CHAPTER **5**

Introduction to Euclid's Geometry

EXERCISE 5A

1. An axiom is a basic fact which s taken for granted without proof.

For example: The whole is grater than each of its parts.

2. An axiom is a fact which is taken for grated and it does not require any proof.

A theorem is a statement that requires a proof.

3. Euclid's five postulates

(*i*) A straight line may be drawn from any one point to any other point.

(*ii*) A terminated line can be produced indefinitely.

(*iii*) A circle can be drawn with any centre and any radius.

(*iv*) All right angles are equal to one another.

(*v*) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

4. Point, line, plane

These terms cannot be defined precisely, Hence, they are considered as undefined terms.

5. (*i*) **Line segment**: The straight path between two points is called a line segment (It has two end points and a definite length).

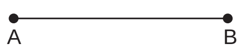


Fig. (*i*)

Figure (*i*) shows line segment $\overline{AB}$ (or $\overline{BA}$) with end points A and B.

(*ii*) **Ray:** A line segment when extended in one direction only is called a ray. It has only one end point, also known as the initial point.

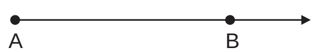


Fig. (*ii*)

Figure (*ii*) represents ray $\overrightarrow{AB}$ with end point A.

(*iii*) **Collinear points**: Three or more points are said to be collinear if there exist a straight line which contains all of them.

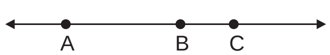


Fig. (*iii*)

Figure (*iii*) shows three collinear points A, B and C.

(*iv*) **Intersecting lines:** Two lines consisting of a common point are said to be intersecting lines. The common point is called 'the point of intersection'.

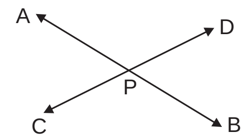


Fig. (*iv*)

Figure (*iv*) shows two intersecting lines AB and CD intersecting at point P.

(*v*) **Concurrent lines:** Three or more lines which intersect at a common point are said to be concurrent.

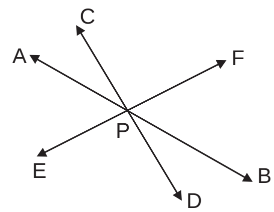


Fig. (*v*)

Figure (*v*) shows three concurrent lines AB, CD and EF intersecting at a common point P.

(*vi*) **Parallel lines:** Two lines in a plans are said to be parallel, if they do not have a common point i.e. they do not intersect at any point.

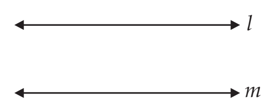


Fig. (*vi*)

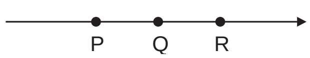
Figure (*vi*) shows parallel lines l and m and we write $l \parallel m$.

6. (*i*) Infinite lines can pass through one point.

(*ii*) Only one line can pass through two distinct points.

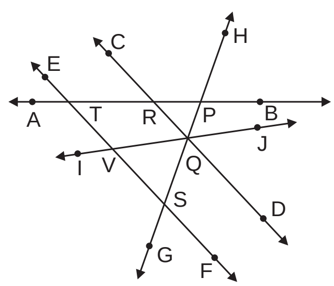
7. Two distinct lines can intersect at most in one point only.

8. The line segments determined by three collinear points P, Q, R such that Q lies between point P and R are $\overline{PQ}$, $\overline{QR}$ and $\overline{PR}$.



$$\overline{\mathbf{PR}} = \overline{\mathbf{PQ}} + \overline{\mathbf{QR}}$$

9. These are sample answers.



(*i*) Twelve points are:
A, B, C, D, E, F, G, H, I, J, P, R

(*ii*) Six line segments are:
$\overline{TR}$, $\overline{RP}$, $\overline{PQ}$, $\overline{QS}$, $\overline{VS}$, $\overline{QV}$

(*iii*) Six rays are:
$\overrightarrow{TE}$, $\overrightarrow{RC}$, $\overrightarrow{PH}$, $\overrightarrow{PB}$, $\overrightarrow{QD}$, $\overrightarrow{SF}$

(*iv*) Five collinear points are:
A, T, R, P, B

(*v*) Three pairs of intersecting lines and their corresponding points of intersection are
$[\overleftrightarrow{AB}, \overleftrightarrow{CD}, R]$, $[\overleftrightarrow{HG}, \overleftrightarrow{IJ}, Q]$, $[\overleftrightarrow{AB}, \overleftrightarrow{HG}, P]$.

