CHAPTER 2

Polynomials

EXERCISE 2A

1. (*i*) $x^3 + \frac{2}{3}x^2 + \sqrt{2}\,x + 1$ has only non-negative integral powers of x. So, it **is a polynomial.**

(*ii*) $\frac{\sqrt{2}}{x^2} + 5 = \sqrt{2}\,x^{-2} + 5$ has one term namely $\sqrt{2}\,x^{-2}$ with negative integral power of x. So, **it is not a polynomial**.

(*iii*) $x + \frac{5}{x} - 2 = x + 5x^{-1} - 2$ has one term namely $5x^{-1}$ with negative integral power of x. So, **it is not a polynomial**.

(*iv*) $x^3 + \frac{2}{x^2} + \frac{3}{x} + 4 = x^3 + 2x^{-2} + 3x^{-1} + 4$ has two terms namely $2x^{-2}$ and $3x^{-1}$ with negative integral powers of x. So, **it is not a polynomial**.

(*v*) $y + 5y^3 + \frac{1}{2}y^2 + 7 = 5y^3 + \frac{1}{2}y^2 + y + 7$ has only non-negative integral powers of y. So, **it is a polynomial.**

(*vi*) $x(x - 2) = x^2 - 2x$ has only non-negative integral powers of x. So, **it is a polynomial.**

(*vii*) $\sqrt{3}\,x^2 - 8x + \sqrt{7}$ has only non-negative integral powers of x. So, **it is a polynomial**.

(*viii*) $\frac{1}{3x^{-2}} + 3x^{-1} + 7 = 3x^2 + 3x^{-1} + 7$ has one term namely $3x^{-1}$ with negative integral power of x. So, **it is not a polynomial.**

(*ix*) $5\sqrt{x} + \sqrt{2}\,y = 5x^{\frac{1}{2}} + \sqrt{2}\,y$ has one term namely $5\sqrt{x}$ in which the power of x is not a whole number. So, **it is not a polynomial**.

(*x*) $8x$ has only non-negative power of x. So, **it is a polynomial**.

(*xi*) $x^2 + y^3 + z^4$ has term consisting of three variables. So, **it is not a polynomial** in one variable.

(*xii*) $\frac{(x-2)(x-4)}{x} = \frac{x^2 - 2x - 4x + 8}{x} = \frac{x^2 - 6x + 8}{x}$

$= \frac{x^2}{x} - \frac{6x}{x} + \frac{8}{x} = x - 6 + 8x^{-1}$ has a term namely $8x^{-1}$ with negative integral power of x. So, **it is not a polynomial**.

2. (*i*) Coefficient of x^2 in the polynomial $4x^2 + 7x$ is 4.

(*ii*) Coefficient of x^2 in the polynomial $4 + 3x - \frac{\pi}{3}x^2$ is $-\frac{\pi}{3}$.

(*iii*) Coefficient of x^2 in the polynomial $\sqrt{5}x + 2$, i.e $0\,x^2 + \sqrt{5}x + 2$ is **0**.

3. (*i*) $x(x - 5) = x^2 - 5x$

Coefficient of x in the given polynomial is **–5**.

(*ii*) $\frac{1}{2x^{-3}} + 3x - 1 = \frac{1}{2}x^3 + 3x - 1$

Coefficient of x^3 in the given polynomial is $\mathbf{\frac{1}{2}}$.

(*iii*) $\frac{\pi}{2}x^2 + 3$

Coefficient of x^2 in the given polynomial is $\mathbf{\frac{\pi}{2}}$.

4.

S.No	Polynomial	Highest power of the variable	Degree of the polynomial
(*i*)	$13 - x + 3x^6$	6	6
(*ii*)	$-7\ (= -7x^0)$	0	0
(*iii*)	$x^3 - \sqrt{2}\,x$	3	3
(*iv*)	$y^3(3 + y^2) = 3y^3 + y^5$	5	5
(*v*)	$x^2 - \frac{1}{5}x + \sqrt{3}$	2	2
(*vi*)	0	Undefined	Undefined

5.

S.No	Polynomial	Highest power of the variable	Degree of the polynomial
(*i*)	$3 - x - x^2$	2	Quadratic
(*ii*)	5	0	Constant
(*iii*)	$3x^3 + 5x^2 + 7x - 4$	3	Cubic
(*iv*)	$x^2 + \sqrt{5}\,x + 3$	2	Quadratic
(*v*)	$y - 5y^3$	3	Cubic
(*vi*)	$\frac{5}{11}x + 3$	1	Linear

6. Answers will vary sample answers are:

(*i*) $3x^{97}$ or $5y^{97}$.

(*ii*) $3x^7 - x^{12} + x$ or $3 - 4x^5 + 7x^{12}$.

EXERCISE 2B

1. Let $p(x) = 5x^3 - 3x^2 - 7x + 11$

(*i*) $p(0) = 5(0)^3 - 3(0)^2 - 7(0) + 11$
$= \mathbf{11}$

(*ii*) $p(1) = 5(1)^3 - 3(1)^2 - 7(1) + 11$
$= 5 - 3 - 7 + 11$
$= 16 - 10$
$= \mathbf{6}$

(*iii*) $p(-1) = 5(-1)^3 - 3(-1)^2 - 7(-1) + 11$
$= -5 - 3 + 7 + 11$
$= -8 + 18$
$= \mathbf{10}$

(*iv*) $p(3) = 5(3)^3 - 3(3)^2 - 7(3) + 11$
$= 5(27) - 3(9) - 21 + 11$
$= 135 - 27 - 21 + 1$
$= 146 - 48$
$= \mathbf{98}$

2. (*i*) Let $p(y) = y^2 - 1$
$p(0) = (0)^2 - 1$
$= \mathbf{-1}$
$p(-1) = (-1)^2 - 1$
$= 1 - 1$
$= \mathbf{0}$
$p(1) = (1)^2 - 1$
$= 1 - 1$
$= \mathbf{0}$
$p(2) = (2)^2 - 1$
$= 4 - 1$
$= \mathbf{3}$

(*ii*) Let $p(x) = 3x(x - 2)$
$= 3x^2 - 6x$
$p(0) = 3(0)^2 - 6(0)$
$= \mathbf{0}$
$p(-1) = 3(-1)^2 - 6(-1)$
$= 3 + 6$
$= \mathbf{9}$
$p(1) = 3(1)^2 - 6(1)$
$= 3 - 6$
$= \mathbf{-3}$
$p(2) = 3(2)^2 - 6(2)$
$= 12 - 12$
$= \mathbf{0}$

(*iii*) Let $p(x) = 3x^4 - 5x^3 + x^2 + 8$
$p(0) = 3(0)^4 - 5(0)^3 + (0)^2 + 8$
$= \mathbf{8}$
$p(-1) = 3(-1)^4 - 5(-1)^3 + (-1)^2 + 8$
$= 3 + 5 + 1 + 8$
$= \mathbf{17}$
$p(1) = 3(1)^4 - 5(1)^3 + (1)^2 + 8$
$= 3 - 5 + 1 + 8$
$= 12 - 5$
$= \mathbf{7}$
$p(2) = 3(2)^4 - 5(2)^3 + (2)^2 + 8$
$= 48 - 40 + 4 + 8$
$= 60 - 40$
$= \mathbf{20}$

(*iv*) Let $p(y) = 2y^3 - 13y^2 + 17y + 12$
$p(0) = 2(0)^3 - 13(0)^2 + 17(0) + 12$
$= \mathbf{12}$
$p(-1) = 2(-1)^3 - 13(-1)^2 + 17(-1) + 12$
$= -2 - 13 - 17 + 12$
$= -32 + 12$
$= \mathbf{-20}$
$p(1) = 2(1)^3 - 13(1)^2 + 17(1) + 12$
$= 2 - 13 + 17 + 12$
$= 31 - 13$
$= \mathbf{18}$
$p(2) = 2(2)^3 - 13(2)^2 + 17(2) + 12$
$= 16 - 52 + 34 + 12$
$= 62 - 52$
$= \mathbf{10}$

3. $p(x) = 4x^3 - 3x^2 + 2x - 4$
$p(2) = 4(2)^3 - 3(2)^2 + 2(2) - 4$
$= 32 - 12 + 4 - 4$
$= \mathbf{20}$
$p(0) = 4(0)^3 - 3(0)^2 + 2(0) - 4$
$= \mathbf{-4}$
$p(1) = 4(1)^3 - 3(1)^2 + 2(1) - 4$
$= 4 - 3 + 2 - 4$
$= \mathbf{-1}$

$$\therefore \frac{p(2)}{p(0) \cdot p(1)} = \frac{20}{(-4)(-1)}$$
$$= \frac{20}{4} = \mathbf{5}$$

4. We know that a real number k is called a zero of the polynomial $p(x)$, if $p(k) = 0$.

(*i*) $p(x) = 9x + 5$
$$p\left(\frac{-5}{9}\right) = 9\left(\frac{-5}{9}\right) + 5$$
$= -5 + 5 = 0$

$\therefore$ $\boldsymbol{x = \frac{-5}{9}}$ **is a zero** of the given polynomial.

Also, $p(0) = 9(0) + 5 = 5 \neq 0$

$\therefore$ $\boldsymbol{x = 0}$ **is not a zero** of the given polynomial.

(*ii*) Let $p(x) = x^2$
$p(0) = (0)^2 = 0$

$\therefore$ $\boldsymbol{x = 0}$ **is a zero** of the given polynomial.

Also, $p(1) = (1)^2 = 1 \neq 0$

$\therefore$ $\boldsymbol{x = 1}$ **is not a zero** of the given polynomial.

(*iii*) Let $p(x) = 2x^2 - x - 1$
$$p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right) - 1$$
$$= 2\left(\frac{1}{4}\right) + \frac{1}{2} - 1$$
$$= \frac{1}{2} + \frac{1}{2} - 1$$
$= 1 - 1$
$= 0$

$\therefore$ $\boldsymbol{x = \frac{-1}{2}}$ **is a zero** of the given polynomial.

Also, $p(1) = 2(1)^2 - (1) - 1 = 2 - 1 - 1 = 0$

$\therefore$ $\boldsymbol{x = 1}$ **is a zero** of the given polynomial.

(*iv*) Let $p(y) = (3y + 2)(y - 1)$
$= 3y^2 + 2y - 3y - 2$
$= 3y^2 - y - 2$
$$p\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 - \left(\frac{-2}{3}\right) - 2$$

$= 3\left(\frac{4}{9}\right) + \frac{2}{3} - 2$

$= \frac{4}{3} + \frac{2}{3} - 2$

$= 2 - 2$

$= 0$

$\therefore$ $x = \frac{-2}{3}$ **is a zero** of the given polynomial.

Also, $p(1) = 3(1)^2 - (1) - 2 = 3 - 1 - 2 = 3 - 3 = 0$

$\therefore$ **$x = 1$ is a zero** of the given polynomial.

(*v*) Let $p(x) = 5x^2 - 1$

$p\left(\frac{-1}{\sqrt{5}}\right) = 5\left(\frac{-1}{\sqrt{5}}\right)^2 - 1$

$= 5\left(\frac{1}{5}\right) - 1$

$= 1 - 1$

$= 0$

$\therefore$ $x = \frac{-1}{\sqrt{5}}$ **is a zero** of the given polynomial.

Also, $p\left(\frac{2}{\sqrt{5}}\right) = 5\left(\frac{2}{\sqrt{5}}\right)^2 - 1$

$= 5\left(\frac{4}{5}\right) - 1$

$= 4 - 1$

$= 3$

$\therefore$ $x = \frac{2}{\sqrt{5}}$ **is not a zero** of the given polynomial.

(*vi*) Let $p(x) = x^2 - 3x$

$p(0) = (0)^2 - 3(0)$

$= 0$

$\therefore$ **$x = 0$ is a zero** of the given polynomial.

Also, $p(3) = (3)^2 - 3(3) = 9 - 9 = 0$

$\therefore$ **$x = 3$ is a zero** of the given polynomial.

(*vii*) Let $p(x) = (x - 1)(x - 3)$

$= x^2 - x - 3x + 3$

$= x^2 - 4x + 3$

$p(1) = (1)^2 - 4(1) + 3$

$= 1 - 4 + 3$

$= 4 - 4$

$= 0$

$\therefore$ **$x = 1$ is a zero** of the given polynomial.

$p(3) = (3)^2 - 4(3) + 3 = 9 - 12 + 3 = 0$

$\therefore$ **$x = 3$ is a zero** of the given polynomial.

(*viii*) Let $p(x) = x^2 - 4x + 4$

$p(4) = 4^2 - 4(4) + 4$

$= 16 - 16 + 4$

$= 4$

$\therefore$ **$x = 4$ is a not zero** of the given polynomial.

$p(-2) = (-2)^2 - 4(-2) + 4 = 4 + 8 + 4 = 16$

$\therefore$ **$x = -2$ is not a zero** of the given polynomial.

5. Let $p(x) = x^2 + 6x + 11$

$= x^2 + 6x + 9 + 2$

$= (x + 3)^2 + 2$

Clearly, $p(x)$ cannot be equal to zero for all real values of x.

$\Rightarrow$ $p(x)$ has no zero.

Hence, $x^2 + 6x + 11$ has no zero.

6. (*i*) Let $p(y) = y - 3$

Zero of the polynomial $p(y) = y - 3$ is given by

$p(y) = 0$

$\Rightarrow$ $y - 3 = 0$

$\Rightarrow$ $y = 3$

Hence, **3** is a zero of the given polynomial.

(*ii*) Let $p(x) = x + 5$

Zero of the polynomial $p(x) = x + 5$ is given by

$p(x) = 0$

$\Rightarrow$ $x + 5 = 0$

$\Rightarrow$ $x = -5$

Hence, **−5** is a zero of the given polynomial.

(*iii*) Let $p(t) = 3t - 5$

Zero of the polynomial $p(t)$ is given by

$p(t) = 0$

$\Rightarrow$ $3t - 5 = 0$

$\Rightarrow$ $3t = 5$

$\Rightarrow$ $t = \frac{5}{3}$

Hence, $\mathbf{\frac{5}{3}}$ is a zero of the given polynomial.

(*iv*) Let $p(x) = 5x + 2$

Zero of the polynomial $p(x) = 5x + 2$ is given by

$p(x) = 0$

$\Rightarrow$ $5x + 2 = 0$

$\Rightarrow$ $5x = -2$

$\Rightarrow$ $x = \frac{-2}{5}$

Hence, $\mathbf{\frac{-2}{5}}$ is a zero of the given polynomial.

(*v*) Let $p(x) = 3 - 4x$

Zero of the polynomial $p(x) = 3 - 4x$ is given by

$p(x) = 0$

$\Rightarrow$ $3 - 4x = 0$

$\Rightarrow$ $4x = 3$

$\Rightarrow$ $x = \frac{3}{4}$

Hence, $\mathbf{\frac{3}{4}}$ is a zero of the given polynomial.

(*vi*) Let $p(x) = 4x - \pi$

Zero of the polynomial $p(x) = 4x - \pi$ is given by

$p(x) = 0$

$\Rightarrow$ $4x - \pi = 0$

$\Rightarrow$ $4x = \pi$

$\Rightarrow$ $x = \frac{\pi}{4}$

Hence, $\mathbf{\frac{\pi}{4}}$ is a zero of the given polynomial.

(*vii*) Let $p(x) = 5x^2$

Zero of the polynomial $p(x) = 5x^2$ is given by

$p(x) = 0$

$\Rightarrow$ $5x^2 = 0$

$\Rightarrow$ $x^2 = 0$

$\Rightarrow$ $x = 0$

Hence, **0** is a zero of the given polynomial.

(*viii*) Let $p(x) = 2x^2 - 5x - 12$

Zeroes of the polynomial $p(x) = 2x^2 - 5x - 12$ are given by

$p(x) = 0$

$\Rightarrow 2x^2 - 5x - 12 = 0$

$\Rightarrow 2x^2 - 8x + 3x - 12 = 0$

$\Rightarrow 2x(x - 4) + 3(x - 4) = 0$

$\Rightarrow (x - 4)(2x + 3) = 0$

$\Rightarrow$ Either $(x - 4) = 0$

or $(2x + 3) = 0$

$\Rightarrow x = 4$

or $x = \frac{-3}{2}$

Hence, **4** and $\mathbf{\frac{-3}{2}}$ are zeroes of the given polynomial.

(*ix*) Let $p(x) = (x + 1)(x + 3)$

$= x^2 + x + 3x + 3$

$= x^2 + 4x + 3$

Zeroes of the polynomial $p(x) = x^2 + 4x + 3$ are given by

$p(x) = 0$

$\Rightarrow x^2 + 4x + 3 = 0$

$\Rightarrow x^2 + x + 3x + 3 = 0$

$\Rightarrow x(x + 1) + 3(x + 1) = 0$

$\Rightarrow (x + 1)(x + 3) = 0$

$\Rightarrow$ Either $(x + 1) = 0$

or $(x + 3) = 0$

$\Rightarrow x = -1$

or $x = -3$

Hence, **–1** and **–3** are zeroes of the given polynomial.

(*x*) Let $p(x) = x^2 - 5$

Zeroes of the polynomial $p(x) = x^2 - 5$ are given by

$p(x) = 0$

$\Rightarrow x^2 - 5 = 0$

$\Rightarrow x^2 = 5$

$\Rightarrow x = \pm\sqrt{5}$

Hence, $\mathbf{\sqrt{5}}$ and $\mathbf{-\sqrt{5}}$ are the zeroes of the given polynomial.

(*xi*) Let $p(x) = 4x^2 - 1$

Zeroes of the polynomial $p(x) = 4x^2 - 1$ are given by

$p(x) = 0$

$\Rightarrow 4x^2 - 1 = 0$

$\Rightarrow 4x^2 = 1$

$\Rightarrow x^2 = \frac{1}{4}$

$\Rightarrow x = \sqrt{\frac{1}{4}}$

$\Rightarrow x = \pm\frac{1}{2}$

Hence, $\mathbf{\frac{1}{2}}$ and $\mathbf{\frac{-1}{2}}$ are the zeroes of the given polynomial.

