $\angle B$ & $\angle C$ respectively]

Thus, $\angle A = 180° - (\angle B + \angle C) = 90°$

So, the triangle is **right triangle.**

41. (*d*) **105°**



$$\angle A = 180° - (40° + 65°) = 180° - 105° = 75° \quad \ldots (1)$$

$$AB \parallel CF$$

$$\Rightarrow \quad \angle FCD = \angle BAC \quad [\text{Corr. } \angle\text{s}]$$

$$\angle FCD = 75° \quad [\text{Using (1)}]$$

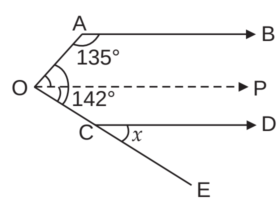
$$\because \quad AD \parallel EF$$

$$\therefore \quad \angle FCD + x = 180° \quad [\text{Cointerior angles}]$$

$$\Rightarrow \quad 75° + x = 180°$$

$$\Rightarrow \quad x = 180° - 75° = \mathbf{105°}$$

42. (*a*) **97°**



Draw OP ∥ AB

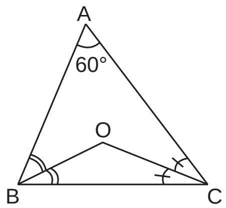
$$\therefore \quad \angle AOP = 180° - 135° = 45°$$

$$\therefore \quad \angle POE = 142° - 45° = 97°$$

$$OP \parallel CD$$

$$\Rightarrow \quad x = \mathbf{97°} \quad [\text{Corr. } \angle\text{s}]$$

43. (*c*) **120°**



$$\angle A = 180° - (B + C)$$
$$\Rightarrow \quad 60° = 180° - (B + C)$$
$$= 90° - \frac{B+C}{2}$$
$$= \frac{60°}{2}$$
$$= 30°$$
$$\Rightarrow \quad \frac{B+C}{2} = 90° - 30°$$
$$= 60° \qquad \dots (1)$$
In $\Delta BOC = \frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC$
$$= 180°$$
$$\Rightarrow \quad \angle BOC = 180° - \frac{B+C}{2}$$
$$= 180° - 60°$$
$$= \mathbf{120°} \qquad \text{[From (1)]}$$

44. (*b*) **90°**

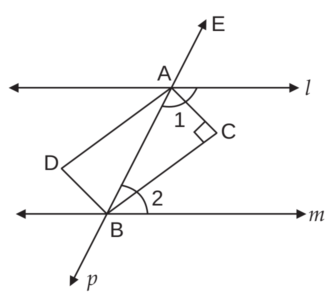
Lines are parallel, cointerior angles are supplementary

$\Rightarrow$ $\frac{1}{2}$ sum of coint. $\angle$s = 90°

Remaining $\angle$ = 90°

So, bisectors of interior $\angle$s on the same sides of transversal intersect at 90°.

45. (*c*) **rectangle**



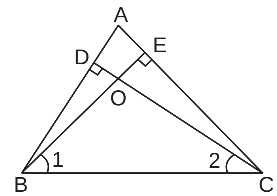
$l \parallel m$ and p is a transversal

$\therefore$ Cointerior angles are supplementary.

$$\therefore \quad \angle 1 + \angle 2 = 90°$$
$$\Rightarrow \quad \angle C = 180° - 90°$$
$$= 90°$$
Similarly $\angle D = 90°$

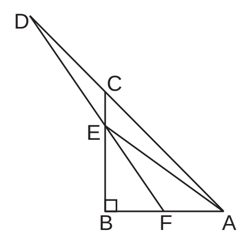
Thus, ADBC is a rectangle.

46. (*b*) **105°**



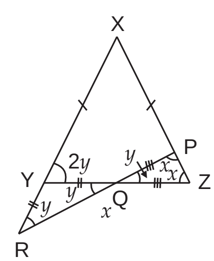
$$\angle A = 75°$$
$$\Rightarrow \quad \angle B + \angle C = 180° - 75°$$
$$= 105°$$
$$\angle C + \angle 1 = 90°$$
$$\Rightarrow \quad \angle 1 = 90° - \angle C$$
Similarly $\angle 2 = 90° - \angle B$
$$\Rightarrow \quad \angle 1 + \angle 2 = 180° - (\angle B + \angle C)$$
$$= 180° - 105°$$
$$= 75°$$
$$\angle BOC + \angle 1 + \angle 2 = 180°$$
$$\angle BOC = 180° - [\angle 1 + \angle 2]$$
$$\Rightarrow \quad 180° - 75° = \mathbf{105°}$$