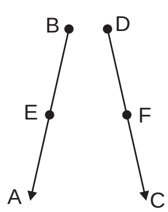
(*vi*) Three concurrent lines are $\overleftrightarrow{CD}$, $\overleftrightarrow{HG}$, $\overleftrightarrow{IJ}$.
Their point of intersection is Q.
Note: The answers of (*i*) to (*vi*) given above are sample answers.
Your answers may vary.

10. (*i*) False
∵ Ray has only one end point .
(*ii*) True
∵ Two distinct lines can intersect at most in one point only.
(*iii*) False
∵ Line is a line segment extended indefinitely in both directions.
(*iv*) True
Parallel line axiom
(*v*) True
∵ Lines which intersect at a common point are concurrent.

11. $AE = DF$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}DC$

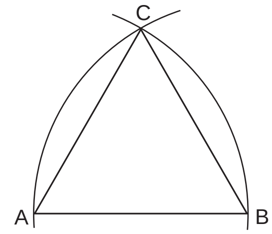
[∵ E and F are the mid-points of AB and DC respectively]



$\Rightarrow \quad AB = DC$

[**Axiom** 6 Things which are double of the same (or equal) things are equal to one another]

12. Let $\overline{AB}$ be any given line segment.
Using Euclid's postulate 3.
[A circle can be drawn with any centre and any radius]
Draw arcs of circles with centres A and B and radius AB.



Let these arcs intersect at C. Join AC and BC to get ΔABC.

Now, $AB = AC$
[Radii of the same circle]

and $AB = BC$
[Radii of the same circle]

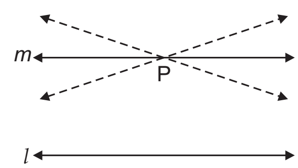
∴ $AB = BC = AC$
[Axiom 1: Things which are equal to the same thing are equal to one another]

∴ ΔABC is an equilateral triangle drawn on line segment $\overline{AB}$.

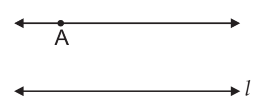
Thus, an equilateral triangle can be constructed on any given line segment.

EXERCISE 5B

1. Two equivalent versions of Euclid's fifth postulate;
(*a*) Two distinct intersecting lines cannot be parallel to the same line.
(*b*) Playfairs Axiom. It states that 'For every line *l* and every point P not lying on *l*, there exists a unique line *m* passing through P and parallel to *l*'.

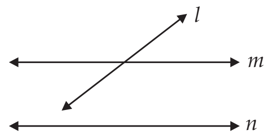


2. Since AB, AC, AD and AE are all parallel to line *l*, therefore, point A lies outside line *l*, through which lines AB, AC, AD and AE are drawn, such that each of them is parallel to *l*.



But by Playfair's Axiom 'For every line *l* and for every point A not lying on *l*, there exists a unique line passing through A and parallel to *l*'. So, AB, AC, AD and AE can be parallel to *l* only if A, B, C, D and E all be on the same line that passes through A and is parallel to line *l*.
Hence, A, B, C, D and E are collinear.

3. $m \parallel n$ [Given] … (1)
l intersects *m*. [Given]
Suppose *l* does not intersect *n*.



Then, $l \parallel n$ … (2)

From (1) and (2), we get

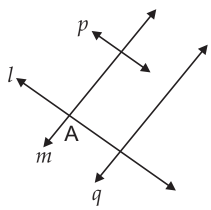
$l \parallel m$

⇒ l and m are non intersecting lines. This contradicts the fact that l intersects m. Hence, l intersects n also.

4. $l \parallel p$ [Given]

l and m are intersecting lines. [Given]

Let l and m intersect at A.



By Playfair's axiom, for every line p and for every point A not lying on p, there exists a unique line l passing through A and parallel to p.

⇒ m cannot be parallel to p

∴ m and p intersect.

Now $m \parallel q$

∴ Line m and p intersect and $m \parallel q$.

∴ By Playfair's axiom p cannot be parallel to q

∴ p and q intersect.