7. Let $p(x) = ax^3 - x^2 + x + 4$

Since (–1) is a zero of $p(x)$,

$\therefore p(-1) = 0$

$\Rightarrow a(-1)^3 - (-1)^2 + (-1) + 4 = 0$

$\Rightarrow -a - 1 - 1 + 4 = 0$

$\Rightarrow -a + 2 = 0$

$\Rightarrow a = \mathbf{2}$

8. Since $\left(\frac{-3}{2}\right)$ is a zero of $p(x) = 2x^3 + 9x^2 - x - a$,

$\therefore p\left(\frac{-3}{2}\right) = 0$

$\Rightarrow 2\left(\frac{-3}{2}\right)^3 + 9\left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right) - a = 0$

$\Rightarrow 2\left(\frac{-27}{8}\right) + 9\left(\frac{9}{4}\right) + \frac{3}{2} - a = 0$

$\Rightarrow \frac{-27}{4} + \frac{81}{4} + \frac{3}{2} = a$

$\Rightarrow a = \frac{-27 + 81 + 6}{4}$

$\Rightarrow a = \frac{60}{4}$

$\Rightarrow a = \mathbf{15}$

9. $p(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$

Since 2 is a zero of $p(x)$,

$\therefore p(2) = 0$

$\Rightarrow a(2)^4 + 2(2)^3 - 3(2)^2 + b(2) - 4 = 0$

$\Rightarrow 16a + 16 - 12 + 2b - 4 = 0$

$\Rightarrow 16a + 2b = 0$

$\Rightarrow 8a + b = 0$ … (1)

Since (–2) is a zero of $p(x)$,

$\therefore p(-2) = 0$

$\Rightarrow a(-2)^4 + 2(-2)^3 - 3(-2)^2 + b(-2) - 4 = 0$

$\Rightarrow 16a - 16 - 12 - 2b - 4 = 0$

$\Rightarrow 16a - 2b = 32$

$\Rightarrow 8a - b = 16$ … (2)

Adding equation (1) and equation (2), we get

$16a = 16$

$\Rightarrow a = 1$

Substituting $a = 1$ in equation (1), we get

$8(1) + b = 0$

$\Rightarrow b = -8$

Hence, $\mathbf{a = 1}$ and $\mathbf{b = -8}$

10. Let $p(x) = ax^2 + 5x + b$

Since 2 is a zero of $p(x)$,

$\therefore p(2) = 0$

$\Rightarrow a(2)^2 + 5(2) + b = 0$

$\Rightarrow 4a + 10 + b = 0$

$\Rightarrow 4a + b = -10$ … (1)

Since $\frac{1}{2}$ is a zero of $p(x)$,

$\therefore p\left(\frac{1}{2}\right) = 0$

$\Rightarrow a\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + b = 0$

$\Rightarrow \frac{a}{4} + \frac{5}{2} + b = 0$

$\Rightarrow a + 10 + 4b = 0$

$\Rightarrow a + 4b = -10$ … (2)

From (1) and (2), we get

$4a + b = a + 4b$

$\Rightarrow 4a - a = 4b - b$

$\Rightarrow 3a = 3b$

$\Rightarrow a = b$

Hence, $\mathbf{a = b}$.

EXERCISE 2C

1. By the remainder theorem, when $p(x)$ is divided by $x - 1$, the remainder is equal to $p(1)$.

Now, $p(x) = 3x^3 + 4x^2 - 4x - 2$

$\Rightarrow$ $p(1) = 3(1)^3 + 4(1)^2 - 4(1) - 2$
$= 3 + 4 - 4 - 2$
$= 1$

Hence, the required remainder is **1**.

2. By the remainder theorem, when $p(x)$ is divided by $x - 3$, the remainder is equal to $p(3)$.

Now, $p(x) = x^6 - 3x^5 + 2x^2 + 8$

$\Rightarrow$ $p(3) = (3)^6 - 3(3)^5 + 2(3)^2 + 8$
$= 3^6 - 3^6 + 18 + 8$
$= 26$

Hence, the required remainder is **26**.

3. By the remainder theorem, when $p(x)$ is divided by $x + 2 = x - (-2)$, the remainder is equal to $p(-2)$.

Now, $p(x) = 4x^3 - 3x^2 + 2x - 4$

$\Rightarrow$ $p(-2) = 4(-2)^3 - 3(-2)^2 + 2(-2) - 4$
$= -32 - 12 - 4 - 4$
$= -52$

Hence, the required remainder is **–52**.

4. By the remainder theorem, when $p(x)$ is divided by $x + 1 = x - (-1)$, the remainder is equal to $p(-1)$.

Now, $p(x) = x^{23} - x^{19} - 1$

$\Rightarrow$ $p(-1) = (-1)^{23} - (-1)^{19} - 1$
$= -1 - (-1) - 1$
$= -1 + 1 - 1$
$= -1$

Hence, the required remainder is **–1**.

5. By the remainder theorem, when $p(x)$ is divided by $x - \frac{1}{2}$, the remainder is equal to $p\left(\frac{1}{2}\right)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$$\Rightarrow p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$
$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$$
$$= \frac{1 + 6 + 12 + 8}{8}$$
$$= \frac{27}{8}$$

Hence, the required remainder is $\mathbf{\frac{27}{8}}$.

6. By the remainder theorem, when $p(x)$ is divided by $x + \frac{1}{2} = x - \left(\frac{-1}{2}\right)$, the remainder is equal to $p\left(\frac{-1}{2}\right)$.

Now, $p(x) = 4x^3 - 3x^2 + 2x - 4$

$$\Rightarrow p\left(\frac{-1}{2}\right) = 4\left(\frac{-1}{2}\right)^3 - 3\left(\frac{-1}{2}\right)^2 + 2\left(\frac{-1}{2}\right) - 4$$
$$= 4\left(\frac{-1}{8}\right) - 3\left(\frac{1}{4}\right) - \frac{2}{2} - 4$$
$$= \frac{-4}{8} - \frac{3}{4} - 1 - 4$$
$$= \frac{-1}{2} - \frac{3}{4} - 5$$
$$= \frac{-2 - 3 - 20}{4}$$
$$= \frac{-25}{4}$$

Hence, the required remainder is $\mathbf{\frac{-25}{4}}$.

7. By the remainder theorem, when $p(x)$ is divided by $2x - 5 = 2\left(x - \frac{5}{2}\right)$, the remainder is equal to $p\left(\frac{5}{2}\right)$.

Now, $p(x) = 2x^3 - 11x^2 + 19x - 10$

$$\Rightarrow p\left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right)^3 - 11\left(\frac{5}{2}\right)^2 + 19\left(\frac{5}{2}\right) - 10$$
$$= 2\left(\frac{125}{8}\right) - 11\left(\frac{25}{4}\right) + \frac{95}{2} - 10$$
$$= \frac{125}{4} - \frac{275}{4} + \frac{95}{2} - 10$$
$$= \frac{125 - 275 + 190 - 40}{4}$$
$$= \frac{315 - 315}{4} = 0$$

Hence, the required remainder is **0**.

8. By the remainder theorem, when $p(x)$ is divided by $5 + 4x = 4x + 5 = 4\left(x + \frac{5}{4}\right) = 4\left[x - \left(\frac{-5}{4}\right)\right]$, the remainder is equal to $p\left(\frac{-5}{4}\right)$.

Now, $p(x) = 4x^4 + 5x^3 - 12x^2 - 11x + 7$

$$\Rightarrow p\left(\frac{-5}{4}\right) = 4\left(\frac{-5}{4}\right)^4 + 5\left(\frac{-5}{4}\right)^3 - 12\left(\frac{-5}{4}\right)^2 - 11\left(\frac{-5}{4}\right) + 7$$
$$= 4\left(\frac{625}{256}\right) - \frac{625}{64} - 12\left(\frac{25}{16}\right) + \frac{55}{4} + 7$$
$$= \frac{625}{64} - \frac{625}{64} - \frac{75}{4} + \frac{55}{4} + 7$$
$$= \frac{-75 + 55 + 28}{4}$$
$$= \frac{-75 + 83}{4}$$
$$= \frac{8}{4} = 2$$

Hence, the required remainder is **2**.

9. By the remainder theorem, when $p(x)$ is divided by $1 - 2x = -2x + 1 = -2\left(x - \frac{1}{2}\right)$, the remainder is equal to $p\left(\frac{1}{2}\right)$.

Now, $p(x) = x^3 - 6x^2 + 2x - 4$

$$\Rightarrow p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4$$
$$= \frac{1}{8} - \frac{6}{4} + 1 - 4$$

$$= \frac{1}{8} - \frac{3}{2} - 3$$

$$= \frac{1 - 12 - 24}{8}$$

$$= \frac{1 - 36}{8}$$

$$= \frac{-35}{8}$$

Hence, the required remainder is $\mathbf{\frac{-35}{8}}$.

10. By the remainder theorem, when $p(x)$ is divided by $x - 1$, the remainder is equal to $p(1)$.

Now, $p(x) = 3x^4 - 4x^3 - 3x + 4$

$\Rightarrow p(1) = 3(1)^4 - 4(1)^3 - 3(1) + 4$
$= 3 - 4 - 3 + 4$
$= 0$

So, when $p(x)$ is divided by $x - 1$, the remainder is 0.
This shows that $x - 1$ is a factor of $p(x)$
Hence, $p(x)$ is a multiple of $x - 1$.

11. By the remainder theorem, when $p(x)$ is divided by $x + 3 = x - (-3)$, the remainder is equal to $p(-3)$.

Now, $p(x) = ax^3 - 3x^2 - 11x + 20$

$\Rightarrow p(-3) = a(-3)^3 - 3(-3)^2 - 11(-3) + 20$
$= -27a - 27 + 33 + 20$
$= -27a + 26$

Since $p(x)$ is a multiple of $x + 3$,

$\therefore$ the remainder = 0
$\therefore -27a + 26 = 0$
$\Rightarrow 27a = 26$
$\Rightarrow a = \mathbf{\frac{26}{27}}$

12. By the remainder theorem, when $p(x)$ is divided by $x - 1$, the remainder is equal to $p(1)$.

Now, $p(x) = 3x^3 + 14x^2 - 2x - 15$

$\Rightarrow p(1) = 3(1)^3 + 14(1)^2 - 2(1) - 15$
$= 3 + 14 - 2 - 15$
$= 0$

Remainder = 0

Hence, $p(x) = 3x^3 + 14x^2 - 2x - 15$ is a multiple of $x - 1$.

13. By the remainder theorem, when $p(x)$ is divided by $x - 2$, the remainder is equal to $p(2)$.

Now, $p(x) = x^4 - 2x^3 + 3x^2 - ax + 8$

$\Rightarrow p(2) = (2)^4 - 2(2)^3 + 3(2)^2 - a(2) + 8$
$= 10$ [Given]

$\Rightarrow 16 - 16 + 12 - 2a + 8 = 10$
$\Rightarrow 20 - 10 = 2a$
$\Rightarrow 10 = 2a$
$\Rightarrow a = \mathbf{5}$

14. By the remainder theorem, when $p(x)$ and $q(x)$ are divided by $x - 2$, the remainders are equal to $p(2)$ and $q(2)$ respectively.

Now, $p(x) = ax^3 + 3x^2 - 13$

$\Rightarrow p(2) = a(2)^3 + 3(2)^2 - 13$
$= 8a + 12 - 13$
$= 8a - 1$

and $q(x) = 2x^3 - 5x + a$

$\Rightarrow q(2) = 2(2)^3 - 5(2) + a$
$= 16 - 10 + a$
$= 6 + a$

Now, $p(2) = q(2)$ [Given]

$\Rightarrow 8a - 1 = 6 + a$
$\Rightarrow 8a - a = 6 + 1$
$\Rightarrow 7a = 7$
$\Rightarrow \mathbf{a = 1}$

15. By the remainder theorem, when $p(x)$ is divided by $(x - 5)$, the remainder is equal to $p(5)$ and when it is divided by $(x - 3)$, the remainder is equal to $p(3)$.

Now, $p(x) = x^3 + ax^2 + bx - 20$

$\Rightarrow p(5) = (5)^3 + a(5)^2 + b(5) - 20$
$= 125 + 25a + 5b - 20$
$= 105 + 25a + 5b$
$= 5(21 + 5a + b)$

and $p(3) = (3)^3 + a(3)^2 + b(3) - 20$
$= 27 + 9a + 3b - 20$
$= 7 + 9a + 3b$

Now, $p(5) = 0$ [Given]

$\Rightarrow 5(21 + 5a + b) = 0$
$\Rightarrow 5a + b = -21$ … (1)

and $p(3) = -2$ [Given]

$\Rightarrow 7 + 9a + 3b = -2$
$\Rightarrow 3a + b = -3$ … (2)

Subtracting equation (2) from equation (1), we get

$2a = -18$
$\Rightarrow a = -9$

Substituting $a = -9$ in equation (2), we get

$3(-9) + b = -3$
$\Rightarrow -27 + b = -3$
$\Rightarrow b = 24$

Hence, $\mathbf{a = -9, b = 24.}$

16. By the remainder theorem, when $p(x)$ is divided by $x + 2 = x - (-2)$ and $x - 2$, the remainders are equal to $p(-2)$ and $p(2)$ respectively.

Now, $p(x) = ax^3 + bx^2 + x - 6$

$\Rightarrow p(-2) = a(-2)^3 + b(-2)^2 + (-2) - 6$
$= -8a + 4b - 2 - 6$
$= -8a + 4b - 8$

and $p(2) = a(2)^3 + b(2)^2 + (2) - 6$
$= 8a + 4b + 2 - 6$
$= 8a + 4b - 4$

$p(-2) = 0$ [Given]

$\Rightarrow -8a + 4b - 8 = 0$
$\Rightarrow -4a + b = 2$ … (1)

and $p(2) = 4$ [Given]

$\Rightarrow 8a + 4b - 4 = 4$
$\Rightarrow 2a + b = 2$ … (2)

Subtracting equation (1) from equation (2), we get

$6a = 0$
$\Rightarrow a = 0$

Substituting $a = 0$ in equation (2), we get

$2(0) + b = 2$
$\Rightarrow b = 2$

Hence, $\mathbf{a = 0, b = 2}$.

17. By the remainder theorem, when $m(x)$ and $n(x)$ are divided by $x - 2$, the remainders are equal to $m(2)$ and $n(2)$ respectively,

Now, $m(x) = ax^3 - 3x^2 + 4$

$\Rightarrow m(2) = a(2)^3 - 3(2)^2 + 4$
$\Rightarrow m(2) = 8a - 12 + 4$
$\Rightarrow m(2) = 8a - 8$

$m(2) = p$ [Given]

$\Rightarrow \quad 8a - 8 = p \quad \dots (1)$

and $\quad n(x) = 2x^3 - 5x + a$

$\Rightarrow \quad n(2) = 2(2)^3 - 5(2) + a$

$\Rightarrow \quad n(2) = 16 - 10 + a$

$\Rightarrow \quad n(2) = 6 + a$

and $\quad n(2) = q \quad$ [Given]

and $\quad 6 + a = q \quad \dots (2)$

Also, $\quad p - 2q = 4 \quad$ [Given]

$\Rightarrow \quad 8a - 8 - 2(6 + a) = 4 \quad$ [Using (1) and (2)]

$\Rightarrow \quad 8a - 8 - 12 - 2a = 4$

$\Rightarrow \quad 6a = 4 + 8 + 12$

$\Rightarrow \quad 6a = 24$

$\Rightarrow \quad a = \mathbf{4}$

18. By the remainder theorem, when polynomials $m(x)$ and $n(x)$ are divided by $(x - 2)$ and $(x + 1)$ respectively, then the remainders are equal to $m(2)$ and $n(-1)$ respectively.

Now, $\quad m(x) = x^3 + 2x^2 - 5ax - 8$

$\Rightarrow \quad m(2) = 2^3 + 2(2)^2 - 5a(2) - 8$

$\Rightarrow \quad m(2) = 8 + 8 - 10a - 8$

$\Rightarrow \quad m(2) = 8 - 10a$

$\Rightarrow \quad p = 8 - 10a$

[$\because m(2) = p$, given] … (1)

and $\quad n(x) = x^3 + ax^2 - 12x - 6$

$\Rightarrow \quad n(-1) = (-1)^3 + a(-1)^2 - 12(-1) - 6$

$\Rightarrow \quad n(-1) = -1 + a + 12 - 6$

$\Rightarrow \quad n(-1) = a + 5$

$\Rightarrow \quad q = a + 5$

[$\because n(-1) = q$, given] … (2)

Also, $\quad q - p = 8$

$\Rightarrow \quad a + 5 - (8 - 10a) = 8 \quad$ [Using (1) and (2)]

$\Rightarrow \quad a + 5 - 8 + 10a = 8$

$\Rightarrow \quad 11a = 8 - 5 + 8$

$\Rightarrow \quad 11a = 11$

$\Rightarrow \quad a = \mathbf{1}$

19. By the remainder theorem, when $m(x)$ and $n(x)$ are divided by $(x + 1)$ and $(x - 2)$, then the remainders are equal to $m(-1)$ and $n(2)$ respectively.

Now, $\quad m(x) = x^3 + 2x^2 - 5ax - 7$

$\Rightarrow \quad m(-1) = (-1)^3 + 2(-1)^2 - 5a(-1) - 7$

$\Rightarrow \quad m(-1) = -1 + 2 + 5a - 7$

$\Rightarrow \quad m(-1) = 5a - 6$

$\Rightarrow \quad p = 5a - 6$

[$\because m(-1) = p$, given] … (1)

and $\quad n(x) = x^3 + ax^2 - 12x + 6$

$\Rightarrow \quad n(2) = (2)^3 + a(2)^2 - 12(2) + 6$

$\Rightarrow \quad n(2) = 8 + 4a - 24 + 6$

$\Rightarrow \quad n(2) = 4a - 10$

$\Rightarrow \quad q = 4a - 10$

[$\because n(2) = q$, given] … (2)

Also, $\quad 2p + q = 6 \quad$ [Given]

$\Rightarrow \quad 2(5a - 6) + 4a - 10 = 6 \quad$ [Using (1) and (2)]

$\Rightarrow \quad 10a - 12 + 4a - 10 = 6$

$\Rightarrow \quad 14a = 6 + 12 + 10$

$\Rightarrow \quad 14a = 28$

$\Rightarrow \quad a = \mathbf{2}$

20.
$$\begin{aligned} 2x^3 - 3x^2 + x &= x(2x^2 - 3x + 1) \\ &= x(2x^2 - 2x - x + 1) \\ &= x[2x(x - 1) - 1(x - 1)] \\ &= x(x - 1)(2x - 1) \end{aligned}$$

By the remainder theorem, when $p(x)$ is divided by $x\ (= x - 0)$, $x - 1$ and $2x - 1\left[= 2\left(x - \frac{1}{2}\right)\right]$, the remainders are given by $p(0)$, $p(1)$ and $p\left(\frac{1}{2}\right)$ respectively.