47. (*c*) **29°**



$$\angle DAE = \angle CAB - \angle FAE$$
$$= 45° - 29°$$
$$= 16°$$
$$\text{Ext. } \angle FEA = \angle ADE + \angle DAE$$
$$= 13° + 16°$$
$$= 29°$$

48. (*c*) **144°**



$$QP = QZ$$
$$\Rightarrow \quad \angle QPZ = \angle QZP = x \text{ (say)}$$
$$(= YZX) \dots (1)$$
Also, $YQ = YR$
$$\Rightarrow \quad \angle YRQ = \angle YQR = y \text{ (say)} \qquad \dots (2)$$
$$\text{Ext. } \angle XYZ = y + y = 2y$$
In ΔXYZ, $XY = XZ$
$$\Rightarrow \quad \angle XYZ = \angle XZY \qquad \dots (3)$$
$$\Rightarrow \quad 2y = x$$
Also $\Delta PQZ = \Delta YQR$ [Vert. opp. $\angle$s]

$\Rightarrow \quad \angle PQZ = y$

In $\Delta PQR, \quad x + x + y = 180°$

$\Rightarrow \quad 2y + 2y + y = 180°$

$\Rightarrow \quad y = 36°$

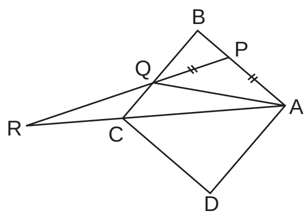
$\angle PQY + \angle PQZ = 180°$ [Linear pair]

$\angle PQY = 180° - 36°$

$= \mathbf{144°}$

49. (*a*) **40°**

ABCD is a square.



$\Rightarrow \quad \angle A = \angle B = \angle C = \angle D = 90°$

Using $\angle PAQ = \angle PQA = x$ say and $\angle QRC$

or $\quad \angle QRA = 35°$

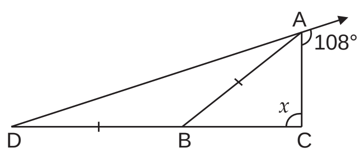
$\angle QAC = 45° - x$

Ext. $\angle PQA = 35° + 45° - x = x$

$\Rightarrow \quad 2x = 80°$

$\Rightarrow \quad x = 40°$

50. (*d*) **90°**



$\angle DAC + 108° = 180°$ [Linear pair]

$\Rightarrow \quad \angle DAC = 180° - 108°$

$= 72°$

DAC is divided into 1 : 3 by AB.

$\therefore \quad \angle DAB = 72° \times \frac{1}{4} = 18°$

and $\quad \angle BAC = 72° \times \frac{3}{4} = 54°$

$\angle BDA = \angle DAB = 18°$

[$\angle$s opp. equal sides]

$\angle ABC = \angle BDA + \angle BAD$

$= 18° + 18°$

$= 36°$

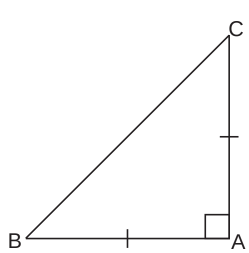
Now, $x + \angle ABC + \angle BAC = 180°$

$\Rightarrow \quad x + 36° + 54° = x + 90°$

$\Rightarrow \quad x = 180° - 90° = \mathbf{90°}$

SHORT ANSWER QUESTIONS

1. $\angle A = 90°$ and AB = AC.



$\Rightarrow \quad \angle B = \angle C$

[$\angle$s opp. equal sides] … (1)

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 180° - 90° = 90°$

$\Rightarrow \quad 2\angle B = 90°$ [Using (1)]

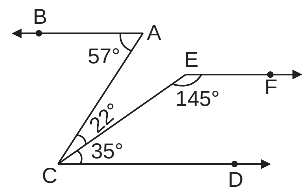
$\Rightarrow \quad \angle B = 45°$

$\Rightarrow \quad \angle B = 45°$

$\angle C = \angle B = 45°$

Thus, $\angle B = \mathbf{45°}$, $\angle C = \mathbf{45°}$

2. $\angle ACD = 22° + 35° = 57°$.



$\therefore \quad \angle ACD = \angle BAC = 57°$

But they from a pair of alternate angles.