CHECK YOUR UNDERSTANDING

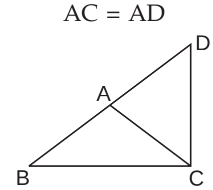
MULTIPLE-CHOICE QUESTIONS

1. (*d*) **Polygon**
The base of a polygon can be of n sides where n can be equal to 3, 4, 5 or more.
2. (*b*) **triangles**
The sides faces of a pyramid are triangles converging to a point at the top.
3. (*b*) **Square and circular**
The geometry of the vedic period originated with the construction of altars (or vedis) for performing vedic rites. Altars used for household rituals were square and circular.
4. (*d*) **rectangles, triangles and trapeziums**
The location of the scared fires had to be in accordance to the clearly laid down instructions about their shapes and areas. So altars whose shapes were combinations rectangles, triangles and trapeziums were required for further worship.
5. (*c*) **nine**
Sriyantra consists of nine interwoven isosceles triangles arranged in such a way that they produce 43 subsidiary triangles.
6. (*c*) **4 : 2 : 1**
The town dwellers of Indus Valley civilization (3000 BC) were spelled in mensuration and practical arithmetic and the dimensions of the bricks used by them were in the ratio 4 : 2 : 1.
7. (*d*) **13 chapters**
Euclid, an Egyptian mathematician arranged all his works in his famous treatise called the *Elements* and divided the elements into thirteen chapters, each called a book.
8. (*d*) **Surfaces**
Boundaries of solids are called surfaces.
9. (*a*) **Solids – surfaces – lines – points**
In each step one dimension is lost.
10. (*d*) **3**
A solid has three dimensions.
11. (*b*) **2**
From solid to surface one dimension is lost, so a surface has 2 dimensions.
12. (*a*) **none**
From line to point one dimension is lost. So, a point has no dimensions.
13. (*c*) **1**
From surface to line one dimension is lost, so a line has only one dimension.
14. (*c*) **universal truths in all branches of Mathematics**
Euclid assumed certain branches which were not to be proved but accepted as universal truths in all branches of Mathematics and called them axioms.
15. (*b*) **Theorems**
By definition, theorem is a statement proved by applying deductive reasoning to previously proved results and some axioms.
16. (*a*) **an axiom**
Axiom 3
17. (*c*) **a postulate**
18. (*d*) **a definition**
19. (*a*) **first axiom**
Axiom 1: Things which are equal to the same thing are equal to one another.
20. (*b*) **43**
The Sriyantra consists of nine interwoven isosceles triangles which are arranged in such a way that they produce 43 subsidiary triangles.

SHORT ANSWER QUESTIONS

1. AB = AD [Given]

AC = AD [Given]

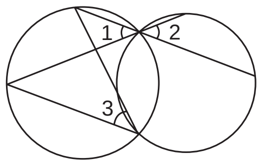


According to Axiom 1, things which are equal to the same thing are equal to one another.

∴ AB = AC

2. $\angle 1 = \angle 2$ [Given]

$\angle 1 = \angle 3$ [Given]



According to Axiom 1, things which are equal to the same thing are equal to one another.

$\therefore \quad \angle 2 = \angle 3$

3. If possible, let M and N be two mid-points of line segment AB.

Then, AB = 2 AM and AB = 2AN.

According to Axiom 1: Things which are equal to the same thing are equal to one another.

$\therefore \quad 2AM = 2AN$

$\Rightarrow \quad AM = AN$

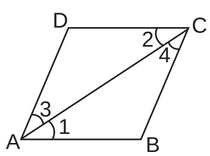
This is possible when M and N coincide.

Hence, a line segment has a unique mid-point.

4. $\angle 1 = \angle 4$ [Given]

$\angle 3 = \angle 2$ [Given]

$\angle 2 = \angle 4$ [Given]

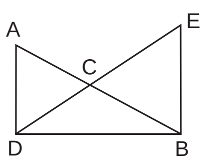


According to axiom 1: Things which are equal to same thing (or equal thing) are equal to one another.

$\therefore \quad \angle 1 = \angle 3$

5. AC = DC [Given] … (1)

CB = CE [Given] … (2)



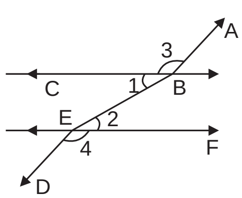
According to Axiom 2: If equals one added to equals, the wholes are equal.

$\therefore \quad AC + CB = DC + CE$ [Using (1) and (2)]

$\therefore \quad AB = DE$

6. $\angle 1 = \angle 2$ [Given] … (1)

$\angle 3 = \angle 4$ [Given] … (2)



According to Axiom 2: If equals are added to equals, the wholes are equal.