Now, $\quad p(x) = (x - 1)^{2a} - x^{2a} + 2x - 1$

$$\begin{aligned} \Rightarrow \quad p(0) &= (0 - 1)^{2a} - (0)^{2a} + 2(0) - 1 \\ &= 1 - 0 + 0 - 1 \\ &= 0 \\ p(1) &= (1 - 1)^{2a} - 1^{2a} + 2(1) - 1 \\ &= 0 - 1 + 2 - 1 \\ &= 0 \end{aligned}$$

and
$$\begin{aligned} p\left(\frac{1}{2}\right) &= \left(\frac{1}{2} - 1\right)^{2a} - \left(\frac{1}{2}\right)^{2a} + 2\left(\frac{1}{2}\right) - 1 \\ &= \left(\frac{-1}{2}\right)^{2a} - \left(\frac{1}{2}\right)^{2a} + 1 - 1 \\ &= \left(\frac{1}{2}\right)^{2a} - \left(\frac{1}{2}\right)^{2a} + 1 - 1 \\ &= 0 \end{aligned}$$

Since $p(0) = 0$, $p(1) = 0$ and $p\left(\frac{1}{2}\right) = 0$,

$\therefore \quad p(x)$ is divisible by $x(x - 1)$ and $(2x - 1)$.

$\Rightarrow \quad p(x)$ is divisible by $x(x - 1)(2x - 1)$.

$\Rightarrow \quad p(x)$ is divisible by $2x^3 - 3x^2 + x$.

EXERCISE 2D

1. By the factor theorem, $(x - 3)$ will be a factor of $f(x)$ if $f(3) = 0$.

Now, $\quad f(x) = x^3 + x^2 - 17x + 15$

$$\begin{aligned} \Rightarrow \quad f(3) &= (3)^3 + (3)^2 - 17(3) + 15 \\ &= 27 + 9 - 51 + 15 \\ &= 51 - 51 \\ &= 0 \end{aligned}$$

$\Rightarrow \quad (x - 3)$ is a factor of $f(x)$.

2. By the factor theorem, $x - 2$ will be a factor of $f(x)$ if $f(2) = 0$.

Now, $\quad f(x) = x^3 - 5x^2 + 2x + 8$

$$\begin{aligned} \Rightarrow \quad f(2) &= (2)^3 - 5(2)^2 + 2(2) + 8 \\ &= 8 - 20 + 4 + 8 \\ &= 20 - 20 \\ &= 0 \end{aligned}$$

$\Rightarrow \quad (x - 2)$ is a factor of $f(x)$.

3. By the factor theorem, $x + 1 = x - (-1)$ will be a factor of $f(x)$ if $f(-1) = 0$.

Now, $\quad f(x) = 2x^3 + 4x + 6$

$$\begin{aligned} \Rightarrow \quad f(-1) &= 2(-1)^3 + 4(-1) + 6 \\ &= -2 - 4 + 6 \\ &= -6 + 6 \\ &= 0 \end{aligned}$$

$\Rightarrow \quad x - (-1)$, i.e. $x + 1$ is a factor of $f(x)$.

4. By the factor theorem, $x + 1 = x - (-1)$ will be a factor of $f(x)$ if $f(-1) = 0$.

Now, $\quad f(x) = x^3 - x^2 - (2 + \sqrt{2})\,x - \sqrt{2}$

$\Rightarrow$ $f(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) - \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} - \sqrt{2}$

$= 0$

$\Rightarrow$ $x - (-1)$, i.e. $x + 1$ is a factor of $f(x)$.

5. By the factor theorem, $x + 2 = x - (-2)$ will be a factor of $f(x)$ if $f(-2) = 0$.

Now, $f(x) = x^3 + 3x^2 + 3x + 2$

$\Rightarrow$ $f(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 2$

$= -8 + 12 - 6 + 2$

$= -14 + 14$

$= 0$

$\Rightarrow$ $x - (-2)$, i.e $x + 2$ is a factor of $f(x)$.

6. By the factor theorem, $2x - 5 = 2\left(x - \frac{5}{2}\right)$ will be a factor of f(x) if $f\left(\frac{5}{2}\right) = 0$.

Now, $f(x) = 2x^3 - 3x^2 - 3x - 5$

$\Rightarrow$ $f\left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right)^3 - 3\left(\frac{5}{2}\right)^2 - 3\left(\frac{5}{2}\right) - 5$

$= 2\left(\frac{125}{8}\right) - 3\left(\frac{25}{4}\right) - \frac{15}{2} - 5$

$= \frac{125}{4} - \frac{75}{4} - \frac{15}{2} - 5$

$= \frac{125 - 75 - 30 - 20}{4}$

$= \frac{125 - 125}{4}$

$= 0$

$\Rightarrow$ $2\left(x - \frac{5}{2}\right)$, i.e $2x - 5$ is a factor of $f(x)$.

7. By the factor theorem, $3x + 2 = 3\left(x + \frac{2}{3}\right) = 3\left[x - \left(\frac{-2}{3}\right)\right]$ will be a factor of $f(x)$ if $f\left(-\frac{2}{3}\right) = 0$.

Now, $f(x) = 6x^3 + 31x^2 + 3x - 10$

$\Rightarrow$ $f\left(-\frac{2}{3}\right) = 6\left(-\frac{2}{3}\right)^3 + 31\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) - 10$

$= 6\left(\frac{-8}{27}\right) + 31\left(\frac{4}{9}\right) - 2 - 10$

$= \frac{-16}{9} + \frac{124}{9} - 12$

$= \frac{-16 + 124 - 108}{9}$

$= \frac{124 - 124}{9}$

$= 0$

$\Rightarrow$ $3x + 2$ is a factor of $f(x)$.

8. By the factor theorem, $x + \sqrt{2} = x - (-\sqrt{2})$ is a factor of $f(x)$ if $f(-\sqrt{2}) = 0$.

Now, $f(x) = 2\sqrt{2}\,x^2 + 5x + \sqrt{2}$

$\Rightarrow$ $f(-\sqrt{2}) = 2\sqrt{2}(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2}$

$= 2\sqrt{2}(2) - 5\sqrt{2} + \sqrt{2}$

$= 4\sqrt{2} + \sqrt{2} - 5\sqrt{2}$

$= 0$

$\Rightarrow$ $x - (-\sqrt{2})$, i.e. $x + \sqrt{2}$ is a factor of $f(x)$.

9. $g(x) = x^2 + 2x - 3$

$= x^2 + 3x - x - 3$

$= x(x + 3) - 1(x + 3)$

$= (x + 3)(x - 1)$

Clearly, $(x - 1)$ and $(x + 3)$ are factors of $g(x)$.

By the factor theorem, $(x - 1)$ will be a factor of $f(x)$ if $f(1) = 0$ and $(x + 3) = x - (-3)$ will be a factor of $f(x)$ if $f(-3) = 0$.

Now, $f(x) = x^4 + 2x^3 - 2x^2 + 2x - 3$

$\Rightarrow$ $f(1) = (1)^4 + 2(1)^3 - 2(1)^2 + 2(1) - 3$

$= 1 + 2 - 2 + 2 - 3$

$= 0$

and $f(-3) = (-3)^4 + 2(-3)^3 - 2(-3)^2 + 2(-3) - 3$

$= 81 - 54 - 18 - 6 - 3$

$= 81 - 81$

$= 0$

Since both $f(1)$ and $f(-3)$ are equal to zero,

$\therefore$ $(x - 1)$ and $(x + 3)$ are factors of $f(x)$.

$\Rightarrow$ $(x - 1)(x + 3) = x^2 + 2x - 3$ is a factor of $f(x)$.

10. By the factor theorem, $(x - 1)$ will be a factor of $p(x)$ if $p(1) = 0$.

(*i*) Let $p(x) = x^4 - 4x^2 + 2x + 1$

$\Rightarrow$ $p(1) = (1)^4 - 4(1)^2 + 2(1) + 1$

$= 1 - 4 + 2 + 1$

$= 4 - 4$

$= 0$

Hence, $(x - 1)$ is a factor of the given polynomial.

(*ii*) Let $p(x) = 2\sqrt{2}\,x^3 + 5\sqrt{2}\,x^2 - 7\sqrt{2}$

$\Rightarrow$ $p(1) = 2\sqrt{2}(1)^3 + 5\sqrt{2}(1)^2 - 7\sqrt{2}$

$= 2\sqrt{2} + 5\sqrt{2} - 7\sqrt{2}$

$= 0$

Hence, $(x - 1)$ is a factor of the given polynomial.

(*iii*) Let $p(x) = x^6 - x^5 + x^4 - x^3 + x^2 - 1$

$\Rightarrow$ $p(1) = (1)^6 - (1)^5 + (1)^4 - (1)^3 + (1)^2 - 1$

$= 1 - 1 + 1 - 1 + 1 - 1$

$= 0$

Hence, $(x - 1)$ is a factor of the given polynomial.

(*iv*) Let $p(x) = 3x^6 - 7x^5 + 7x^4 - 3x^3 + 2x^2 - 2$

$\Rightarrow$ $p(1) = 3(1)^6 - 7(1)^5 + 7(1)^4 - 3(1)^3 + 2(1)^2 - 2$

$= 3 - 7 + 7 - 3 + 2 - 2$

$= 0$

Hence, $(x - 1)$ is a factor of the given polynomial.

11. By the factor theorem, $(x + 1) = x - (-1)$ will be a factor of $p(x)$ if $p(-1) = 0$ and $(x + 10) = x - (-10)$ will be a factor of $p(x)$ if $p(-10) = 0$.

Let $p(x) = x^3 + 13x^2 + 32x + 20$

$\Rightarrow$ $p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33$

$= 0$

and $p(-10) = (-10)^3 + 13(-10)^2 + 32(-10) + 20$

$= -1000 + 1300 - 320 + 20$

$= 1320 - 1320$

$= 0$

Both $p(-1)$ and $p(-10)$ are equal to zero.

$\therefore$ $(x + 1)$ and $(x + 10)$ are factors of the given polynomial.

12. By the factor theorem, $(x - 1)$ is a factor of $p(x)$ if $p(1) = 0$, $x + 1 = x - (-1)$ is a factor of $p(x)$ if $p(-1) = 0$ and $2x + 1 = 2\left(x + \frac{1}{2}\right) = 2\left[x - \left(-\frac{1}{2}\right)\right]$ is a factor of $p(x)$ if $p\left(-\frac{1}{2}\right) = 0$.

Let $p(x) = 2x^3 + x^2 - 2x - 1$

$\Rightarrow$ $p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$
$= 2 + 1 - 2 - 1$
$= 0$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$= -2 + 1 + 2 - 1$
$= 0$

and $p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 + \left(\frac{-1}{2}\right)^2 - 2\left(\frac{-1}{2}\right) - 1$
$= 2\left(\frac{-1}{8}\right) + \frac{1}{4} + 1 - 1$
$= -\frac{1}{4} + \frac{1}{4} + 1 - 1$
$= 0$

$\therefore$ $(x - 1)$, $(x + 1)$ and $(2x + 1)$ are factors of the given polynomial.

13. Let $p(x) = 3x^4 - 2x^3 - 2x^2 - 2x - 5$

and $g(x) = 3x^2 - 2x - 5$

Now, $g(x) = 3x^2 - 2x - 5$
$= 3x^2 - 5x + 3x - 5$
$= x(3x - 5) + 1(3x - 5)$
$= (3x - 5)(x + 1)$

Clearly, $(x + 1)$ and $(3x - 5)$ are factors of $g(x)$.

$\therefore$ $p(x)$ will be exactly divisible by $g(x)$ if both $(x + 1)$ and $(3x - 5)$ are its factors.

By the factor theorem, $(x + 1) = x - (-1)$ and $3x - 5 = 3\left(x - \frac{5}{3}\right)$ will be factors of $p(x)$ if $p(-1) = 0$ and $p\left(\frac{5}{3}\right) = 0$.

Now, $p(x) = 3x^4 - 2x^3 - 2x^2 - 2x - 5$

$\Rightarrow$ $p(-1) = 3(-1)^4 - 2(-1)^3 - 2(-1)^2 - 2(-1) - 5$
$= 3 + 2 - 2 + 2 - 5 = 0$

and $p\left(\frac{5}{3}\right) = 3\left(\frac{5}{3}\right)^4 - 2\left(\frac{5}{3}\right)^3 - 2\left(\frac{5}{3}\right)^2 - 2\left(\frac{5}{3}\right) - 5$
$= 3\left(\frac{625}{81}\right) - 2\left(\frac{125}{27}\right) - 2\left(\frac{25}{9}\right) - \frac{10}{3} - 5$
$= \frac{625}{27} - \frac{250}{27} - \frac{50}{9} - \frac{10}{3} - 5$
$= \frac{625 - 250 - 150 - 90 - 135}{27}$
$= \frac{625 - 625}{27}$
$= 0$

Since both $p(-1)$ and $p\left(\frac{5}{3}\right)$ are equal to zero,

$\therefore$ $(x + 1)$ and $(3x - 5)$ are factors of $p(x)$.

$\Rightarrow$ $g(x) = (x + 1)(3x - 5) = 3x^2 - 2x - 5$ is a factor of $p(x)$.

Hence, given polynomial is exactly divisible by $3x^2 - 2x - 5$.

14. (*i*) Let $p(x) = x^2 + x + a$

Since $(x - 1)$ is a factor of $p(x)$, therefore by the factor theorem, we have

$p(1) = 0$

$\Rightarrow$ $(1)^2 + (1) + a = 0$

$\Rightarrow$ $2 + a = 0$

$\Rightarrow$ $\boldsymbol{a = -2}$

(*ii*) Let $p(x) = 2x^2 + ax + \sqrt{2}$

Since $(x - 1)$ is a factor of $p(x)$, therefore by the factor theorem, we have

$p(1) = 0$

$\Rightarrow$ $2(1)^2 + a(1) + \sqrt{2} = 0$

$\Rightarrow$ $2 + a + \sqrt{2} = 0$

$\Rightarrow$ $\boldsymbol{a = -2 - \sqrt{2}}$

(*iii*) Let $p(x) = a^2x^3 - 4ax + 4a - 1$

Since $(x - 1)$ is a factor of $p(x)$, therefore by the factor theorem, we have

$p(1) = 0$

$\Rightarrow$ $a^2(1)^3 - 4a(1) + 4a - 1 = 0$

$\Rightarrow$ $a^2 - 4a + 4a - 1 = 0$

$\Rightarrow$ $a^2 - 1 = 0$

$\Rightarrow$ $\boldsymbol{a = \pm 1}$

15. Let $p(x) = 2x^3 + kx^2 + x - 10$

Since $x + 2 = x - (-2)$ is a factor of $p(x)$, therefore by the factor theorem, we have

$p(-2) = 0$

$\Rightarrow$ $2(-2)^3 + k(-2)^2 + (-2) - 10 = 0$

$\Rightarrow$ $-16 + 4k - 12 = 0$

$\Rightarrow$ $4k = 28$

$\Rightarrow$ $\boldsymbol{k = 7}$

16. Let $p(x) = a^2x^3 - ax^2 + 3ax - a$

Since $x - 3$ is a factor of $p(x)$, therefore by the factor theorem, we have

$p(3) = 0$

$\Rightarrow$ $a^2(3)^3 - a(3)^2 + 3a(3) - a = 0$

$\Rightarrow$ $a[27a - 9 + 9 - 1] = 0$

$\Rightarrow$ $a(27a - 1) = 0$

$\Rightarrow$ Either $a = 0$ or $27a - 1 = 0$

Hence, $\boldsymbol{a = 0}$ or $\boldsymbol{a = \frac{1}{27}}$.

17. Let $p(x) = 2x^3 - 9x^2 + x + a$ and let $g(x) = 2x - 3$.

By the factor theorem, $p(x)$ will be exactly divisible by $g(x) = 2x - 3 = 2\left(x - \frac{3}{2}\right)$, if $p\left(\frac{3}{2}\right) = 0$.

$\Rightarrow$ $2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + a = 0$

$\Rightarrow$ $2\left(\frac{27}{8}\right) - 9\left(\frac{9}{4}\right) + \left(\frac{3}{2}\right) + a = 0$

$\Rightarrow$ $\frac{27}{4} - \frac{81}{4} + \frac{3}{2} + a = 0$

$\Rightarrow$ $a = \frac{81}{4} - \frac{27}{4} - \frac{3}{2}$
$= \frac{81 - 27 - 6}{4}$

$$= \frac{48}{4}$$

$\Rightarrow$ $\boldsymbol{a = 12}$

18. Let $p(x) = 5x^3 - x^2 + 4x + a$ and $g(x) = 1 - 5x$.
By the factor theorem, $p(x)$ will be exactly divisible by $g(x) = 1 - 5x = -(5x - 1) = -5\left(x - \frac{1}{5}\right)$, if $p\left(\frac{1}{5}\right) = 0$.

$$\Rightarrow \quad 5\left(\frac{1}{5}\right)^3 - \left(\frac{1}{5}\right)^2 + 4\left(\frac{1}{5}\right) + a = 0$$

$$\Rightarrow \quad \frac{5}{125} - \frac{1}{25} + \frac{4}{5} + a = 0$$

$$\Rightarrow \quad \frac{1}{25} - \frac{1}{25} + \frac{4}{5} + a = 0$$

$$\Rightarrow \quad \boldsymbol{a = \frac{-4}{5}}$$

19. By the factor theorem, if $(x - 2)$ and $(x + 3)$ are factors of $p(x)$, then

$p(2) = 0$
and $p(-3) = 0$
$\Rightarrow$ $a(2)^3 + 3(2)^2 - b(2) - 12 = 0$
and $a(-3)^3 + 3(-3)^2 - b(-3) - 12 = 0$
$\Rightarrow$ $8a + 12 - 2b - 12 = 0$
and $-27a + 27 + 3b - 12 = 0$
$\Rightarrow$ $8a - 2b = 0$
and $-27a + 3b + 15 = 0$
$\Rightarrow$ $4a - b = 0$ … (1)
and $-9a + b = -5$ … (2)

Adding equation (1) and equation (2), we get
$-5a = -5$
$\Rightarrow$ $a = 1$
Substituting $a = 1$ in equation (1), we get
$4(1) - b = 0$
$\Rightarrow$ $b = 4$
Hence, $\boldsymbol{a = 1, b = 4}$.