$\therefore \quad AB \parallel CD$ … (1)

Again $\quad 145° + 35° = 180°$

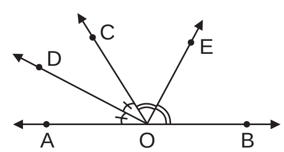
But they from a pair of cointerior angles

$\therefore \quad EF \parallel CD$ … (2)

From (1) and (2) AB ∥ CD ∥ EF

Hence, AB ∥ EF

3. A, O and B will be collinear when AOB is straight line, i.e. $\angle AOB = 180°$.



$\angle DOC = \frac{1}{2}\angle AOC$

and $\quad \angle EOC = \frac{1}{2}\angle BOC$

$\Rightarrow \quad (\angle DOC + \angle EOC) = \frac{1}{2}[\angle AOC + \angle BOC$

$= \frac{1}{2}\angle AOB$ … (1)

$OD \perp OE$

$\Rightarrow \quad [\angle DOC + \angle BOC] = 90°$ … (2)

From (1) and (2), we have

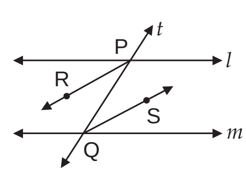
$\frac{1}{2}\angle AOB = 90°$

$\Rightarrow \quad \angle AOB = 90° \times 2 = 180°$

Thus, AOB is a straight line.

$\therefore$ **A, O and B are collinear.**

4. PR ∥ QS



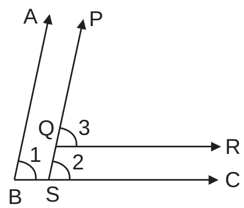
$\Rightarrow \quad \angle RPQ = \angle SQP$ [Alt. angles]

$\Rightarrow \quad 2\angle RPQ = 2\angle SQP$

$\Rightarrow$ Alternate angles formed when l and m are cut by transversal t are equal.

$\Rightarrow \quad l \parallel m$

5. Produce PQ to intersect BC at S.



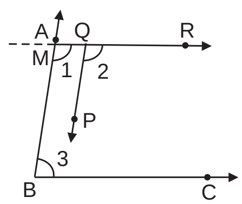
$\angle 1 = \angle 2$ [Corr. $\angle$s, AB $\parallel$ PQS]

$\angle 3 = \angle 2$ [Corr. $\angle$s, QR $\parallel$ AC]

$\therefore \quad \angle 1 = \angle 3$

Hence, $\angle$**ABC** = $\angle$**PQR**

6. Extend RQ to meet AB at M.



$\because \quad$ QP $\parallel$ AB

$\angle 1 = \angle 2$ [Corr. angle] … (1)

$\angle 1 + \angle 3 = 180°$

[Coint. angle AQR $\parallel$ BC] … (2)

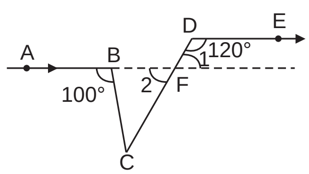
From (1) and (2),

$\angle 2 + \angle 3 = 180°$

$\Rightarrow \quad \angle PQR + \angle ABC = 180°$

Thus, $\angle$**ABC** + $\angle$**PQR = 180°**

7. Produce AB of meet DC at F.



Now, DE $\parallel$ AF

$\therefore \quad \angle 1 + 120° = 180°$ [Cointerior angles]

$\Rightarrow \quad \angle 1 = 180° - 120°$

$= 60°$

But $\angle 2 = \angle 1$ [Vert. opp. angles]

In ΔBCF, Ext $\angle ABC = \angle BCE + \angle 2$

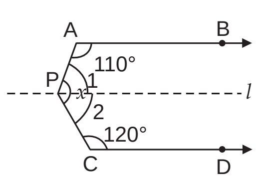
$\Rightarrow \quad 100° = \angle BCE + 60°$

$\Rightarrow \quad \angle BCD = 100 - 60°$

$= 40°$

Thus, $\angle$BCD = **40°.**

8. Though P draw $l \parallel$ AB or CD.



Now, $l \parallel$ AB

$\because \quad \angle 1 + 110° = 180°$

[Cointerior angles] … (1)

Similarly, $\angle 2 + 120° = 180°$ … (2)

From (1) and (2) we get

$\angle 1 = 180° - 110° = 70°$

and $\angle 2 = 180° - 120° = 60°$

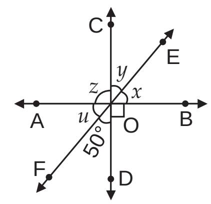
Thus, $x = \angle 1 + \angle 2$

$= 70° + 60°$

$= 130°$

Thus, x **= 130°.**

9. Since the three coplanar lines intersect at O.



$\therefore \quad \angle AOC = \angle BOD = 90°$ [opp. angles]

$\Rightarrow \quad z = 90°$

Similarly, $y = 50°$

$x + y = 90°$

$\Rightarrow \quad x + 50° = 90°$ [vert. opp. angles]

$\Rightarrow \quad z = 40°$

$u = x$ [vert. opp. $\angle$s]

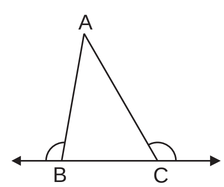
$\Rightarrow \quad u = 40°$

Thus, $x = 40°$, $y = 50°$, $z = 90°$ and $u = 40°$.