$\therefore \quad \angle 1 + \angle 3 = \angle 2 + \angle 4$ [Using (1) and (2)]

$\therefore \quad \angle ABE = \angle BED$

7. Let Vinita's weight = x kg and let Vijay's weight = y kg

Vinita's weight = Vijay weight [Given]

$\therefore \quad x = y$ … (1)

Vinita's gain in weight = 2.5 kg

and Vijay's gain in weight = 2.5 kg

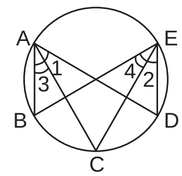
According to Axiom 2: If equals are added to equals, the wholes are equal.

$\therefore \quad x + 2.5 = y + 2.5$

$\therefore$ Vinita's new weight = Vijay's new weight

8. $\angle 1 = \angle 2$ [Given] … (1)

$\angle 3 = \angle 4$ [Given] … (2)

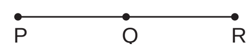


According to Axiom 3: If equals are subtracted from equals, the remainders are equals.

$\therefore \quad \angle 1 - \angle 3 = \angle 2 - \angle 4$

$\therefore \quad \angle CAD = \angle CED$

9. In the given figure



PR coincides with PQ + QR

According to Axiom 4: Things which coincide with one another are equal to one another. So it can be deduced that PQ + QR = PR.

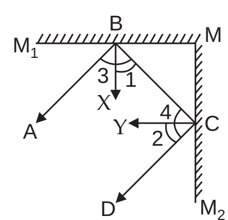
10. $m(\text{rt}\angle) = 90^\circ$

$m(\text{acute } \angle) < 90^\circ$

$\therefore$ Acute angle is a part of a right angle.

According to Axiom 5: The whole is greater than the part. So it can be deduced that a right angle is greater than an acute angle.

11. BX bisects $\angle ABC$



$\therefore \quad \angle 1 = \frac{1}{2} \angle ABC$

$\Rightarrow \quad 2\angle 1 = \angle 3$

BY bisects $\angle BCD$

$\therefore \quad \angle 2 = \frac{1}{2} \angle BCD$

$\Rightarrow \quad 2\angle 2 = \angle 4$

$\angle 1 = \angle 2$ [Given]

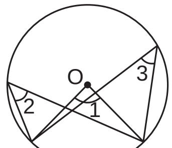
According to Axiom 6 : Things which are double of the same thing (or equal things) are equal to one another.

$\therefore \quad \angle 3 = \angle 4$

Hence, $\angle ABC = \angle BCD$.

12. $\angle 2 = \frac{1}{2}\angle 1$ [Given]

$\angle 3 = \frac{1}{2}\angle 1$ [Given]



According to Axiom 7: Things which are halves of the same thing are equal to one another.

$\therefore$ $\angle 2 = \angle 3$

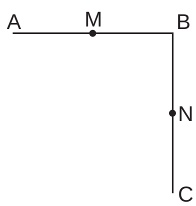
13. (*i*) M is the mid-point of AB [Given]

$\therefore$ $AM = \frac{1}{2} AM$

N is the mid-point of BC [Given]

$\therefore$ $NC = \frac{1}{2} BC$

AB = BC [Given]



According to Axiom 7: Things which are halves of the same thing (or equal things) are equal to one another.

$\therefore$ AM = NC

(*ii*) M is the mid-point of AB [Given]

$\therefore$ AB = 2BM

N is the mid-point of BC [Given]

$\therefore$ BC = 2BN

BM = BN [Given]

According to Axiom 6: Things which are double of the same thing (or equal things) are equal to one another.

$\therefore$ AB = BC

UNIT TEST

1. (*i*) True

A line segment has two end points and therefore it has a definite length.

(*ii*) False

A ray has one end point and therefore it can be intended indefinitely only in one direction.

(*iii*) True

Infinite number of lines can pass through a point.

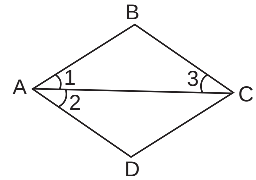
(*iv*) False

Ray OA has O as the initial point and it extends indefinitely in direction OA.

So ray OA is not same as ray AO.

2. $\angle 1 = \angle 2$ [Given]

$\angle 3 = \angle 2$ [Given]

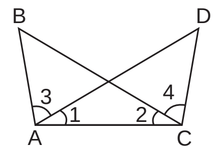


According to Axiom 1: Things which are equal to the same thing are equal to one another.

$\therefore$ $\angle \mathbf{1} = \angle \mathbf{3}$

3. $\angle 1 = \angle 2$ [Given] …(1)

$\angle 3 = \angle 4$ [Given] …(2)



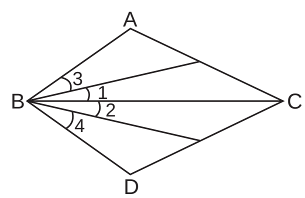
According to Axiom 2: If equals are added to equals, the wholes are equal.