20. Let $p(x) = ax^2 + 5x + b$.
By the factor theorem, if $(x - 3)$ and $\left(x - \frac{1}{3}\right)$ are factors of $p(x)$,

Then, $p(3) = 0$ and $p\left(\frac{1}{3}\right) = 0$

$\Rightarrow$ $a(3)^2 + 5(3) + b = 0$
and $a\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) + b = 0$
$\Rightarrow$ $9a + 15 + b = 0$ … (1)
and $a + 15 + 9b = 0$ … (2)
From (1) and (2), we get
$9a + 15 + b = a + 15 + 9b$
$\Rightarrow$ $8a = 8b$
$\Rightarrow$ $\boldsymbol{a = b}$

EXERCISE 2E

1. (*i*) $(2x + 3y)(2x + 3y) = (2x + 3y)^2$
$= (2x)^2 + 2(2x)(3y) + (3y)^2$
$= \boldsymbol{4x^2 + 12xy + 9y^2}$
[Using $(x + y)^2 = x^2 + 2xy + y^2$]

(*ii*) $(3 - 2x)(3 - 2x) = (3 - 2x)^2$
$= (3)^2 - 2(3)(2x) + (2x)^2$
$= \boldsymbol{9 - 12x + 4x^2}$
[Using $(x - y)^2 = x^2 - 2xy + y^2$]

(*iii*) $\left(3x - \frac{1}{x}\right)^2 = (3x)^2 - 2(3x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$
$= \boldsymbol{9x^2 - 6 + \frac{1}{x^2}}$
[Using $(x - y)^2 = x^2 - 2xy + y^2$]

(*iv*) $\left(x - \frac{1}{10}\right)\left(x + \frac{1}{10}\right) = (x)^2 - \left(\frac{1}{10}\right)^2 = \boldsymbol{x^2 - \frac{1}{100}}$
[Using $(x - y)(x + y) = x^2 - y^2$]

(*v*) $\left(x + \frac{4}{3}\right)\left(x + \frac{14}{3}\right) = x^2 + \left(\frac{4}{3} + \frac{14}{3}\right)x + \left(\frac{4}{3}\right)\left(\frac{14}{3}\right)$
$= \boldsymbol{x^2 + 6x + \frac{56}{9}}$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]

(*vi*) $(z^2 + 2)(z^2 - 3)$
Let $z^2 = x$. Then,
$(z^2 + 2)(z^2 - 3) = (x + 2)(x - 3)$
$= (x + 2)[(x) + (-3)]$
$= (x)^2 + [(2) + (-3)]x + (2)(-3)$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
$= x^2 - x - 6$
$= (z^2)^2 - z^2 - 6$ [Putting $x = z^2$]
$= \boldsymbol{z^4 - z^2 - 6}$

2. (*i*) $\left(x - \frac{3}{2}\right)\left(x + \frac{3}{2}\right)\left(x^2 + \frac{9}{4}\right)$
$= \left[(x)^2 - \left(\frac{3}{2}\right)^2\right]\left(x^2 + \frac{9}{4}\right)$
[Using $(x - y)(x + y) = x^2 - y^2$]
$= \left(x^2 - \frac{9}{4}\right)\left(x^2 + \frac{9}{4}\right)$
$= (x^2)^2 - \left(\frac{9}{4}\right)^2$ [Using $(x - y)(x + y) = x^2 - y^2$]
$= \boldsymbol{x^4 - \frac{81}{16}}$

(*ii*) $(x + 1 + y)(x - 1 - y)$
$= [(x) + (1 + y)][(x) - (1 + y)]$
$= (x)^2 - (1 + y)^2$ [Using $(x - y)(x + y) = x^2 - y^2$]
$= x^2 - (1 + 2y + y^2)$ [Using $(x + y)^2 = x^2 + 2xy + y^2$]
$= \boldsymbol{x^2 - 1 - 2y - y^2}$

(*iii*) $(2x^2 + 5x + 1)(2x^2 + 5x - 1)$
$= [(2x^2 + 5x) + 1][(2x^2 + 5x) - 1]$
$= (2x^2 + 5x)^2 - 1^2$ [Using $(x + y)(x - y) = x^2 - y^2$]
$= \boldsymbol{4x^4 + 20x^3 + 25x^2 - 1}$
[Using $(x + y)^2 = x^2 + 2xy + y^2$]

(*iv*) $(x^2 + 1 + y)(x^2 + 1 - y)$
$= [(x^2 + 1) + y][(x^2 + 1) - y]$

$= [(x^2 + 1)^2 - y^2]$ [Using $(x + y)(x - y) = x^2 - y^2$]
$= \mathbf{x^4 + 2x^2 + 1 - y^2}$ [Using $(x + y)^2 = x^2 + 2xy + y^2$]

(*v*) $\left(x^3 + \frac{y^2}{2} - 3\right)\left(x^3 + \frac{y^2}{2} + 3\right)$

$= \left[\left(x^3 + \frac{y^2}{2}\right) - 3\right]\left[\left(x^3 + \frac{y^2}{2}\right) + 3\right]$

$= \left(x^3 + \frac{y^2}{2}\right)^2 - (3)^2$ [Using $(x + y)(x - y) = x^2 - y^2$]

$= x^6 + 2(x^3)\left(\frac{y^2}{2}\right) + \frac{y^4}{4} - 9$

[Using $(x + y)^2 = x^2 + 2xy + y^2$]

$= \mathbf{x^6 + x^3y^2 + \frac{y^4}{4} - 9}$

(*vi*) $(2z - 3 + x - y)(2z - 3 - x + y)$
$= [(2z - 3) + (x - y)][(2z - 3) - (x - y)]$
$= (2z - 3)^2 - (x - y)^2$
[Using $(x + y)(x - y) = x^2 - y^2$]
$= (4z^2 - 12z + 9) - (x^2 - 2xy + y^2)$
[Using $(x - y)^2 = x^2 - 2xy + y^2$]
$= \mathbf{4z^2 - 12z + 9 - x^2 + 2xy - y^2}$

3. (*i*) $(104)^2$
$= (100 + 4)^2$
$= (100)^2 + 2 \times 100 \times 4 + (4)^2$
[Using $(x + y^2) = x^2 + 2xy + y^2$]
$= 10000 + 800 + 16$
$= \mathbf{10816}$

(*ii*) $(499)^2$
$= (500 - 1)^2$
$= (500)^2 - (2)(500)(1) + (1)^2$
[Using $(x - y)^2 = x^2 - 2xy + y^2$]
$= 250000 - 1000 + 1$
$= \mathbf{249001}$

(*iii*) $\frac{100 \times 100 - 83 \times 83}{17}$

$= \frac{(100)^2 - 83^2}{17}$

$= \frac{(100 + 83)(100 - 83)}{17}$ [Using $x^2 - y^2 = (x + y)(x - y)$]

$= \frac{(183)(17)}{(17)} = \mathbf{183}$

(*iv*) 1.92×2.08
$= (2 - 0.08) \times (2 + 0.08)$
$= (2)^2 - (0.08)^2$ [Using $(x - y)(x + y) = x^2 - y^2$]
$= 4 - 0.0064$
$= \mathbf{3.9936}$

(*v*) $103 \times 96 = (100 + 3)(100 - 4)$
$= [100 + 3][100 + (-4)]$
$= (100)^2 + [(3) + (-4)] 100 + (3)(-4)$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
$= 10000 - 100 - 12$
$= \mathbf{9888}$

(*vi*) $95 \times 97 = (100 - 5)(100 - 3)$
$= [(100) + (-5)][100 + (-3)]$
$= (100)^2 + [(-5) + (-3)] 100 + (-5)(-3)$
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
$= 10000 - 800 + 15$
$= 10015 - 800$
$= \mathbf{9215}$

4. (*i*) $225 \times 225 + 2 \times 225 \times 75 + 75 \times 75$
$= (225)^2 + 2 \times (225) \times (75) + (75)^2$
$= (225 + 75)^2$ [Using $x^2 + 2xy + y^2 = (x + y)^2$]
$= (300)^2$
$= \mathbf{90000}$

(*ii*) $2.2 \times 2.2 - 2 \times 2.2 \times 0.2 + 0.2 \times 0.2$
$= (2.2)^2 - 2(2.2)(0.2) + (0.2)^2$
$= (2.2 - 0.2)^2$ [Using $x^2 - 2xy + y^2 = (x - y)^2$]
$= 2^2 = \mathbf{4}$

(*iii*) $\frac{3.7 \times 3.7 - 2.9 \times 2.9}{0.8} = \frac{(3.7)^2 - (2.9)^2}{0.8}$

$= \frac{(3.7 + 2.9)(3.7 - 2.9)}{0.8}$

[Using $x^2 - y^2 = (x + y)(x - y)$]

$= \frac{(6.6)(0.8)}{0.8} = \mathbf{6.6}$

(*iv*) $5.1 \times 5.1 - 0.1 \times 0.1 = (5.1)^2 - (0.1)^2$
$= (5.1 + 0.1)(5.1 - 0.1)$
[Using $x^2 - y^2 = (x + y)(x - y)$]
$= (5.2)(5) = \mathbf{26}$

5. $x + \frac{1}{x} = 2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 4$ [squaring both sides]

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = 4 - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = \mathbf{2}$

6. $x - \frac{1}{x} = 4$

$\Rightarrow x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 16$ [Squaring both sides]

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 16$

$\Rightarrow x^2 + \frac{1}{x^2} = 16 + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = \mathbf{18}$

7. $x + \frac{1}{x} = \sqrt{7}$

$\Rightarrow x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 7$ [Squaring both sides]

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 7$

$\Rightarrow x^2 + \frac{1}{x^2} = 7 - 2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = \mathbf{5}$

Now, $\quad x^2 + \frac{1}{x^2} = 5$

$\Rightarrow x^4 + \frac{1}{x^4} + 2(x^2)\left(\frac{1}{x^2}\right) = 25$ [Squaring both sides]

$\Rightarrow \quad x^4 + \frac{1}{x^4} + 2 = 25$

$\Rightarrow \quad x^4 + \frac{1}{x^4} = 25 - 2$

$\Rightarrow \quad x^4 + \frac{1}{x^4} = \mathbf{23}$

8. $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)$

$= \left(x^2 + \frac{1}{x^2}\right) + 2$

$= 38 + 2$
$= 40$

$\Rightarrow \quad x + \frac{1}{x} = \sqrt{40}$

$= \pm \mathbf{2\sqrt{10}}$

$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right)$

$= \left(x^2 + \frac{1}{x^2}\right) - 2$

$= 38 - 2$
$= 36$

$\Rightarrow \quad \left(x - \frac{1}{x}\right) = \sqrt{36} = \pm \mathbf{6}$

9. $2x - \sqrt{7}\,y = 10$

$\Rightarrow \quad (2x - \sqrt{7}\,y)^2 = (10)^2$

$\Rightarrow \quad 4x^2 + 7y^2 - 2(2x)(\sqrt{7}\,y) = 100$

$\Rightarrow \quad 4x^2 + 7y^2 - 4\sqrt{7}\,xy = 100$

$\Rightarrow \quad 4x^2 + 7y^2 - 4\sqrt{7}(-\sqrt{7}) = 100$

[$\because xy = -\sqrt{7}$, given]

$\Rightarrow \quad 4x^2 + 7y^2 + 28 = 100$

$\Rightarrow \quad 4x^2 + 7y^2 = 100 - 28$

$\Rightarrow \quad 4x^2 + 7y^2 = \mathbf{72}$

10. $(2x + 3y)^2 = 4x^2 + 2(2x)(3y) + 9y^2$

$= 4x^2 + 9y^2 + 12xy$

$= 69 + 12(1)$

$= 69 + 12$

$= 81$

$\therefore \quad 2x + 3y = \sqrt{81}$

$\Rightarrow \quad 2x + 3y = \pm \mathbf{9}$

EXERCISE 2F

1. (*i*) $(2x + 5y + 1)^2$

$= (2x)^2 + (5y)^2 + (1)^2 + 2(2x)(5y) + 2(5y)(1) + 2(1)(2x)$

$= 4x^2 + 25y^2 + 1 + 20xy + 10y + 4x$

$= \mathbf{4x^2 + 25y^2 + 1 + 20xy + 4x + 10y}$

(*ii*) $(-2x + 3y - 2z)^2$

$= [(-2x) + 3y + (-2z)]^2$

$= (-2x)^2 + (3y)^2 + (-2z)^2 + 2(-2x)(3y) + 2(3y)(-2z) + 2(-2z)(-2x)$

$= \mathbf{4x^2 + 9y^2 + 4z^2 - 12xy - 12yz + 8zx}$

(*iii*) $(xy + yz + zx)^2$

$= (xy)^2 + (yz)^2 + (zx)^2 + 2(xy)(yz) + 2(yz)(zx) + 2(zx)(xy)$

$= x^2y^2 + y^2z^2 + z^2x^2 + 2xy^2z + 2xyz^2 + 2x^2yz$

$= \mathbf{x^2y^2 + y^2z^2 + z^2x^2 + 2x^2yz + 2xy^2z + 2xyz^2}$

(*iv*) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 = \left[\frac{a}{4} + \left(\frac{-b}{2}\right) + 1\right]^2$

$= \left(\frac{a}{4}\right)^2 + \left(\frac{-b}{2}\right)^2 + (1)^2 + 2\left(\frac{a}{4}\right)\left(\frac{-b}{2}\right) + 2\left(\frac{-b}{2}\right)(1) + 2(1)\left(\frac{a}{4}\right)$

$= \mathbf{\frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}}$

2. $(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ ... (1)

$(a - b + c)^2 = [a + (-b) + c]^2 = a^2 + (-b)^2 + c^2 + 2(a)(-b) + 2(-b)(c) + 2(c)(a)$

$\Rightarrow (a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$ ... (2)

and $(a + b - c)^2 = [a + b + (-c)]^2$

$= a^2 + b^2 + (-c)^2 + 2(a)(b) + 2b(-c) + 2(-c)(a)$

$\Rightarrow (a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$ ... (3)

Adding the corresponding sides of (1), (2) and (3), we get

$(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2$

$= \mathbf{3a^2 + 3b^2 + 3c^2 + 2ab - 2bc + 2ca}$

3. $x + y + z = 7$

$\Rightarrow \quad (x + y + z)^2 = 7^2$

$\Rightarrow \quad x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 49$

$\Rightarrow \quad x^2 + y^2 + z^2 + 2(xy + yz + zx) = 49$

$\Rightarrow \quad x^2 + y^2 + z^2 + 2(6) = 49$

$\Rightarrow \quad x^2 + y^2 + z^2 + 12 = 49$

$\Rightarrow \quad x^2 + y^2 + z^2 = 49 - 12$

$\Rightarrow \quad x^2 + y^2 + z^2 = \mathbf{37}$

4. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\Rightarrow \quad (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow \quad (x + y + z)^2 = 40 + 2(30)$

$= 40 + 60 = 100$

$\Rightarrow \quad (x + y + z)^2 = 100$

$\Rightarrow \quad x + y + z = \sqrt{100}$

$\Rightarrow \quad x + y + z = \pm \mathbf{10}$

5. (*i*) $(3x + 4y)^3 = (3x)^3 + (4y)^3 + 3(3x)(4y)(3x + 4y)$

[Using $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$]

$\Rightarrow \quad (3x + 4y)^3 = 27x^3 + 64y^3 + 36xy(3x + 4y)$

$\Rightarrow \quad (3x + 4y)^3 = \mathbf{27x^3 + 64y^3 + 108x^2y + 144xy^2}$

(*ii*) $\left(2x+\frac{1}{3x}\right)^3 = (2x)^3 + \left(\frac{1}{3x}\right)^3 + 3(2x)\left(\frac{1}{3x}\right)\left(2x+\frac{1}{3x}\right)$

[Using $(x+y)^3 = x^3+y^3+3xy(x+y)$]

$\Rightarrow \left(2x+\frac{1}{3x}\right)^3 = 8x^3 + \frac{1}{27x^3} + 2\left(2x+\frac{1}{3x}\right)$

$\Rightarrow \left(2x+\frac{1}{3x}\right)^3 = \mathbf{8x^3 + \frac{1}{27x^3} + 4x + \frac{2}{3x}}$

(*iii*) $(5x-3y)^3 = (5x)^3 - (3y)^3 - 3(5x)(3y)(5x-3y)$

[Using $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$]

$\Rightarrow (5x-3y)^3 = 125x^3 - 27y^3 - 45xy\,(5x-3y)$

$\Rightarrow (5x-3y)^3 = \mathbf{125x^3 - 27y^3 - 225x^2y + 135xy^2}$

(*iv*) $\left(\frac{1}{3x}-\frac{2}{5y}\right)^3 = \left(\frac{1}{3x}\right)^3 - \left(\frac{2}{5y}\right)^3 - 3\left(\frac{1}{3x}\right)\left(\frac{2}{5y}\right)\left(\frac{1}{3x}-\frac{2}{5y}\right)$

$\Rightarrow \left(\frac{1}{3x}-\frac{2}{5y}\right) = \frac{1}{27x^3} - \frac{8}{125y^3} - \frac{2}{5xy}\left(\frac{1}{3x}-\frac{2}{5y}\right)$

$\Rightarrow \left(\frac{1}{3x}-\frac{2}{5y}\right) = \mathbf{\frac{1}{27x^3} - \frac{8}{125y^3} - \frac{2}{15x^2y} + \frac{4}{25xy^2}}$

(*v*) $\left(\frac{4}{3}x+\frac{3}{4}y\right)^3 = \left(\frac{4}{3}x\right)^3 + \left(\frac{3}{4}y\right)^3 + 3\left(\frac{4}{3}x\right)\left(\frac{3}{4}y\right)\left(\frac{4}{3}x+\frac{3}{4}y\right)$

$\Rightarrow \left(\frac{4}{3}x+\frac{3}{4}y\right)^3 = \frac{64}{27}x^3 + \frac{27}{64}y^3 + 3xy\left(\frac{4}{3}x+\frac{3}{4}y\right)$

$\Rightarrow \left(\frac{4}{3}x+\frac{3}{4}y\right)^3 = \mathbf{\frac{64}{27}x^3 + \frac{27}{64}y^3 + 4x^2y + \frac{9}{4}xy^2}$

(*vi*) $\left(x^2-\frac{3}{2}y^2\right)^3 = (x^2)^3 - \left(\frac{3}{2}y^2\right)^3 - 3(x^2)\left(\frac{3}{2}y^2\right)\left(x^2-\frac{3}{2}y^2\right)$

$\Rightarrow \left(x^2-\frac{3}{2}y^2\right)^3 = x^6 - \frac{27}{8}y^6 - \frac{9}{2}x^2y^2\left(x^2-\frac{3}{2}y^2\right)$

$\Rightarrow \left(x^2-\frac{3}{2}y^2\right)^3 = \mathbf{x^6 - \frac{27}{8}y^6 - \frac{9}{2}x^4y^2 + \frac{27}{4}x^2y^4}$

6. (*i*) $(3x+4y)^3 + (3x-4y)^3$

$= (3x+4y+3x-4y)[(3x+4y)^2 - (3x+4y)(3x-4y) + (3x-4y)^2]$

[Using $a^3+b^3 = (a+b)(a^2-ab+b^2)$]