10. $\because \quad \angle B + 100° = 180°$ [Linear pair]

$\therefore \quad \angle B = 180° - 100$

$= 80°$



Similarly, $\angle C = 180° - 120°$

$= 60°$

Now, $\angle A = 180° - [\angle B + \angle C]$

[Sum of $\angle$s of a Δ is 180°]

$= 180° - [80° + 60°]$

$= 180° - 140°$

$= 40°$

Hence, the angles of the given triangle are 80°, 60°, 40°.

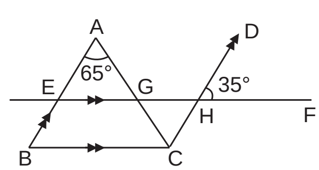
11. AB $\parallel$ CD

$\Rightarrow \quad \angle AEG = 35°$ [Corr. angles]

or, Ext. $\angle AGH = \angle EAG + \angle AEG$

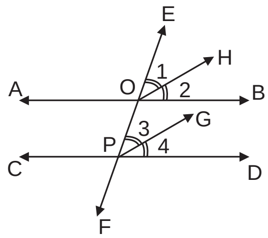
$\angle AGH = 65° + 35° = 100°$

[$\because$ It is given that, $\angle EAG = 65°$]



Thus, $\angle AGH = 100°$.

12. AB ∥ CD and EF is a transversal.



$\therefore$ $\angle EOB = \angle OPD$ [Corr. angles]

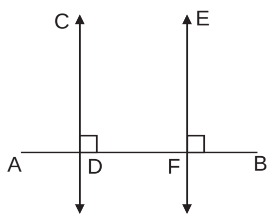
$\therefore$ $\frac{1}{2}\angle EOB = \frac{1}{2}\angle OPD$

$\Rightarrow$ $\angle 1 = \angle 3$

But they from a pair of corresponding angles.

$\therefore$ OH ∥ PG

13. We have, CD ⊥ AB and EF ⊥ AB.



$\therefore$ $\angle CDF = 90°$

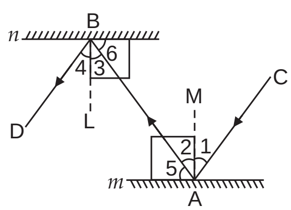
$\angle EFB = 90°$

$\Rightarrow$ $\angle CDF = \angle EFB$

But they from a pair of corresponding angles.

$\Rightarrow$ CD ∥ EF

14. $\because$ $m \parallel n$ and AB is a transversal.



$\therefore$ $\angle 5 = \angle 6$ [Alternate angles]

But AM ⊥ m and BL ⊥ n

$\therefore$ $(90° - \angle 5) = (90° - \angle 6)$

$\Rightarrow$ $\angle 2 = \angle 3$

or, $2\angle 2 = 2\angle 3$

[$\angle 2 = \angle 1$ and $\angle 3 = \angle 4$ $\because$ $\angle$ of incidence $= \angle$ of reflection]

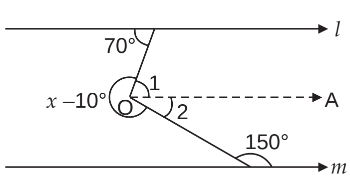
$\Rightarrow$ $(\angle 1 + \angle 2) = (\angle 3 + \angle 4)$

$\Rightarrow$ $\angle BAC = \angle ABD$

But they are a pair of alternate angles.