$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$ [Using (1) and (2)]

Hence, $\angle$**A** = $\angle$**C**

4. $\angle 1 = \angle 2$ [Given] …(1)

$\angle 3 = \angle 4$ [Given] …(2)



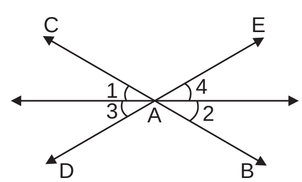
According to Axiom 2: If equals are added to equals, the wholes are equal.

$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$ [Using (1) and (2)]

Hence, $\angle$**ABC** = $\angle$**DBC**

5. $\angle 1 = \angle 2$ [Given] … (1)

$\angle 3 = \angle 4$ [Given] … (2)



According to Axiom 2: If equals are added to equals, the wholes are equal.

$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$ [Using (1) and (2)]

Hence, $\angle$**DAC** = $\angle$**BAE**.

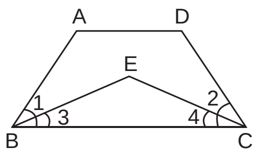
6. $x - 12 = 18$ [Given]

According to Axiom 2: If equals are added to equals, the wholes are equal.

$\therefore$ $x - 12 + 12 = 18 + 12$

Hence, $\boldsymbol{x} = \mathbf{30}$

7. $\angle 1 = \angle 2$ [Given] … (1)
$\angle 3 = \angle 4$ [Given] … (2)

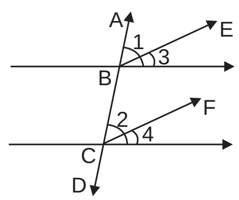


According to Axiom 3: If equals are subtracted from equals, the remainders are equal.

$\therefore$ $\angle 1 - \angle 3 = \angle 2 - \angle 4$ [Using (1) and (2)]

Hence, $\angle$**ABE** = $\angle$**DCE**

8. $\angle 1 = \angle 2$ [Given] … (1)
$\angle 8 = \angle 4$ [Given] … (2)

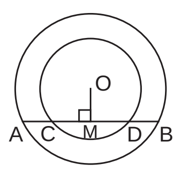


According to Axiom 3: If equals are subtracted from equals, the remainders are equal.

$\therefore$ $\angle 1 - \angle 3 = \angle 2 - \angle 4$ [Using (1) and (2)]

Hence, $\angle$**ABE** = $\angle$**BCF**

9. AM = BM [Given] … (1)
CM = DM [Given] … (2)

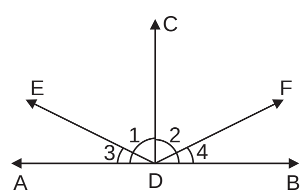


According to Axiom 3: If equals are subtracted from equals, the remainders are equals.

$\therefore$ AM − CM = BM − DM

Hence, **AC = BD**

10. $\angle 1 = \angle 2$ (= 90°) [Given] … (1)
$\angle 3 = \angle 4$ [Given] … (2)



According to Axiom 3: If equals are subtracted from equals, the remainders are equal.

$\therefore$ $\angle 1 - \angle 3 = \angle 2 - \angle 4$ [Using (1) and (2)]

Hence, $\angle$**CDE** = $\angle$**CDF**

11. AH = AB + BC + CD + DE + EF + FG + GH

$\therefore$ (AB + BC + CD + EF) is a part of AH

According to Axiom 5: The whole is greater than the part.

$\therefore$ AH > (AB + BC + CD + EF)

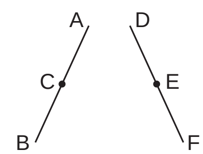
12. C is the mid-point of AB

$\therefore$ AB = 2 BC

E is the mid-point of DF

$\therefore$ DF = 2DE

BC = DE [Given]



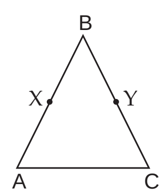
According to Axiom 6: Things which are double of the same thing (or equal things) are equal to one another.

$\therefore$ **AB = DF**

13. $BX = \frac{1}{2} AB$ [Given]

$BY = \frac{1}{2} BC$ [Given]

AB = BC [Given]



According to Axiom 7: Things which are halves of the same thing (or equal things) are equal to one another.

$\therefore$ **BX = BY**