$= (6x)\,[(9x^2+24xy+16y^2) - (9x^2-16y^2) + (9x^2-24xy+16y^2)]$

$= 6x(9x^2+24xy+16y^2-9x^2+16y^2+9x^2-24xy+16y^2)$

$= 6x(9x^2+48y^2)$

$= \mathbf{54x^3 + 288xy^2}$

(*ii*) $\left(\frac{x}{3}+\frac{y}{5}\right)^3 - \left(\frac{x}{3}-\frac{y}{5}\right)^3$

$= \left(\frac{x}{3}+\frac{y}{5}-\frac{x}{3}+\frac{y}{5}\right)\left[\left(\frac{x}{3}+\frac{y}{5}\right)^2 + \left(\frac{x}{3}+\frac{y}{5}\right)\left(\frac{x}{3}-\frac{y}{5}\right) + \left(\frac{x}{3}-\frac{y}{5}\right)^2\right]$

[Using: $a^3-b^3 = (a-b)(a^2+ab+b^2)$]

$= \left(\frac{2y}{5}\right)\left[\left(\frac{x^2}{9} + 2\times\frac{x}{3}\times\frac{y}{5} + \frac{y^2}{25}\right) + \left(\frac{x^2}{9} - \frac{y^2}{25}\right) + \left(\frac{x^2}{9} - \frac{2xy}{15} + \frac{y^2}{25}\right)\right]$

$= \frac{2y}{5}\left(\frac{x^2}{9} + \frac{2xy}{15} + \frac{y^2}{25} + \frac{x^2}{9} - \frac{y^2}{25} + \frac{x^2}{9} - \frac{2xy}{15} + \frac{y^2}{25}\right)$

$= \frac{2y}{5}\left(\frac{x^2}{3} + \frac{y^2}{25}\right) = \mathbf{\frac{2x^2y}{15} + \frac{2y^3}{125}}$

7. (*i*) $(23)^3 = (20+3)^3$

$= (20)^3 + (3)^3 + 3(20)(3)\,(20+3)$

[Using: $(x+y)^3 = x^3+y^3+3xy\,(x+y)$]

$\Rightarrow (23)^3 = 8000 + 27 + 180(23)$

$= 8000 + 27 + 4140$

$= \mathbf{12167}$

(*ii*) $(102)^3$

$= (100+2)^3$

$= (100)^3 + (2)^3 + 3(100)\,(2)\,(100+2)$

[Using $(x+y)^3 = x^3+y^3+3xy\,(x+y)$]

$= 1000000 + 8 + 600\,(102)$

$= 1000000 + 8 + 61200$

$= \mathbf{1061208}$

(*iii*) $(995)^3$

$= (1000-5)^3$

$= (1000)^3 - (5)^3 - 3(1000)\,(5)\,(1000-5)$

[Using $(x-y)^3 = x^3-y^3-3xy\,(x-y)$]

$= 1000000000 - 125 - 15000\,(1000-5)$

$= 1000000000 - 125 - 15000000 + 75000$

$= 1000075000 - 15000125$

$= \mathbf{985074875}$

8. $x + \frac{1}{x} = 7$

$\Rightarrow \left(x+\frac{1}{x}\right)^3 = 7^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right) = 343$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(7) = 343$

$\Rightarrow x^3 + \frac{1}{x^3} + 21 = 343$

$\Rightarrow x^3 + \frac{1}{x^3} = 343 - 21 = \mathbf{322}$

9. $\left(x-\frac{1}{x}\right) = 5$

$\Rightarrow \left(x-\frac{1}{x}\right)^3 = 5^3$

$$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 125$$

$$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3(5) = 125$$

$$\Rightarrow \quad x^3 - \frac{1}{x^3} - 15 = 125$$

$$\Rightarrow \quad x^3 - \frac{1}{x^3} = 125 + 15 = \mathbf{140}$$

10. $(3x + 2y) = 10$

$\Rightarrow (3x + 2y)^3 = 10^3$

$\Rightarrow (3x)^3 + (2y)^3 + 3(3x)(2y)(3x + 2y) = 1000$

$\Rightarrow 27x^3 + 8y^3 + 18xy(3x + 2y) = 1000$

$\Rightarrow 27x^3 + 8y^3 + 18(2)(10) = 1000$

$\Rightarrow 27x^3 + 8y^3 + 360 = 1000$

$\Rightarrow 27x^3 + 8y^3 = 1000 - 360$

$\Rightarrow 27x^3 + 8y^3 = \mathbf{640}$

11. $$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = \left(x^2 + \frac{1}{x^2}\right) + 2$$

$= 23 + 2$

$= 25$

$$\Rightarrow \left(x + \frac{1}{x}\right) = \sqrt{25}$$

$= +5$ or -5

When $x + \dfrac{1}{x} = 5$

$$\left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - x \times \frac{1}{x}\right)$$

$= 5(23 - 1)$

$= 5(22)$

$= 110$

When $x + \dfrac{1}{x} = -5$

$$\left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$$

$= -5(23 - 1)$

$= -5(22)$

$= -110$

Hence, $x^3 + \dfrac{1}{x^3} = \mathbf{\pm 110.}$

12. $$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2x \times \frac{1}{x}$$

$$= \left(x^2 + \frac{1}{x^2}\right) - 2$$

$= 18 - 2$

$= 16$

$$\Rightarrow \left(x - \frac{1}{x}\right) = \sqrt{16} = +4 \text{ or } -4$$

When $\left(x - \dfrac{1}{x}\right) = 4$

$$\left(x^3 - \frac{1}{x^3}\right) = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + x \times \frac{1}{x}\right)$$

$$= \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$$

$= (4)(18 + 1) = 4(19) = 76$

When $\left(x - \dfrac{1}{x}\right) = -4$

$$\left(x^3 - \frac{1}{x^3}\right) = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + x \times \frac{1}{x}\right)$$

$$= \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$$

$= -4(18 + 1)$

$= -4(19)$

$= -76$

Hence, $x^3 - \dfrac{1}{x^3} = \mathbf{\pm 76.}$

13. (*i*) $(5x - 3y)(25x^2 + 15xy + 9y^2)$

$= (5x - 3y)[(5x)^2 + (5x)(3y) + (3y)^2]$

$= (5x)^3 - (3y)^3$ [Using: $(x - y)(x^2 + xy + y^2) = x^3 - y^3$]

$= \mathbf{125x^3 - 27y^3}$

(*ii*) $(2xy + 3z)(4x^2y^2 - 6xyz + 9z^2)$

$= (2xy + 3z)[(2xy)^2 - (2xy)(3z) + (3z)^2]$

$= (2xy)^3 + (3z)^3$ [Using: $(x + y)(x^2 - xy + y^2) = x^3 + y^3$]

$= \mathbf{8x^3y^3 + 27z^3}$

14. (*i*) $(25)^3 + (5)^3 = (25 + 5)[(25)^2 - (25) \times (5) + (5)^2]$

[Using: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$]

$= 30(625 - 125 + 25) = 30(525) = \mathbf{15750}$

(*ii*) $(1100)^3 - (100)^3 = (1100 - 100)[(1100)^2 + 1100 \times 100 + (100)^2]$

[Using: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$]

$= (1000)(1210000 + 110000 + 10000)$

$= 1000(330000)$

$= \mathbf{1330000000}$

15. Simplify

(*i*) $$\frac{27 \times 27 \times 27 - 7 \times 7 \times 7}{27 \times 27 + 27 \times 7 + 7 \times 7}$$

$$= \frac{(27)^3 - (7)^3}{(27 \times 27 + 27 \times 7 + 7 \times 7)}$$

$$= \frac{(27 - 7)[(27)^2 + 27 \times 7 + (7)^2]}{(27 \times 27 + 27 \times 7 + 7 \times 7)}$$

$$= \frac{(20)(27 \times 27 + 27 \times 7 + 7 \times 7)}{(27 \times 27 + 27 \times 7 + 7 \times 7)}$$

$= \mathbf{20}$

(*ii*) $$\frac{3.9 \times 3.9 \times 3.9 + 2.1 \times 2.1 \times 2.1}{3.9 \times 3.9 - 3.9 \times 2.1 + 2.1 \times 2.1}$$

$$= \frac{(3.9)^3 + (2.1)^3}{(3.9 \times 3.9 - 3.9 \times 2.1 + 2.1 \times 2.1)}$$

$$= \frac{(3.9 + 2.1) \times [(3.9)^2 - 3.9 \times 2.1 + (2.1)^2]}{(3.9 \times 3.9 - 3.9 \times 2.1 + 2.1 \times 2.1)}$$

$$= \frac{(6)(3.9 \times 3.9 - 3.9 \times 2.1 + 2.1 \times 2.1)}{(3.9 \times 3.9 - 3.9 \times 2.1 + 2.1 \times 2.1)}$$

$= \mathbf{6}$

16.

$$x + y = 3$$
$$\Rightarrow \quad (x + y)^2 = 3^2$$
$$\Rightarrow \quad x^2 + y^2 + 2xy = 9$$
$$\Rightarrow \quad x^2 + y^2 + 2(2) = 9$$
$$\Rightarrow \quad x^2 + y^2 + 4 = 9$$
$$\Rightarrow \quad x^2 + y^2 = 5 \quad \ldots (1)$$
$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$= (x + y)(x^2 + y^2 - xy)$$
$$= (3)(5 - 2) \quad \text{[Using (1)]}$$
$$= (3)(3)$$
$$= \mathbf{9}$$

17.

$$x - y = 5$$
$$\Rightarrow \quad (x - y)^2 = 5^2$$
$$\Rightarrow \quad x^2 + y^2 - 2xy = 25$$
$$\Rightarrow \quad x^2 + y^2 - 2(-6) = 25$$
$$\Rightarrow \quad x^2 + y^2 + 12 = 25$$
$$\Rightarrow \quad x^2 + y^2 = 25 - 12$$
$$\Rightarrow \quad x^2 + y^2 = 13 \quad \ldots (1)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$
$$\Rightarrow \quad x^3 - y^3 = 5(x^2 + y^2 + xy)$$
$$= 5[(13) + (-6)] \quad \text{[Using (1)]}$$
$$= 5\,(7)$$
$$= \mathbf{35}$$

18. (*i*) $(p - 3q + 2r)(p^2 + 9q^2 + 4r^2 + 3pq + 6qr - 2pr)$

$$= [p + (-3q) + 2r]\,[(p)^2 + (-3q)^2 + (2r)^2 - (p)(-3q) - (-3q)(2r) - (2r)(p)]$$
$$= (p)^3 + (-3q)^3 + (2r)^3 - 3(p)(-3q)(2r)$$
[Using: $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$]
$$= \boldsymbol{p^3 - 27q^3 + 8r^3 + 18pqr}$$

(*ii*) $(2x - y - 1)(4x^2 + y^2 + 1 + 2xy - y + 2x)$

$$= [2x + (-y) + (-1)]\,[(2x)^2 + (-y)^2 + (-1)^2 - 2x(-y) - (-y)(-1) - (-1)(2x)]$$
$$= (2x)^3 + (-y)^3 + (-1)^3 - 3(2x)(-y)(-1)$$
[Using: $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$]
$$= \mathbf{8x^3 - y^3 - 1 - 6xy}$$

(*iii*) $(\sqrt{2}x + 2\sqrt{2y} + z)\ (2x^2 + 8y^2 + z^2 - 4xy - 2\sqrt{2}\,yz - \sqrt{2}\,xz)$

$$= (\sqrt{2}\,x + 2\sqrt{2}\,y + z)\,[(\sqrt{2}\,x)^2 + (2\sqrt{2}\,y)^2 + (z)^2 - (\sqrt{2}\,x)(2\sqrt{2}\,y) - (2\sqrt{2}\,y)(z) - z(\sqrt{2}\,x)]$$
$$= (\sqrt{2}\,x)^3 + (2\sqrt{2}\,y)^3 + z^3 - 3(\sqrt{2}\,x)(2\sqrt{2}\,y)(z)$$
[Using: $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$]
$$= \mathbf{2\sqrt{2}\,x^3 + 16\sqrt{2}\,y^3 + z^3 - 12xyz}$$

19. (*i*) $28 + (-15) + (-13) = 28 - 28 = 0$

$\therefore \quad (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

[If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$]

$\therefore \quad (28)^3 + (-15)^3 + (-13)^3 = \mathbf{16380}$

(*ii*) $(0.1)^3 + (0.2)^3 - (0.3)^3 = (0.1)^3 + (0.2)^3 + (-0.3)^3$

Here, $0.1 + 0.2 + (-0.3) = 0$

$\therefore \quad (0.1)^3 + (0.2)^3 + (-0.3)^3 = 3(0.1)(0.2)(-0.3)$

[If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$]

$\therefore \quad (0.1)^3 + (0.2)^3 + (-0.3)^3 = -0.018$

$\Rightarrow \quad (0.1)^3 + (0.2)^3 - (0.3)^3 = \mathbf{-0.018}$

(*iii*) $\left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{7}{12}\right)^3 = \left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(\frac{-7}{12}\right)^3$

Here, $\left(\frac{1}{4}\right) + \left(\frac{1}{3}\right) - \left(\frac{7}{12}\right) = \frac{1}{4} + \frac{1}{3} - \frac{7}{12}$

$$= \frac{3 + 4 - 7}{12}$$
$$= \frac{7 - 7}{12} = 0$$

$$\therefore \quad \left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(\frac{-7}{12}\right)^3 = 3\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{-7}{12}\right)$$

[If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$]

$$\therefore \quad \left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(\frac{-7}{12}\right)^3 = -\frac{7}{48}$$
$$\Rightarrow \quad \left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{7}{12}\right)^3 = \mathbf{\frac{-7}{48}}$$

20. $\left(\frac{8}{15}\right)^3 - \left(\frac{1}{3}\right)^3 - \left(\frac{1}{5}\right)^3 = \left(\frac{8}{15}\right)^3 + \left(\frac{-1}{3}\right)^3 + \left(-\frac{1}{5}\right)^3$

Here, $\frac{8}{15} + \left(-\frac{1}{3}\right) + \left(-\frac{1}{5}\right) = \frac{8}{15} - \frac{1}{3} - \frac{1}{5}$

$$= \frac{8 - 5 - 3}{15}$$
$$= \frac{8 - 8}{15}$$
$$= 0$$

$$\therefore \left(\frac{8}{15}\right)^3 + \left(\frac{-1}{3}\right)^3 + \left(\frac{-1}{5}\right)^3 = 3\left(\frac{8}{15}\right)\left(\frac{-1}{3}\right)\left(\frac{-1}{5}\right)$$

[If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$]

$$\therefore \quad \left(\frac{8}{15}\right)^3 + \left(\frac{-1}{3}\right)^3 + \left(\frac{-1}{5}\right)^3 = \frac{8}{75}$$
$$\Rightarrow \quad \left(\frac{8}{15}\right)^3 - \left(\frac{1}{3}\right)^3 - \left(\frac{1}{5}\right)^3 = \mathbf{\frac{8}{75}}$$
$$\therefore \text{ LHS} = \left(\frac{8}{15}\right)^3 - \left(\frac{1}{3}\right)^3 - \left(\frac{1}{5}\right)^3 = \frac{8}{75} = \text{RHS}$$

21.

$$x + y + z = 8$$
$$\Rightarrow \quad (x + y + z)^2 = 8^2$$
$$\Rightarrow \quad x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 64$$
$$\Rightarrow \quad x^2 + y^2 + z^2 + 2(xy + yz + zx) = 64$$
$$\Rightarrow \quad x^2 + y^2 + z^2 + 2(20) = 64$$
$$\Rightarrow \quad x^2 + y^2 + z^2 = 64 - 40$$
$$\Rightarrow \quad x^2 + y^2 + z^2 = 24 \quad \ldots (1)$$

Now, $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)\,[(x^2 + y^2 + z^2) - (xy + yz + zx)]$

$= 8(24 - 20)$ [Using (1)]

$= 8(4) = \mathbf{32}$

22.

$$x + y + z = 15$$
$$\Rightarrow \quad (x + y + z)^2 = 15^2$$
$$\Rightarrow \quad x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 225$$
$$\Rightarrow \quad (x^2 + y^2 + z^2) + 2(xy + yz + zx) = 225$$
$$\Rightarrow \quad 33 + 2(xy + yz + zx) = 225$$
$$\Rightarrow \quad 2(xy + yz + zx) = 225 - 33 = 192$$
$$\Rightarrow \quad xy + yz + zx = \frac{192}{2} = 96 \quad \ldots (1)$$

Now, $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)\ [(x^2 + y^2 + z^2) - (xy + yz + zx)]$
$= (15)\ (33 - 96)$ [Using (1)]
$= 15(-63)$
$= \mathbf{-945}$

23. $x + y = 5$
$\Rightarrow (x + y)^3 = 5^3$
$\Rightarrow x^3 + y^3 + 3xy(x + y) = 125$
$\Rightarrow x^3 + y^3 + 3xy\ (5) = 125$
$\Rightarrow x^3 + y^3 + 15xy = 125$
$\Rightarrow x^3 + y^3 + 15xy - 125 = \mathbf{0}$

24. $(2 - a)^3 + (2 - b)^3 + (2 - c)^3 - 3(2 - a)\ (2 - b)\ (2 - c)$
$= [(2 - a) + (2 - b) + (2 - c)]\ [(2 - a)^2 + (2 - b)^2 + (2 - c)^2 - (2 - a)\ (2 - b) - (2 - b)\ (2 - c) - (2 - c)\ (2 - a)]$
[Using $x^3 + y^3 + z^3 = (x + y + z)\ (x^2 + y^2 + z^2 - xy - yz - zx)$]
$= [6 - (a + b + c)]\ [(2 - a)^2 + (2 - b)^2 + (2 - c)^2 - (2 - a)\ (2 - b) - (2 - b)\ (2 - c) - (2 - c)\ (2 - a)]$
$= [6 - 6]\ [(2 - a)^2 + (2 - b)^2 + (2 - c)^2 - (2 - a)\ (2 - b) - (2 - b)\ (2 - c) - (2 - c)\ (2 - a)]$
$= \mathbf{0}$

25. $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)\ (x - b)\ (x - c)$
$= (x - a + x - b + x - c)\ [(x - a)^2 + (x - b)^2 + (x - c)^2 - (x - a)\ (x - b) - (x - b)(x - c) - (x - c)(x - a)]$
[Using $x^3 + y^3 + z^3 - 3xyz = (x + y + z)\ (x^2 + y^2 + z^2 - xy - yz - zx)$]
$= [3x - (a + b + c)]\ [(x - a)^2 + (x - b)^2 + (x - c)^2 - (x - a)(x - b) - (x - b)(x - c) - (x - c)(x - a)]$
$= (3x - 3x)\ [(x - a)^2 + (x - b)^2 + (x - c)^2 - (x - a)\ (x - b) - (x - b)(x - c) - (x - c)(x - a)]$
$= \mathbf{0}$