$\Rightarrow$ AC ∥ BD

15. Though O draw OA ∥ l ∥ m.



$\angle 1 = 70°$ [Alt. ∠s, l ∥ OA] … (1)

$\angle 2 + 150° = 180°$ [Coint. ∠s, OA ∥ m]

$\Rightarrow$ $\angle 2 = 30°$ … (2)

$\angle 1 + \angle 2 + x - 10° = 360°$ [Angles about point O]

$70° + 30° + x - 10° = 360°$

$\Rightarrow$ $x = 270°$

$\Rightarrow$ $x = 3 \times 90°$

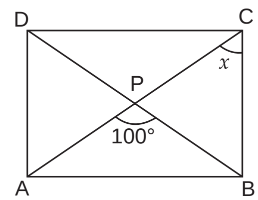
$\Rightarrow$ $x = 3$ rt ∠s

UNIT TEST

1. (*b*) **50°**

Given $\angle APB = 100°$. Then, $\angle CPB = 80°$ [Linear pair]

Also, since diagonals of rectangle are equal and bisect each other.



$\therefore$ PC = PB

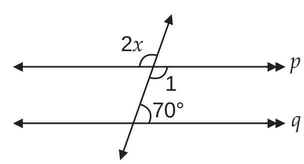
$\Rightarrow$ $\angle PCB = \angle PBC = x$

In ΔPBC, $x + x + 80° = 180°$

$\Rightarrow$ $2x = 100°$

$\Rightarrow$ $x =$ **50°**

2. (*a*) **55°**



$p \parallel q$

$\Rightarrow$ $\angle 1$ and 70° are cointerior angles.

$\therefore$ $\angle 1 + 70° = 180°$

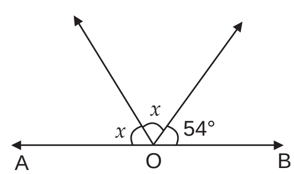
$\Rightarrow$ $\angle 1 = 180° - 70° = 110°$

$\angle 1 = \angle 2x$ [Vert. opp. angles]

$\Rightarrow$ $2x = 110°$

or, $x =$ **55°**

3. (*c*) **63°**



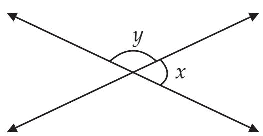
AOB is a straight line

$\therefore$ $x + x + 54° = 180°$

$\Rightarrow$ $2x + 54° = 180°$

$\Rightarrow$ $x = \frac{180° - 54°}{2} = 63°$

4. (*a*) **36° and 144°**



$x : y = 1 : 4$

Let $x = a$ and $y = 4a$

$\therefore \quad a + 4a = 180°$

$\Rightarrow \quad 5a = 180°$

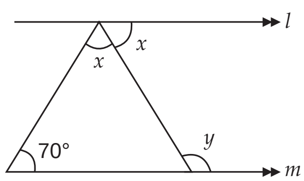
$\Rightarrow \quad a = 36°$

$x = a = \mathbf{36°}$

$y = 4a = 4 \times 36°$

$= 144°$

5. (*b*) **55° , 125°**



$l \parallel m$

$\Rightarrow \quad (x + x) + 70° = 180°$ [Cointerior angles]

$\Rightarrow \quad 2x + 70° = 180°$

$\Rightarrow \quad 2x = 180° - 70°$

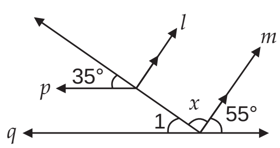
$= 110°$

$\Rightarrow \quad x = \frac{110°}{2} = \mathbf{55°}$

Also, $x + y = 180°$ [Cointerior angles]

$\Rightarrow y = 180° - 55° = \mathbf{125°}$

6. (*b*) **90°**



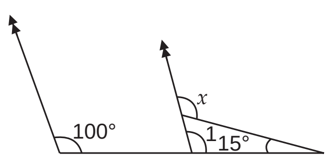
$\because \quad \angle 1 = 35°$ [Corr. ∠s, $p \parallel q$)

$\angle 1 + x + 55° = 180°$

$\Rightarrow \quad 35° + x + 55° = 180°$

$\Rightarrow \quad x = 180° - 90° = \mathbf{90°}$

7. (*a*) **115°**



$\angle 1 = 100°$ [Corresponding angles]

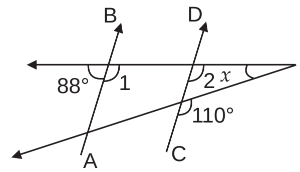
Ext. $\angle x$

$\Rightarrow \quad \angle 1 + 15° = x$

$100 + 15° = x$

or, $\quad x = 115°$

8. (*d*) 18°



$88° + \angle 1 = 180°$ [Linear pair]

$\Rightarrow \quad \angle 1 = 180° - 88° = 92°$

$\angle 2 = \angle 1 = 92°$ [Corr. ∠s, AB ∥ CD]

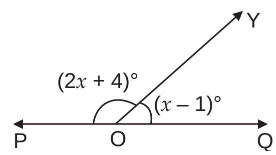
$\angle 2 + x = 110°$

[Ext. ∠ = Sum of int. opp. ∠s]

$92° + x = 110°$

$\Rightarrow \quad x = 110° - 92° = 18°$

9. ∵ POQ is a straight line.