EXERCISE 2G

1. $3a^2 + 6ab = \mathbf{3a(a + 2b)}$
2. $5xy - 25x^3y^2 = \mathbf{5xy(1 - 5x^2y)}$
3. $46x^2 + 2xy + 10y^2 = \mathbf{2(23x^2 + xy + 5y^2)}$
4. $7x^3y - 21x^2y^2 + 35y^2 = \mathbf{7y(x^3 - 3x^2y + 5y)}$
5. $8(3x + 2y)^2 - 16(3x + 2y) = \mathbf{8(3x + 2y)\ (3x + 2y - 2)}$
6. $2x(x^2 + y^2) - 4y(x^2 + y^2) = \mathbf{2(x^2 + y^2)(x - 2y)}$
7. $p^2(q - r) + q(r - q) = p^2(q - r) - q(q - r) = \mathbf{(q - r)\ (p^2 - q)}$
8. $30a\ (b - c) - 25(c - b) = 30a(b - c) + 25(b - c)$
$= \mathbf{5(b - c)\ (6a + 5)}$
9. $a^2(a^2 + b^2 - c^2) - b^2(c^2 - a^2 - b^2)$
$= a^2(a^2 + b^2 - c^2) + b^2(a^2 + b^2 - c^2)$
$= \mathbf{(a^2 + b^2 - c^2)\ (a^2 + b^2)}$
10. $(a + b)\ (x + y) + (2a + 3b)\ (x + y) + (3a + 4b)\ (x + y)$
$= (x + y)\ (a + b + 2a + 3b + 3a + 4b)$
$= (x + y)\ (6a + 8b) = (x + y)\ 2(3a + 4b)$
$= \mathbf{2(x + y)\ (3a + 4b)}$
11. $2a(x - y) + 3b(5x - 5y) + 4c(2y - 2x)$
$= 2a(x - y) + 3b \times 5(x - y) + 4c \times 2(y - x)$
$= 2a(x - y) + 15b(x - y) - 8c(x - y)$
$= \mathbf{(x - y)(2a + 15b - 8c)}$
12. $ap^2 + bp^2 + aq^2 + bq^2 = p^2(a + b) + q^2(a + b)$
$= \mathbf{(a + b)\ (p^2 + q^2)}$
13. $1 + x^2y^2 + x^2 + y^2 = 1 + x^2 + x^2y^2 + y^2$
$= (1 + x^2) + y^2(x^2 + 1)$
$= \mathbf{(1 + x^2)\ (1 + y^2)}$
14. $4a^3 - 8a^2 + 3a - 6 = 4a^2(a - 2) + 3(a - 2)$
$= \mathbf{(a - 2)\ (4a^2 + 3)}$
15. $x^3 - x^2y - xy + y^2 = x^2(x - y) - y(x - y)$
$= \mathbf{(x - y)\ (x^2 - y)}$
16. $x^3 - x^2 + x - 1 = x^2(x - 1) + 1(x - 1)$
$= \mathbf{(x - 1)\ (x^2 + 1)}$
17. $a^2xy + abx^2 + b^2xy + aby^2 = ax(ay + bx) + by\ (bx + ay)$
$= \mathbf{(bx + ay)\ (ax + by)}$
18. $a^2x^2 + (ax^2 + 1)\ x + a = a^2x^2 + ax^3 + x + a$
$= ax^2(a + x) + 1(x + a)$
$= \mathbf{(x + a)\ (ax^2 + 1)}$
19. $x^3 + xy(1 - 3x) - 3y^2 = x^3 + xy - 3x^2y - 3y^2$
$= x(x^2 + y) - 3y(x^2 + y)$
$= \mathbf{(x^2 + y)\ (x - 3y)}$
20. $abc^2 + (ac - b)c - c = abc^2 + ac^2 - bc - c$
$= ac^2(b + 1) - c(b + 1)$
$= \mathbf{c(b + 1)\ (ac - 1)}$
21. $xy - ay - ax + a^2 + b(x - a)$
$= y(x - a) - a(x - a) + b(x - a)$
$= \mathbf{(x - a)\ (y - a + b)}$
22. $ab^2 + ab - ac - abc + xy + bxy$
$= ab(b + 1) - ac(1 + b) + xy(1 + b)$
$= \mathbf{(1 + b)\ (ab - ac + xy)}$
23. $x^3 - x^2 - ax + x + a - 1 = x^3 - x^2 - ax + a + x - 1$
$= x^2\ (x - 1) - a(x - 1) + 1(x - 1)$
$= \mathbf{(x - 1)\ (x^2 - a + 1)}$
24. $x^2 + \dfrac{1}{x^2} + 2 - 5x - \dfrac{5}{x} = \left(x^2 + \dfrac{1}{x^2} + 2\right) - 5\left(x + \dfrac{1}{x}\right)$
$= \left(x + \dfrac{1}{x}\right)^2 - 5\left(x + \dfrac{1}{x}\right)$
$= \left(\mathbf{x + \dfrac{1}{x}}\right)\left(\mathbf{x + \dfrac{1}{x} - 5}\right)$
25. $\dfrac{a}{b}x^2 + \left(\dfrac{a}{b} + \dfrac{c}{d}\right)x + \dfrac{c}{d}$, $[b \neq 0, d \neq 0]$
$= \dfrac{a}{b}x^2 + \dfrac{a}{b}x + \dfrac{c}{d}x + \dfrac{c}{d}$
$= \dfrac{ax}{b}(x + 1) + \dfrac{c}{d}(x + 1)$
$= \mathbf{(x + 1)}\left(\mathbf{\dfrac{ax}{b} + \dfrac{c}{d}}\right)$
26. $2p(a - b) + 3q(5a - 5b) + 4r(2b - 2a)$
$= 2p(a - b) + 3q \times 5(a - b) + 4r \times 2(b - a)$
$= 2p(a - b) + 15q(a - b) - 8r(a - b)$
$= \mathbf{(a - b)\ (2p + 15q - 8r)}$

EXERCISE 2H

1. $x^2 - 4y^2 = (x)^2 - (2y)^2 = \mathbf{(x + 2y)\ (x - 2y)}$
2. $25x^2 - 36y^2 = (5x)^2 - (6y)^2 = \mathbf{(5x + 6y)\ (5x - 6y)}$
3. $100 - 9x^2 = (10)^2 - (3x)^2 = \mathbf{(10 + 3x)\ (10 - 3x)}$

4. $3x^2 - 4 = (\sqrt{3}\,x)^2 - (2)^2 = \mathbf{(\sqrt{3}\,x + 2)\ (\sqrt{3}\,x - 2)}$

5. $1 - (x - y)^2 = (1)^2 - (x - y)^2 = \mathbf{(1 + x - y)\ (1 - x + y)}$

6. $4(x + y)^2 - 1 = [2(x + y)]^2 - (1)^2$
$= \mathbf{(2x + 2y + 1)\ (2x + 2y - 1)}$

7. $16 - 9(x + y)^2 = (4)^2 - [3(x + y)]^2$
$= \mathbf{(4 + 3x + 3y)\ (4 - 3x - 3y)}$

8. $4(2x - 3)^2 - 9(y + 1)^2 = [2(2x - 3)]^2 - [3(y + 1)]^2$
$= (4x - 6 + 3y + 3)(4x - 6 - 3y - 3)$
$= \mathbf{(4x + 3y - 3)\ (4x - 3y - 9)}$

9. $x^2 - \frac{1}{25} = (x)^2 - \left(\frac{1}{5}\right)^2 = \mathbf{\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right)}$

10. $\frac{3x^2}{25} - \frac{27y^2}{16} = 3\left[\frac{x^2}{25} - \frac{9y^2}{16}\right] = 3\left[\left(\frac{x}{5}\right)^2 - \left(\frac{3y}{4}\right)^2\right]$

$= \mathbf{3\left(\frac{x}{5} + \frac{3y}{4}\right)\left(\frac{x}{5} - \frac{3y}{4}\right)}$

11. $3 - 12(a - b)^2 = 3[1 - 4(a - b)^2]$
$= 3\{(1)^2 - [2(a - b)]^2\}$
$= \mathbf{3(1 + 2a - 2b)(1 - 2a + 2b)}$

12. $x^2y^2 - 9x^4y^4 = x^2y^2\,(1 - 9x^2y^2)$
$= x^2y^2\,[(1)^2 - (3xy)^2]$
$= \mathbf{x^2y^2(1 + 3xy)\ (1 - 3xy)}$

13. $162x^4 - 50 = 2(81x^4 - 25)$
$= 2[(9x^2)^2 - (5)^2]$
$= 2(9x^2 + 5)\ (9x^2 - 5)$
$= 2(9x^2 + 5)\ [(3x)^2 - (\sqrt{5}\,)^2]$
$= \mathbf{2(9x^2 + 5)(3x + \sqrt{5}\,)(3x - \sqrt{5}\,)}$

14. $16x^4 - 625 = (4x^2)^2 - (25)^2$
$= (4x^2 + 25)\ (4x^2 - 25)$
$= (4x^2 + 25)\ [(2x)^2 - (5)^2]$
$= \mathbf{(4x^2 + 25)\ (2x + 5)\ (2x - 5)}$

15. $81 - 256x^4 = (9)^2 - (16x^2)^2$
$= (9 + 16x^2)\ (9 - 16x^2)$
$= (9 + 16x^2)\ [(3)^2 - (4x)^2]$
$= \mathbf{(9 + 16x^2)\ (3 + 4x)\ (3 - 4x)}$

16. $x^9y - xy^9 = xy(x^8 - y^8)$
$= xy[(x^4)^2 - (y^4)^2]$
$= xy(x^4 + y^4)\ (x^4 - y^4)$
$= xy(x^4 + y^4)\ [(x^2)^2 - (y^2)^2]$
$= xy(x^4 + y^4)\ (x^2 + y^2)\ (x^2 - y^2)$
$= \mathbf{xy(x^4 + y^4)\ (x^2 + y^2)\ (x + y)\ (x - y)}$

17. $x - y - x^2 + y^2 = (x - y) - (x^2 - y^2)$
$= (x - y) - (x + y)(x - y)$
$= \mathbf{(x - y)\ (1 - x - y)}$

18. $4a^2 - 4b^2 + 4a + 1 = (4a^2 + 4a + 1) - 4b^2$
$= (2a + 1)^2 - (2b)^2$
$= (2a + 1 + 2b)\ (2a + 1 - 2b)$
$= \mathbf{(2a + 2b + 1)\ (2a - 2b + 1)}$

19. $4a^2 - 25b^2 + 30b - 9 = 4a^2 - (25b^2 - 30b + 9)$
$= (2a)^2 - (5b - 3)^2$
$= \mathbf{(2a + 5b - 3)\ (2a - 5b + 3)}$

20. $25x^2 - 10x + 1 - 36z^2 = (25x^2 - 10x + 1) - 36z^2$
$= (5x - 1)^2 - (6z)^2$
$= \mathbf{(5x - 1 + 6z)\ (5x - 1 - 6z)}$

21. $9a^2 - 25b^2 - 36c^2 + 16d^2 + 2(12ad + 30bc)$
$= 9a^2 - 25b^2 - 36c^2 + 16d^2 + 24ad + 60bc$
$= 9a^2 + 24ad + 16d^2 - 25b^2 + 60bc - 36c^2$
$= (9a^2 + 24ad + 16d^2) - (25b^2 - 60bc + 36c^2)$
$= (3a + 4d)^2 - (5b - 6c)^2$
$= \mathbf{(3a + 4d + 5b - 6c)\ (3a + 4d - 5b + 6c)}$

22. $x^4 + 7x^2 + 16 = x^4 + 7x^2 + 16 + x^2 - x^2$
[Adding and subtracting x^2]
$= (x^4 + 8x^2 + 16) - x^2$
$= (x^2 + 4)^2 - (x)^2$
$= \mathbf{(x^2 + 4 + x)\ (x^2 + 4 - x)}$

23. $(1 - 4x^2)\ (1 - 4y^2) + 16xy$
$= 1 - 4x^2 - 4y^2 + 16x^2y^2 + 8xy + 8xy$
$= 1 + 8xy + 16x^2y^2 - 4x^2 - 4y^2 + 8xy$
$= (1 + 8xy + 16x^2y^2) - 4(x^2 + y^2 - 2xy)$
$= [(1 + 4xy)]^2 - [2(x - y)]^2$
$= \mathbf{(1 + 4xy + 2x - 2y)\ (1 + 4xy - 2x + 2y)}$

24. $a^8 + a^4b^4 + b^8$
$= a^8 + a^4b^4 + b^8 + a^4b^4 - a^4b^4$
[Adding and subtracting a^4b^4]
$= (a^8 + 2a^4b^4 + b^8) - a^4b^4$
$= (a^4 + b^4)^2 - (a^2b^2)^2$
$= (a^4 + b^4 + a^2b^2)\ (a^4 + b^4 - a^2b^2)$
$= (a^4 + b^4 + a^2b^2 + a^2b^2 - a^2b^2)\ (a^4 + b^4 - a^2b^2)$
$= (a^4 + 2a^2b^2 + b^4 - a^2b^2)\ (a^4 + b^4 - a^2b^2)$
$= [(a^2 + b^2)^2 - (ab)^2]\ (a^4 + b^4 - a^2b^2)$
$= \mathbf{(a^2 + b^2 + ab)\ (a^2 + b^2 - ab)\ (a^4 + b^4 - a^2b^2)}$

EXERCISE 2I

1. $x^2 + 19x + 88 = x^2 + 8x + 11x + 88$
$= x(x + 8) + 11(x + 8)$
$= \mathbf{(x + 8)\ (x + 11)}$

2. $x^2 + 14x + 45 = x^2 + 5x + 9x + 45$
$= x(x + 5) + 9(x + 5)$
$= \mathbf{(x + 5)\ (x + 9)}$

3. $x^2 + 2x - 3 = x^2 + 3x - x - 3$
$= x(x + 3) - 1(x + 3)$
$= \mathbf{(x + 3)\ (x - 1)}$

4. $x^2 + 9x - 36 = x^2 + 12x - 3x - 36$
$= x(x + 12) - 3(x + 12)$
$= \mathbf{(x + 12)\ (x - 3)}$

5. $12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$
$= 3x(4x - 1) - 1(4x - 1)$
$= \mathbf{(4x - 1)\ (3x - 1)}$

6. $x^2 - x - 132 = x^2 - 12x + 11x - 132$
$= x(x - 12) + 11(x - 12)$
$= \mathbf{(x - 12)\ (x + 11)}$

7. $x^2 - 11x - 42 = x^2 - 14x + 3x - 42$
$= x(x - 14) + 3(x - 14)$
$= \mathbf{(x - 14)\ (x + 3)}$

8. $y^2 - 11y + 18 = y^2 - 9y - 2y + 18$
$= y(y - 9) - 2(y - 9)$
$= \mathbf{(y - 9)\ (y - 2)}$

9. $x^2 - 24x + 108 = x^2 - 6x - 18x + 108$
$= x(x - 6) - 18(x - 6)$
$= \mathbf{(x - 6)\ (x - 18)}$

10. $x^2 - 2x - 15 = x^2 - 5x + 3x - 15$
$= x(x - 5) + 3(x - 5)$
$= \mathbf{(x - 5)(x + 3)}$

11. $6x^2 + 19x + 10 = 6x^2 + 4x + 15x + 10$
$= 2x(3x + 2) + 5(3x + 2)$
$= \mathbf{(3x + 2)(2x + 5)}$

12. $12x^2 - 25x + 12 = 12x^2 - 16x - 9x + 12$
$= 4x(3x - 4) - 3(3x - 4)$
$= \mathbf{(3x - 4)(4x - 3)}$

13. $4y^2 - 17y - 21 = 4y^2 + 4y - 21y - 21$
$= 4y(y + 1) - 21(y + 1)$
$= \mathbf{(y + 1)(4y - 21)}$

14. $10x^2 + 3x - 4 = 10x^2 + 8x - 5x - 4$
$= 2x(5x + 4) - 1(5x + 4)$
$= \mathbf{(5x + 4)(2x - 1)}$

15. $4x^2 - 25x + 21 = 4x^2 - 21x - 4x + 21$
$= x(4x - 21) - 1(4x - 21)$
$= \mathbf{(4x - 21)(x - 1)}$

16. $3x^2 - 10x + 8 = 3x^2 - 4x - 6x + 8$
$= x(3x - 4) - 2(3x - 4)$
$= \mathbf{(3x - 4)(x - 2)}$

17. $\frac{1}{2}x^2 + 3x + 4 = \frac{1}{2}(x^2 + 6x + 8)$
$= \frac{1}{2}(x^2 + 2x + 4x + 8)$
$= \frac{1}{2}[x(x + 2) + 4(x + 2)]$
$= \mathbf{\frac{1}{2}(x + 2)(x + 4)}$

18. $\frac{1}{5}x^2 + 2x - 15 = \frac{1}{5}(x^2 + 10x - 75)$
$= \frac{1}{5}(x^2 - 5x + 15x - 75)$
$= \frac{1}{5}[x(x - 5) + 15(x - 5)]$
$= \mathbf{\frac{1}{5}(x - 5)(x + 15)}$

19. $9x^2 - 2x - \frac{1}{3} = \frac{1}{3}(27x^2 - 6x - 1)$
$= \frac{1}{2}(27x^2 + 3x - 9x - 1)$
$= \frac{1}{3}[3x(9x + 1) - 1(9x + 1)]$
$= \mathbf{\frac{1}{3}(9x + 1)(3x - 1)}$

20. $\sqrt{3}x^2 + 5x + 2\sqrt{3} = \sqrt{3}x^2 + 2x + 3x + 2\sqrt{3}$
$= x(\sqrt{3}x + 2) + \sqrt{3}(\sqrt{3}x + 2)$
$= \mathbf{(\sqrt{3}x + 2)(x + \sqrt{3})}$

21. $4\sqrt{3}x^2 + 10x + 2\sqrt{3} = 2(2\sqrt{3}x^2 + 5x + \sqrt{3})$
$= 2(2\sqrt{3}x^2 + 2x + 3x + \sqrt{3})$
$= 2[2x(\sqrt{3}x + 1) + \sqrt{3}(\sqrt{3}x + 1)]$
$= \mathbf{2(\sqrt{3}x + 1)(2x + \sqrt{3})}$

22. $4\sqrt{5}x^2 + 17x - 3\sqrt{5} = 4\sqrt{5}x^2 + 20x - 3x - 3\sqrt{5}$
$= 4\sqrt{5}x(x + \sqrt{5}) - 3(x + \sqrt{5})$
$= \mathbf{(x + \sqrt{5})(4\sqrt{5}x - 3)}$

23. $5\sqrt{3}x^2 - 32x - 7\sqrt{3} = 5\sqrt{3}x^2 + 3x - 35x - 7\sqrt{3}$
$= \sqrt{3}x(5x + \sqrt{3}) - 7(5x + \sqrt{3})$
$= \mathbf{(5x + \sqrt{3})(\sqrt{3}x - 7)}$

24. $3(x + 5)^2 - 2(x + 5) - 8$
Let $(x + 5) = a$.
Then, the given polynomial becomes $3a^2 - 2a - 8$.
Now, $3a^2 - 2a - 8 = 3a^2 - 6a + 4a - 8$
$= 3a(a - 2) + 4(a - 2)$
$= (a - 2)(3a + 4)$
$= (x + 5 - 2)[3(x + 5) + 4]$
[Putting $a = x + 5$]
$= (x + 3)(3x + 15 + 4)$
$= (x + 3)(3x + 19)$

25. $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$
Let $a^2 - 2a = x$.
Then, the given polynomial becomes $x^2 - 23x + 120$.
Now,
$x^2 - 23x + 120 = x^2 - 8x - 15x + 120$
$= x(x - 8) - 15(x - 8)$
$= (x - 8)(x - 15)$
$= (a^2 - 2a - 8)(a^2 - 2a - 15)$
[Putting $x = a^2 - 2a$]
$= (a^2 - 4a + 2a - 8)(a^2 - 5a + 3a - 15)$
$= [a(a - 4) + 2(a - 4)][a(a - 5) + 3(a - 5)]$
$= \mathbf{(a - 4)(a + 2)(a - 5)(a + 3)}$

26. $12(x^2 + 7x)^2 - 8(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$
Let $(x^2 + 7x) = a$ and $(2x - 1) = b$.
Then, the given polynomial becomes
$12a^2 - 8ab - 15b^2$
$= 12a^2 - 18ab + 10ab - 15b^2$
$= 6a(2a - 3b) + 5b(2a - 3b)$
$= (2a - 3b)(6a + 5b)$
$= [2(x^2 + 7x) - 3(2x - 1)][6(x^2 + 7x) + 5(2x - 1)]$
[Putting $a = x^2 + 7x$ and $b = 2x - 1$]
$= (2x^2 + 14x - 6x + 3)(6x^2 + 42x + 10x - 5)$
$= \mathbf{(2x^2 + 8x + 3)(6x^2 + 52x - 5)}$

27. $8(x + 1)^2 - 2(x + 1)(y + 2) - 15(y + 2)^2$
Let $(x + 1) = a$ and $(y + 2) = b$.
Then, the given polynomial becomes
$8a^2 - 2ab - 15b^2$
$= 8a^2 - 12ab + 10ab - 15b^2$
$= 4a(2a - 3b) + 5b(2a - 3b)$
$= (2a - 3b)(4a + 5b)$
$= [2(x + 1) - 3(y + 2)][4(x + 1) + 5(y + 2)]$
[Putting $a = x + 1$ and $b = y + 2$]
$= (2x + 2 - 3y - 6)(4x + 4 + 5y + 10)$
$= \mathbf{(2x - 3y - 4)(4x + 5y + 14)}$

28. $4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2$
Let $(x - y) = a$ and $(x + y) = b$.
Then, the given polynomial becomes
$4a^2 - 12ab + 9b^2$
$= 4a^2 - 6ab - 6ab + 9b^2$

$= 2a(2a - 3b) - 3b(2a - 3b)$
$= (2a - 3b)(2a - 3b)$
$= [2(x - y) - 3(x + y)][2(x - y) - 3(x + y)]$
[Putting $a = (x - y)$ and $b = (x + y)$]
$= (2x - 2y - 3x - 3y)(2x - 2y - 3x - 3y)$
$= (-x - 5y)(-x - 5y)$
$= \mathbf{(x + 5y)(x + 5y)}$

29. $x^4 + 19x^2 - 150$
Let $x^2 = a$
$\Rightarrow x^4 = a^2$
Then, the given polynomial becomes
$a^2 + 19a - 150 = a^2 + 25a - 6a - 150$
$= a(a + 25) - 6(a + 25)$
$= (a + 25)(a - 6)$
$= \mathbf{(x^2 + 25)(x^2 - 6)}$
[Putting $a = x^2$]

30. $x^4 + 3x^2 - 28$
Let $x^2 = a$
$\Rightarrow x^4 = a^2$
Then, the given polynomial becomes
$a^2 + 3a - 28 = a^2 + 7a - 4a - 28$
$= a(a + 7) - 4(a + 7)$
$= (a + 7)(a - 4)$
$= (x^2 + 7)(x^2 - 4)$
[Putting $a = x^2$]
$= \mathbf{(x^2 + 7)(x - 2)(x + 2)}$

31. $(x^2 - 4x)(x^2 - 4x - 1) - 20$
Let $x^2 - 4x = a$
Then, the given polynomial becomes
$a(a - 1) - 20$
$= a^2 - a - 20$
$= a^2 - 5a + 4a - 20$
$= a(a - 5) + 4(a - 5)$
$= (a - 5)(a + 4)$
$= (x^2 - 4x - 5)(x^2 - 4x + 4)$ [Putting $a = x^2 - 4x$]
$= (x^2 - 5x + x - 5)(x^2 - 2x - 2x + 4)$
$= [x(x - 5) + 1(x - 5)][x(x - 2) - 2(x - 2)]$
$= \mathbf{(x - 5)(x + 1)(x - 2)(x - 2)}$

32. $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$
Let $\left(5x - \frac{1}{x}\right) = a$
Then, the given polynomial becomes
$a^2 + 4a + 4 = a^2 + 2a + 2a + 4$
$= a(a + 2) + 2(a + 2)$
$= (a + 2)(a + 2)$
$= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)$
[Putting $a = 5x - \frac{1}{x}$]

33. $y^2 + 5y - 24 = y^2 + 8y - 3y - 24$
$= y(y + 8) - 3(y + 8)$
$= (y + 8)(y - 3)$
One possible answer is: **Length = $y + 8$, breadth = $y - 3$**.

EXERCISE 2J

1. $4p^2 + 9q^2 + 4r^2 + 12pq + 12qr + 8pr$
$= (2p)^2 + (3q)^2 + (2r)^2 + 2(2p)(3q) + 2(3q)(2r) + 2(2r)(2p)$
$= (2p + 3q + 2r)^2$
[Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= \mathbf{(2p + 3q + 2r)(2p + 3q + 2r)}$

2. $x^2 + 4y^2 + z^2 - 4xy - 4yz + 2xz$
$= (x)^2 + (-2y)^2 + (z)^2 + 2(x)(-2y) + 2(-2y)(z) + 2(z)(x)$
$= [(x) + (-2y) + z]^2$
[Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= (x - 2y + z)^2$
$= \mathbf{(x - 2y + z)(x - 2y + z)}$

3. $x^2 + y^2 + 4z^2 - 2xy + 4yz - 4zx$
$= (x)^2 + (-y)^2 + (-2z)^2 + 2(x)(-y) + 2(-y)(-2z) + 2(-2z)(x)$
$= [(x) + (-y) + (-2z)]^2$
[Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= (x - y - 2z)^2$
$= \mathbf{(x - y - 2z)(x - y - 2z)}$

4. $a^2 + \frac{1}{4}b^2 + \frac{1}{9}c^2 - ab - \frac{1}{3}bc + \frac{2}{3}ca$
$= (a)^2 + \left(\frac{-1}{2}b\right)^2 + \left(\frac{1}{3}c\right)^2 + 2(a)\left(\frac{-1}{2}b\right) + 2\left(\frac{-1}{2}b\right)\left(\frac{1}{3}c\right) + 2\left(\frac{1}{3}c\right)(a)$
$= \left[a + \left(\frac{-1}{2}b\right) + \left(\frac{1}{3}c\right)\right]^2$
[Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= \left(a - \frac{1}{2}b + \frac{1}{3}c\right)^2$
$= \mathbf{\left(a - \frac{1}{2}b + \frac{1}{3}c\right)\left(a - \frac{1}{2}b + \frac{1}{3}c\right)}$

5. $27 + 9x^2 + \frac{1}{9x^2} - 30x - \frac{10}{3x}$
$= 25 + 9x^2 + \frac{1}{9x^2} - 30x + 2 - \frac{10}{3x}$
$= (5)^2 + (-3x)^2 + \left(-\frac{1}{3x}\right)^2 + 2(5)(-3x) + 2(-3x)\left(-\frac{1}{3x}\right) + 2\left(-\frac{1}{3x}\right)(5)$
$= \left[(5) + (-3x) + \left(-\frac{1}{3x}\right)\right]^2$
[Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= \left(5 - 3x - \frac{1}{3x}\right)^2$
$= \mathbf{\left(5 - 3x - \frac{1}{3x}\right)\left(5 - 3x - \frac{1}{3x}\right)}$

6. $9x^4 + y^2 + z^2 + 6x^2y - 2yz - 6x^2z$
$= (3x^2)^2 + (y)^2 + (-z)^2 + 2(3x^2)(y) + 2(y)(-z) + 2(-z)(3x^2)$
$= [(3x^2) + (y) + (-z)]^2$
[Using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$]
$= (3x^2 + y - z)^2$
$= \mathbf{(3x^2 + y - z)(3x^2 + y - z)}$

EXERCISE 2K

1. $x^3 + 6x^2y + 12xy^2 + 8y^3$
$= (x)^3 + 3(x)^2(2y) + 3(x)(2y)^2 + (2y)^3$
$= (x + 2y)^3$ [Using $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$]
$= \mathbf{(x + 2y)(x + 2y)(x + 2y)}$

2. $8x^3 - 36x^2y + 54xy^2 - 27y^3$
$= (2x)^3 - 3(2x)^2(3y) + 3(2x)(3y)^2 - (3y)^2$
$= (2x - 3y)^3$ [Using $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$]
$= \mathbf{(2x - 3y)(2x - 3y)(2x - 3y)}$

3. $\frac{64}{27}x^3 + \frac{27}{64}y^3 + 4x^2y + \frac{9}{4}xy^2$

$= \left(\frac{4}{3}x\right)^3 + \left(\frac{3}{4}y\right)^3 + 3\left(\frac{4}{3}x\right)^2\left(\frac{3}{4}y\right) + 3\left(\frac{4}{3}x\right)\left(\frac{3}{4}y\right)^2$

$= \left(\frac{4}{3}x + \frac{3}{4}y\right)^3$ [Using $(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$]

$= \mathbf{\left(\frac{4}{3}x + \frac{3}{4}y\right)\left(\frac{4}{3}x + \frac{3}{4}y\right)\left(\frac{4}{3}x + \frac{3}{4}y\right)}$

4. $\frac{1}{8}a^3 + \frac{1}{4}a^2b + \frac{1}{6}ab^2 + \frac{1}{27}b^3$

$= \left(\frac{a}{2}\right)^3 + 3\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right) + 3\left(\frac{a}{2}\right)\left(\frac{b}{3}\right)^2 + \left(\frac{b}{3}\right)^2$

$= \left(\frac{a}{2} + \frac{b}{3}\right)^3$ [Using $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$]

$= \mathbf{\left(\frac{a}{2} + \frac{b}{3}\right)\left(\frac{a}{2} + \frac{b}{3}\right)\left(\frac{a}{2} + \frac{b}{3}\right)}$

5. $\frac{8x^3}{27} - \frac{28x^2}{3} + 98x - 343$

$= \left(\frac{2x}{3}\right)^3 - 3\left(\frac{2x}{3}\right)^2(7) + 3\left(\frac{2x}{3}\right)(7)^2 - (7)^3$

$= \left(\frac{2x}{3} - 7\right)^3$ [Using $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$]

$= \mathbf{\left(\frac{2x}{3} - 7\right)\left(\frac{2x}{3} - 7\right)\left(\frac{2x}{3} - 7\right)}$

6. $p^6 - \frac{27}{8}q^6 - \frac{9}{2}p^4q^2 + \frac{27}{4}p^2q^4$

$= (p^2)^3 - \left(\frac{3}{2}q^2\right)^3 - 3(p^2)^2\left(\frac{3}{2}q^2\right) + 3(p^2)\left(\frac{3}{2}q^2\right)^2$

$= \left(p^2 - \frac{3}{2}q^2\right)^3$ [Using $(x - y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$]

$= \mathbf{\left(p^2 - \frac{3}{2}q^2\right)\left(p^2 - \frac{3}{2}q^2\right)\left(p^2 - \frac{3}{2}q^2\right)}$

EXERCISE 2L

Q.1 to Q.13 have been solved using
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

1. $x^3 + 64 = (x)^3 + (4)^3$
$= \mathbf{(x + 4)(x^2 - 4x + 16)}$

2. $8a^3 + b^3 = (2a)^3 + (b)^3$
$= \mathbf{(2a + b)(4a^2 - 2ab + b^2)}$

3. $64x^3 + 343y^3 = (4x)^3 + (7y)^3$
$= \mathbf{(4x + 7y)(16x^2 - 28xy + 49y^2)}$

4. $512a^3 + \frac{1}{729b^3} = (8a)^3 + \left(\frac{1}{9b}\right)^3$
$= \left(8a + \frac{1}{9b}\right)\left(64a^2 - \frac{8a}{9b} + \frac{1}{81b^2}\right)$

5. $125x^3 + \frac{1}{216} = (5x)^3 + \left(\frac{1}{6}\right)^3$
$= \mathbf{\left(5x + \frac{1}{6}\right)\left(25x^2 - \frac{5x}{6} + \frac{1}{36}\right)}$

6. $32x^3 + 108y^3 = 4(8x^3 + 27y^3)$
$= 4[(2x)^3 + (3y)^3]$
$= \mathbf{4(2x + 3y)(4x^2 - 6xy + 9y^2)}$

7. $54x^6y + 2x^3y^4 = 2x^3y(27x^3 + y^3)$
$= 2x^3y[(3x)^3 + (y)^3]$
$= \mathbf{2x^3y(3x + y)(9x^2 - 3xy + y^2)}$

8. $3x^5y^3 + 24x^2 = 3x^2(x^3y^3 + 8)$
$= 3x^2[(xy)^3 + (2)^3]$
$= \mathbf{3x^2(xy + 2)(x^2y^2 - 2xy + 4)}$

9. $1 + 125x^3 = (1)^3 + (5x)^3$
$= \mathbf{(1 + 5x)(1 - 5x + 25x^2)}$

10. $0.343 + 8a^3 = (0.7)^3 + (2a)^3$
$= \mathbf{(0.7 + 2a)(0.49 - 1.4a + 4a^2)}$

11. $8x^3 + 0.125 = (2x)^3 + (0.5)^3$
$= \mathbf{(2x + 0.5)(4x^2 - x + 0.25)}$

12. $125x^6 + y^6 = (5x^2)^3 + (y^2)^3$
$= \mathbf{(5x^2 + y^2)(25x^4 - 5x^2y^2 + y^4)}$

13. $x^7y + xy^7 = xy(x^6 + y^6)$
$= xy[(x^2)^3 + (y^2)^3]$
$= \mathbf{xy(x^2 + y^2)(x^4 - x^2y^2 + y^4)}$

Q.14 to Q.20 have been solved using
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

14. $54x^3y - 128y^4 = 2y(27x^3 - 64y^3)$
$= 2y[(3x)^3 - (4y)^3]$
$= \mathbf{2y(3x - 4y)(9x^2 + 12xy + 16y^2)}$

15. $x^3 - 125 = (x)^3 - (5)^3$
$= \mathbf{(x - 5)(x^2 + 5x + 25)}$

16. $1331 - 343x^3 = (11)^3 - (7x)^3$
$= \mathbf{(11 - 7x)(121 + 77x + 49x^2)}$

17. $\frac{1}{8}x^3 - 216y^3 = \left(\frac{1}{2}x\right)^3 - (6y)^3$
$= \mathbf{\left(\frac{1}{2}x - 6y\right)\left(\frac{x^2}{4} + 3xy + 36y^2\right)}$

18. $a^3 - 2\sqrt{2}b^3 = (a)^3 - (\sqrt{2}b)^3$
$= \mathbf{(a - \sqrt{2}b)(a^2 + \sqrt{2}ab + 2b^2)}$

19. $250x^3 - 16y^3 = 2(125x^3 - 8y^3)$
$= 2[(5x)^3 - (2y)^3]$
$= \mathbf{2(5x - 2y)(25x^2 + 10xy + 4y^2)}$

20. $8x^3 - (2x - y)^3$
$= (2x)^3 - (2x - y)^3$
$= (2x - 2x + y)[4x^2 + 2x(2x - y) + (2x - y)^2]$
$= y(4x^2 + 4x^2 - 2xy + 4x^2 + y^2 - 4xy)$
$= \mathbf{y(12x^2 + y^2 - 6xy)}$

21. $5a + 20b + a^3 + 64b^3$
$= 5(a + 4b) + (a)^3 + (4b)^3$
$= 5(a + 4b) + (a + 4b)(a^2 - 4ab + 16b^2)$
[Using $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$]
$= \mathbf{(a + 4b)(5 + a^2 - 4ab + 16b^2)}$

22. $8a^3 - 27b^3 - 4ax + 6bx$
$= (2a)^3 - (3b)^3 - 2x(2a - 3b)$
$= (2a - 3b)(4a^2 + 6ab + 9b^2) - 2x(2a - 3b)$
[Using $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$]
$= \mathbf{(2a - 3b)(4a^2 + 6ab + 9b^2 - 2x)}$

23. $2x - 3y - 8x^3 + 27y^3$
$= (2x - 3y) - (8x^3 - 27y^3)$
$= (2x - 3y) - [(2x)^3 - (3y)^3]$
$= (2x - 3y) - [(2x - 3y)(4x^2 + 6xy + 9y^2)]$
[Using $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$]
$= \mathbf{(2x - 3y)(1 - 4x^2 - 6xy - 9y^2)}$

24. $x^8 - x^2y^6$
$= x^2(x^6 - y^6)$
$= x^2[(x^2)^3 - (y^2)^3]$
$= x^2(x^2 - y^2)(x^4 + x^2y^2 + y^4)$
[Using $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$]
$= x^2(x^2 - y^2)(x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2)$
$= x^2(x^2 - y^2)(x^4 + 2x^2y^2 + y^4 - x^2y^2)$
$= x^2(x^2 - y^2)[(x^2 + y^2)^2 - (xy)^2]$
$= x^2(x + y)(x - y)(x^2 + y^2 - xy)(x^2 + y^2 + xy)$
[Using $x^2 - y^2 = (x + y)(x - y)$]
$= \mathbf{x^2(x + y)(x - y)(x^2 - xy + y^2)(x^2 + xy + y^2)}$