$\therefore \quad (2x + 4)° + (x - 1)° = 180°$ [Linear pair]

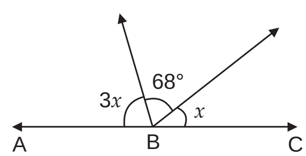
$\Rightarrow \quad 2x + x = 180 - 4° + 1°$

$\Rightarrow \quad 3x = 177°$

$\Rightarrow \quad x = \frac{177°}{3} = \mathbf{59°}$

Hence, x = **59.**

10. ∵ ABC is a straight line.



$\therefore \quad 3x + 68° + x = 180°$

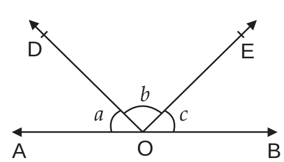
$\Rightarrow \quad 4x + 68° = 180°$

$\Rightarrow \quad 4x = 180° - 68° = 112°$

$\Rightarrow \quad x = \frac{112°}{4} = 28°$

Thus, x = **28°**

11. Let $a = 2x$, $b = 3x$ and $c = 3x$.



Since, AOB is a straight line

$\therefore \quad 2x + 5x + 3x = 180°$

$\Rightarrow \quad x = 18°$

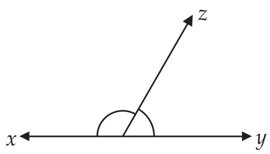
$\therefore \quad a = 2x = 2 \times 18° = \mathbf{36°}$

$b = 5x = 5 \times 18° = \mathbf{90°}$

$c = 3x = 3 \times 18° = \mathbf{54°}$

Hence, a = **36°**, b = **90°**, c = **54°.**

12.

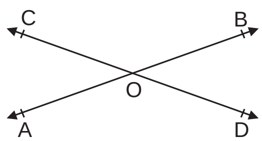


∵	$\angle XOZ + \angle ZOY = 180°$	[Linear pair]
But	$\angle XOZ = \angle ZOY$	
⇒	$2\angle XOZ = 180°$	
⇒	$\angle XOZ = \frac{180°}{2} = 90°$	

13. ∵ $\angle BOC + \angle AOD = 290°$

and $\angle BOC = \angle AOD$ [V. opp. angles]

∴ $\angle BOC = \angle AOD = \frac{290°}{2} = 145°$



Now, $\angle AOC + \angle COB = 180°$ [Linear pair]

⇒ $\angle AOC = 180° - \angle BOC$

$= 180° - 145°$

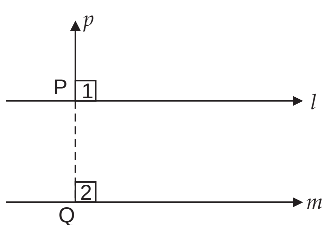
⇒ $\angle AOC = 35°$

But $\angle AOC = \angle BOD$ [V. opp. Angles]

⇒ $BOD = 35°$

Thus, **∠BOC = 145°, ∠AOC = 35°, ∠AOD = 145°**
∠BOD= 35°.

14. We have, $l \parallel m$ and $p \perp l$.



Produce p to m

∵ $p \perp l$

∴ $\angle 1 = 90°$ … (1)

$l \parallel m$ and PQ is a transversal

∴ $\angle 1 = \angle 2$

[Corresponding angles] … (2)

From (1) and (2)

$\angle 2 = 90°$

⇒ $PQ \perp m$

Thus a line perpendicular to one of the two parallel lines, then it is perpendicular the other line.

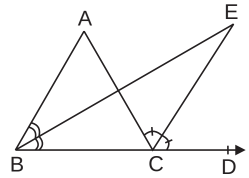
15. Ext. $\angle ACD = \angle ABC + \angle BAC$

⇒ $\frac{1}{2}\angle ACD = \frac{1}{2}\angle ABC + \frac{1}{2}\angle BAC$ … (1)

Also, Ext. $\angle ECD = \angle EBC + \angle BEC$

⇒ $\angle BEC = \angle ECD - \angle EBC$

⇒ $\angle BEC = \frac{1}{2}\angle ACD - \frac{1}{2}\angle ABC$ … (2)



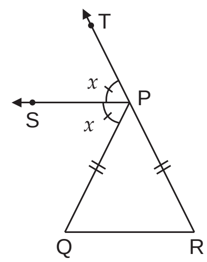
From (1) and (2) we have

$\angle BEC = [\frac{1}{2}\angle ABC + \frac{1}{2}\angle BAC] - \frac{1}{2}\angle ABC$

⇒ $\angle BEC = \frac{1}{2}\angle ABC + \frac{1}{2}\angle BAC - \frac{1}{2}\angle ABC$

⇒ $\mathbf{\angle BEC = \frac{1}{2}\angle BAC}$

16. ∵ ΔPQR is an isosceles triangle.



∴ $\angle Q = \angle R$

Now, Ext. $\angle TPQ = \angle Q + \angle R$

$= \angle Q + \angle Q$

$= 2\angle Q$

⇒ $\frac{1}{2}\angle TPQ = \frac{1}{2}[2\angle Q]$

⇒ $\frac{1}{2}\angle TPQ = \angle Q$

or $x = \angle Q$

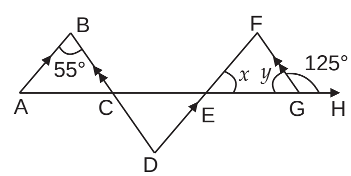
[∵ PS is bisector of∠TPQ]

But they are a pair of alternate angles.

∴ $PS \parallel RQ$

17. $\angle B = 55°$

$AB \parallel DE$ and BD is a transversal.



∴ $\angle B = \angle CDE$ [Alternate angles]

⇒ $\angle CDE = 55°$

Since, $BD \parallel FG$ and DF is a transversal

∴ $\angle BDE = \angle EFG$ [Alternate angles]

⇒ $\angle EFG = 55°$

Now, in ΔEFG,

Ext. $\angle FGH = 125°$

$= x + \angle EFG$

[Ext. ∠ = sum of int. opp. ∠s]

⇒ $x + \angle EFG = 125°$

$\Rightarrow \quad x + 55° = 125°$

$\Rightarrow \quad x = 125° - 55° = 70°$

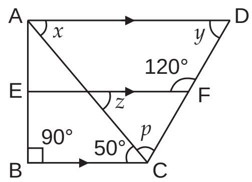
Also, $\quad y + 125° = 180°$ [Linear pair]

$\therefore \quad y = 180° - 125°$

$= 55°$

Thus, x **= 70°** , y **= 55°.**

18. EF ∥ BC



$\Rightarrow \quad \angle z = 50°$ [Alternate angles]

AD ∥ EF

$\Rightarrow \quad \angle x = \angle z$ [Corr. angles]

$\Rightarrow \quad x = 50°$

AD ∥ EF

$\Rightarrow \quad y + 120° = 180°$ [Cointerior angles]

$\Rightarrow \quad y = 180° - 120° = 60°$

Now, $\quad$ Ext. $120° = z + p$

$= 50° + p$

$\Rightarrow \quad p = 120° - 50°$

$= 70°$

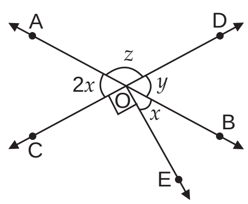
Thus , x **= 50°,** y **= 60°,** z **= 50°,** p **= 70°**

19. $\angle AOC + \angle COE + \angle EOB = 180°$ [∵ AOB is a st. line]

$\Rightarrow \quad 2x + 90° + x = 180°$

$\Rightarrow \quad 3x = 180° - 90° = 90°$

$\Rightarrow \quad x = \dfrac{90°}{3} = 30°$



Again, $\quad \angle COE + x + y = 180°$ [∵ COD is a st. line]

$\Rightarrow \quad 90° + 30° + y = 180°$

$\Rightarrow \quad y = 180° - 90° - 30° = 60°$

Now, $\quad y + z = 180°$ [∵ AOB is a st. line]

$60° + z = 180°$

$\Rightarrow \quad z = 180° - 60° = 120°$

Thus, x **= 30°,** y **= 60°,** z **= 120°**

20. $\angle 1 = \angle 3$ [Vert. opp. angles]

$\therefore \quad 3x + 15 = x + 5y$

$\Rightarrow \quad 3x - x - 5y = -15$

$\Rightarrow \quad 2x - 5y = -15$ … (1)

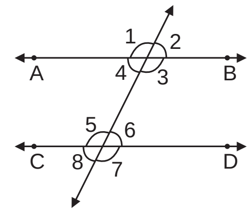
∵ AB ∥ CD

$\Rightarrow \quad \angle 3 = \angle 5$ [Alt. angles]

$\therefore \quad x + 5y = 7y + 2$

$\Rightarrow \quad x + 5y - 7y = 2$

$\Rightarrow \quad x - 2y = 2$ … (2)



Solving (1) and (2) we get

$x = 40$

and $\quad y = 19$

$\therefore \quad \angle 3 = (x + 5y)°$

$= [40 + 5(19)]°$

$= (40 + 95)°$

$= 135°$

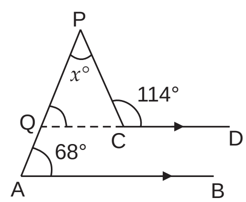
Now, $\quad \angle 3 + \angle 6 = 180°$ [Cointerior angles]

$\Rightarrow \quad 135° + \angle 6 = 180°$

$\Rightarrow \quad \angle 6 = 180° - 135° =$ **45°**

Hence, $\angle 6$ **= 45°.**

21. Produce DC to meet AP at Q.



∵ $\quad$ AB ∥ CD

$\Rightarrow$ $\quad$ AB ∥ QD

$\therefore \quad \angle PQD = 68°$ [Corr. ∠s]

Now, in ΔPQC,

Ext. $\angle 114° = x + 68°$

[Ext. ∠ = Sum of the opp. ∠s]

$\Rightarrow \quad x = 114° - 68° = 46°$

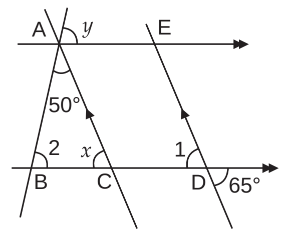
Thus, x **= 46°**

22. Here, $\quad \angle 1 = 60°$ [Vert. opp. ∠s]

CA ∥ DE

$\therefore \quad \angle 1 = \angle x$ [Corr. angles]

$\therefore \quad \angle x = 65°$



In ΔABC,

$\angle 2 = 180° - 50° - x$

[Sum of ∠s of a Δ]

$\Rightarrow \quad \angle 2 = 180° - 50° - 65° = 65°$

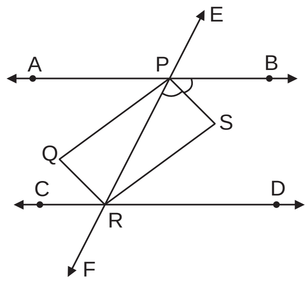
Now $\quad$ AE ∥ BD [Corr. angles]

$\therefore \quad \angle y = \angle 2$

$\Rightarrow \quad y = 65°$

Thus, x **= 65°,** y **= 65°**.

23. Let AB ∥ CD and transversal EF cut AB at P and CD at R. Let the bisectors of interior ∠RPB and ∠DRP intersect at S and the bisectors of interior ∠APR and ∠PRC intersect at Q.



∠BPR = ∠PRC   [Alt. ∠s, AB ∥ CD]

⇒ $\frac{1}{2}$∠BPR = $\frac{1}{2}$∠PRC

⇒ ∠SPR = ∠PRQ

[∵ PS and RQ are bisectors of ∠BPR and ∠PRC respectively]

But there are alt. ∠s formed when transversal EF cuts PS at P and QR at R.

∴ PS ∥ QR

Similarly PQ ∥ SR

∴ PQRS is a parallelogram.

Also, ∠APR + ∠BPR = 180°   [Linear Pair]

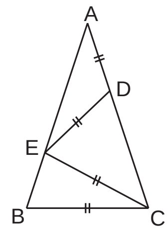
⇒ $\frac{1}{2}$∠APR + $\frac{1}{2}$∠BPR = 90°

⇒ ∠QPR + ∠SPR = 90°

⇒ Thus, PQRS is a ∥gm with one angle 90°.

Hence, PQRS is a rectangle.

24.



∠B = ∠C = y (*say*)

[∠s opp. to equal sides of ΔABC]

Let ∠A = x

In ΔABC,

$x + y + y = 180°$

$x = 180° - 2y$ … (1)

∠AED = ∠A [∠s opp. to equal sides]

⇒ ∠AED = x

$= 180° - 2y$ [Using (1)] … (2)

∠B = ∠BEC = y

[∠s opp. to equal sides]

In ΔBEC, we have

$y + y$ + ∠BCE = 180°   [Sum of ∠s of a Δ]

⇒ ∠BCE = $180° - 2y$ … (3)

From (2) and (3), we get

∠AED = ∠BCE