25. $a^9 - b^9$
$= (a^3)^3 - (b^3)^3$
$= (a^3 - b^3)(a^6 + a^3b^3 + b^6)$
$= \mathbf{(a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6)}$

26. $x^6 - 26x^3 - 27$
Let $x^3 = y$.
Then, the given polynomial becomes
$y^2 - 26y - 27$
$= y^2 - 27y + y - 27$
$= y(y - 27) + 1(y - 27)$
$= (y - 27)(y + 1)$
$= (x^3 - 27)(x^3 + 1)$ [Putting $y = x^3$]
$= [(x)^3 - (3)^3][(x)^3 + (1)^3]$
$= (x - 3)(x^2 + 3x + 9)(x + 1)(x^2 - x + 1)$
$= \mathbf{(x - 3)(x + 1)(x^2 + 3x + 9)(x^2 - x + 1)}$

27. $(3x + 4)^3 + (7 - 3x)^3$
$= (3x + 4 + 7 - 3x)[(3x + 4)^2 - (3x + 4)(7 - 3x) + (7 - 3x)^2]$
$= 11(9x^2 + 24x + 16 - 21x - 28 + 9x^2 + 12x + 49 - 42x + 9x^2)$
$= \mathbf{11(27x^2 - 27x + 37)}$

28. $(2x + 1)^3 - (x + 1)^3$
$= (2x + 1 - x - 1)[(2x + 1)^2 + (2x + 1)(x + 1) + (x + 1)^2]$
$= x(4x^2 + 4x + 1 + 2x^2 + x + 2x + 1 + x^2 + 2x + 1)$
$= \mathbf{x(7x^2 + 9x + 3)}$

29. $\left(\frac{a}{3} + \frac{b}{5}\right)^3 - \left(\frac{a}{3} - \frac{b}{5}\right)^3$

$$= \left(\frac{a}{3} + \frac{b}{5} - \frac{a}{3} + \frac{b}{5}\right)^3$$
$$\left[\left(\frac{a}{3} + \frac{b}{5}\right)^2 + \left(\frac{a}{3} + \frac{b}{5}\right)\left(\frac{a}{3} - \frac{b}{5}\right) + \left(\frac{a}{3} - \frac{b}{5}\right)^2\right]$$

$$= \frac{2b}{5}\left(\frac{a^2}{9} + \frac{2ab}{15} + \frac{b^2}{25} + \frac{a^2}{9} - \frac{b^2}{25} + \frac{a^2}{9} - \frac{2ab}{15} + \frac{b^2}{25}\right)$$

$$= \frac{2b}{5}\left(\frac{3a^2}{9} + \frac{b^2}{25}\right)$$

$$= \mathbf{\frac{2b}{5}\left(\frac{a^2}{3} + \frac{b^2}{25}\right)}$$

30. $x^3 - 3x^2 + 3x + 7$
$= (x^3 - 3x^2 + 3x - 1) + 8$
$= [(x)^3 - 3(x)^2(1) + 3(x)(1)^2 - (1)^3] + 8$
$= (x - 1)^3 + (2)^3$
$= (x - 1 + 2)[(x - 1)^2 - (x - 1)(2) + (2)^2]$
$= (x + 1)(x^2 - 2x + 1 - 2x + 2 + 4)$
$= \mathbf{(x + 1)(x^2 - 4x + 7)}$

EXERCISE 2M

1. $8x^3 + 27y^3 + 64z^3 - 72xyz$
$= (2x)^3 + (3y)^3 + (4z)^3 - 3(2x)(3y)(4z)$
$= (2x + 3y + 4z)[(2x)^2 + (3y)^2 + (4z)^2 - (2x)(3y) - (3y)(4z) - (4z)(2x)]$
$= \mathbf{(2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx)}$

2. $8x^3 - 27y^3 + z^3 + 18xyz$
$= (2x)^3 + (-3y)^3 + (z)^3 - 3(2x)(-3y)(z)$
$= [(2x) + (-3y) + (z)][(2x)^2 + (-3y)^2 + (z)^2 - (2x)(-3y) - (-3y)(z) - (z)(2x)]$
$= \mathbf{(2x - 3y + z)(4x^2 + 9y^2 + z^2 + 6xy + 3yz - 2zx)}$

3. $27a^3 + 125b^3 - c^3 + 45abc$
$= (3a)^3 + (5b)^3 + (-c)^3 - 3(3a)(5b)(-c)$
$= [(3a) + (5b) + (-c)][(3a)^2 + (5b)^2 + (-c)^2 - (3a)(5b) - (5b)(-c) - (-c)(3a)]$
$= \mathbf{(3a + 5b - c)(9a^2 + 25b^2 + c^2 - 15ab + 5bc + 3ca)}$

4. $x^3 - 27y^3 - 1 - 9xy$
$= (x)^3 + (-3y)^3 + (-1)^3 - 3(x)(-3y)(-1)$
$= [(x) + (-3y) + (-1)][(x)^2 + (-3y)^2 + (-1)^2 - (x)(-3y) - (-3y)(-1) - (-1)(x)]$
$= (x - 3y - 1)(x^2 + 9y^2 + 1 + 3xy - 3y + x)$
$= \mathbf{(x - 3y - 1)(x^2 + 9y^2 + 3xy - 3y + x + 1)}$

5. $-27x^3 + y^3 - z^3 - 9xyz$
$= (-3x)^3 + (y)^3 + (-z)^3 - 3(-3x)(y)(-z)$
$= [(-3x) + (y) + (-z)][(-3x)^2 + y^2 + (-z)^2 - (-3x)(y) - (y)(-z) - (-z)(-3x)]$
$= \mathbf{(-3x + y - z)(9x^2 + y^2 + z^2 + 3xy + yz - 3zx)}$

6. $\frac{1}{8}x^3 - 64y^3 + 27z^3 + 18xyz$

$$= \left(\frac{x}{2}\right)^3 + (-4y)^3 + (3z)^3 - 3\left(\frac{x}{2}\right)(-4y)(3z)$$

$$= \left[\frac{x}{2} + (-4y) + (3z)\right]$$
$$\left[\frac{x^2}{4} + 16y^2 + 9z^2 - \left(\frac{x}{2}\right)(-4y) - (-4y)(3z) - (3z)\left(\frac{x}{2}\right)\right]$$

$$= \mathbf{\left(\frac{x}{2} - 4y + 3z\right)\left(\frac{x^2}{4} + 16y^2 + 9z^2 + 2xy + 12yz - \frac{3}{2}zx\right)}$$

7. $2\sqrt{2}x^3 + 3\sqrt{3}y^3 + \sqrt{5}(5 - 3\sqrt{6}xy)$
$= 2\sqrt{2}x^3 + 3\sqrt{3}y^3 + 5\sqrt{5} - 3\sqrt{5}\sqrt{6}xy$
$= (\sqrt{2}x)^3 + (\sqrt{3}y)^3 + (\sqrt{5})^3 - 3(\sqrt{2}x)(\sqrt{3}y)(\sqrt{5})$
$= (\sqrt{2}x + \sqrt{3}y + \sqrt{5})[(2x^2 + 3y^2 + 5) - (\sqrt{2}x)(\sqrt{3}y) - (\sqrt{3}y)(\sqrt{5}) - (\sqrt{5})(\sqrt{2}x)]$
$= \mathbf{(\sqrt{2}x + \sqrt{3}y + \sqrt{5})(2x^2 + 3y^2 + 5 - \sqrt{6}xy - \sqrt{15}y - \sqrt{10}x)}$

8. $8x^3 - y^3 - 1 - 6xy$
$= (2x)^3 + (-y)^3 + (-1)^3 - 3(2x)(-y)(-1)$
$= [(2x) + (-y) + (-1)]\,[(2x)^2 + (-y)^2 + (-1)^2 - (2x)(-y) - (-y)(-1) - (-1)(2x)]$
$= \mathbf{(2x - y - 1)(4x^2 + y^2 + 1 + 2xy - y + 2x)}$

9. $-a^6 + 8b^6 + c^6 + 6a^2b^2c^2$
$= (-a^2)^3 + (2b^2)^3 + (c^2)^3 - 3(-a^2)(2b^2)(c^2)$
$= [(-a^2) + (2b^2) + c^2]\,[(-a^2)^2 + (2b^2)^2 + (c^2)^2 - (-a^2)(2b^2) - (2b^2)(c^2) - (c^2)(-a^2)]$
$= \mathbf{(-a^2 + 2b^2 + c^2)(a^4 + 4b^4 + c^4 + 2a^2b^2 - 2b^2c^2 + c^2a^2)}$

10. $27x^3 - 8y^6 + 125z^3 + 90xy^2z$
$= (3x)^3 + (-2y^2)^3 + (5z)^3 - 3(3x)(-2y^2)(5z)$
$= [(3x) + (-2y^2) + (5z)][(3x)^2 + (-2y^2)^2 + (5z)^2 - (3x)(-2y^2) - (-2y^2)(5z) - (5z)(3x)]$
$= \mathbf{(3x - 2y^2 + 5z)(9x^2 + 4y^4 + 25z^2 + 6xy^2 + 10y^2z - 15xz)}$

11. $16a^3 - 54b^6 - 2c^3 - 36ab^2c$
$= 2(8a^3 - 27b^6 - c^3 - 18ab^2c)$
$= 2[(2a)^3 + (-3b^2)^3 + (-c)^3 - 3(2a)(-3b^2)(-c)]$
$= 2[(2a) + (-3b^2) + (-c)]\,[4a^2 + 9b^4 + c^2 - (2a)(-3b^2) - (-3b^2)(-c) - (-c) - (-c)(2a)]$
$= \mathbf{2(2a - 3b^2 - c)(4a^2 + 9b^4 + c^2 + 6ab^2 - 3b^2c + 2ca)}$

12. $x^6 - \dfrac{1}{x^6} - 14$

$= x^6 - \dfrac{1}{x^6} - 8 - 6$

$= (x^2)^3 + \left(\dfrac{-1}{x^2}\right)^3 + (-2)^3 - 3(x^2)\left(-\dfrac{1}{x^2}\right)(-2)$

$= \left[(x^2) + \left(-\dfrac{1}{x^2}\right) + (-2)\right]\left[x^4 + \dfrac{1}{x^4} + 4 - (x^2)\left(-\dfrac{1}{x^2}\right) - \left(-\dfrac{1}{x^2}\right)(-2) - (-2)\right](x^2)]$

$= \left(x^2 - \dfrac{1}{x^2} - 2\right)\left(x^4 + \dfrac{1}{x^4} + 4 + 1 - \dfrac{2}{x^2} + 2x^2\right)$

$= \mathbf{\left(x^2 - \dfrac{1}{x^2} - 2\right)\left(x^4 + \dfrac{1}{x^4} + 5 - \dfrac{2}{x^2} + 2x^2\right)}$

13. $(x - 3y)^3 + (3y - 7z)^3 + (7z - x)^3$
Let $a = (x - 3y)$, $b = (3y - 7z)$ and $c = (7z - x)$
Then, $a + b + c = x - 3y + 3y - 7z + 7z - x = 0$
$\therefore\ a^3 + b^3 + c^3 = 3abc$
$\therefore\ (x - 3y)^3 + (3y - 7z)^3 + (7z - x)^3$
$= \mathbf{3(x - 3y)(3y - 7z)(7z - x)}$

14. $(5a - 6b)^3 + (7c - 5a)^3 + (6b - 7c)^3$
Let $x = 5a - 6b$, $y = 7c - 5a$ and $z = 6b - 7c$
Then, $x + y + z = 5a - 6b + 7c - 5a + 6b - 7c = 0$
$\therefore\ x^3 + y^3 + z^3 = 3xyz$
$\therefore\ (5a - 6b)^3 + (7c - 5a)^3 + (6b - 7c)^3$
$= \mathbf{3(5a - 6b)(7c - 5a)(6b - 7c)}$

15. $(x + y - 2z)^3 + (y + z - 2x)^3 + (z + x - 2y)^3$
Let $a = (x + y - 2z)$, $b = (y + z - 2x)$ and $c = (z + x - 2y)$
Then, $a + b + c = x + y - 2z + y + z - 2x + z + x - 2y = 0$
$\therefore\ a^3 + b^3 + c^3 = 3ab$
$\therefore\ (x + y - 2z)^3 + (y + z - 2x)^3 + (z + x - 2y)^3$
$= 3(x + y - 2z)(y + z - 2x) + (z + x - 2y)$

16. Numerator $= (9x^2 - 4y^2)^3 + (4y^2 - 25z^2)^3 + (25z^2 - 9x^2)^3$
Let $a = (9x^2 - 4y^2)$, $b = (4y^2 - 25z^2)$ and $c = (25z^2 - 9x^2)$
Then, $a + b + c = 9x^2 - 4y^2 + 4y^2 - 25z^2 + 25z^2 - 9x^2 = 0$
$\therefore\ a^3 + b^3 + c^3 = 3abc$
$\therefore\ (9x^2 - 4y^2)^3 + (4y^2 - 25z^2)^3 + (25z^2 - 9x^2)^3$
$= 3(9x^2 - 4y^2)(4y^2 - 25z^2)(25z^2 - 9x^2)$
$= 3[(3x)^2 - (2y)^2][(2y)^2 - (5z)^2][(5z)^2 - (3x)^2]$
$= 3(3x + 2y)(3x - 2y)(2y + 5z)(2y - 5z)(5z + 3x)(5z - 3x)$ … (1)
Denominator $= (3x - 2y)^3 + (2y - 5z)^3 + (5z - 3x)^3$
Let, $a = (3x - 2y)$, $b = (2y - 5z)$ and $c = (5z - 3x)$
Then, $a + b + c = 3x - 2y + 2y - 5z + 5z - 3x = 0$
$\therefore\ a^3 + b^3 + c^3 = 3abc$
$\therefore\ (3x - 2y)^3 + (2y - 5z)^3 + (5z - 3x)^3$
$= 3(3x - 2y)(2y - 5z)(5z - 3x)$ … (2)
The given expression
$= \dfrac{3(3x+2y)(3x-2y)(2y+5z)(2y-5z)(5z+3x)(5z-3x)}{3(3x-2y)(2y-5z)(5z-3x)}$ [Using (1) and (2)]
$= \mathbf{(3x + 2y)(2y + 5z)(5z + 3x)}$

17. LHS $= (x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x)$
$= [(x + y) + (y + z) + (z + x)][(x + y)^2 + (y + z)^2 + (z + x)^2 - (x + y)(y + z) - (y + z)(z + x) - (z + x)(x + y)]$
$= (2x + 2y + 2z)(x^2 + 2xy + y^2 + y^2 + 2yz + z^2 + z^2 + 2xz + x^2 - xy - y^2 - xz - yz - yz - z^2 - xy - xz - zx - x^2 - zy - xy)$
$= 2(x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$
$= 2(x^3 + y^3 + z^3 - 3xyz) =$ RHS

18. $3x + y + z = 0$ [Given]
$\therefore\ (3x)^3 + (y)^3 + (z)^3 = 3(3x)(y)(z)$
[If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]
$\therefore\ 27x^3 + y^3 + z^3 = 9xyz$

19. Volume of cuboid $= x^3 + 2x^2 - x - 2 = x^2(x + 2) - 1(x + 2)$
$= (x + 2)(x^2 - 1)$
$= (x + 2)(x + 1)(x - 1)$
Possible expressions for the dimensions of the cuboid are $(x + 1)$, $(x - 1)$ and $(x + 2)$.

20. Volume of cube $= x^3 - 9x^2 + 27x - 27$
$= (x)^3 - 3(x)^2(3) + 3(x)(3)^2 - (3)^3$
$= (x - 3)^3$ [Using $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$]
$= (x - 3)(x - 3)(x - 3)$
Possible expressions for the side of the cube is $(x - 3)$.

EXERCISE 2N

1. (*i*) Let us divide $x^3 + 13x^2 + 31x - 45$ by $x + 9$ to get the other factor

$$
\begin{array}{r|l|l}
x+9 & x^3 + 13x^2 + 31x - 45 & x^2 + 4x - 5 \\
 & \underline{\overset{-}{x^3} + \overset{-}{9x^2}} & \\
 & 4x^2 + 31x - 45 & \\
 & \underline{\overset{-}{4x^2} + \overset{-}{36x}} & \\
 & -5x - 45 & \\
 & \underline{\overset{+}{-5x} \overset{+}{-45}} & \\
 & 0 &
\end{array}
$$

$\therefore \quad x^3 + 13x^2 + 31x - 45 = (x^2 + 4x - 5)(x + 9)$
$= (x^2 + 5x - x - 5)(x + 9)$
$= [x(x + 5) - 1(x + 5)(x + 9)]$
$= (x + 5)\ (x - 1)(x + 9)$

(*ii*) Let us divide $3x^3 - 4x^2 - 12x + 16$ by $x - 2$ to get the other factors

$$\begin{array}{r|l} x-2 & 3x^3 - 4x^2 - 12x + 16 \;(3x^2 + 2x - 8 \\ & \underline{3x^3 - 6x^2} \\ & \quad\; -\quad + \\ & \qquad 2x^2 - 12x + 16 \\ & \qquad \underline{2x^2 - 4x} \\ & \qquad -\quad + \\ & \qquad\quad -8x + 16 \\ & \qquad\quad \underline{-8x + 16} \\ & \qquad\quad +\quad - \\ & \qquad\qquad 0 \end{array}$$

$\therefore \quad 3x^3 - 4x^2 - 12x + 16 = (3x^2 + 2x - 8)(x - 2)$
$= [3x^3 + 6x - 4x - 8](x - 2)$
$= [3x\ (x + 2) - 4(x + 2)](x - 2)$
$= (x + 2)(3x - 4)(x - 2)$

(*iii*) Let us divide $3x^3 + x^2 - 20x + 12$ by $(3x - 2)$ to get the other factors

$$\begin{array}{r|l} 3x-2 & 3x^3 + x^2 - 20x + 12 \;(x^2 + x - 6 \\ & \underline{3x^3 - 2x^2} \\ & \quad\; -\quad + \\ & \qquad 3x^2 - 20x + 12 \\ & \qquad \underline{3x^2 - 2x} \\ & \qquad -\quad + \\ & \qquad\quad -18x + 12 \\ & \qquad\quad \underline{-18x + 12} \\ & \qquad\quad +\quad - \\ & \qquad\qquad 0 \end{array}$$

$\therefore 3x^3 + x^2 - 20x + 12 = (x^2 + x - 6)(3x - 2)$
$= [x^2 + 3x - 2x - 6](3x - 2)$
$= [x\ (x + 3) -2(x + 3)](3x - 2)$
$= (x + 3)(x - 2)(3x - 2)$

2. By splitting method

$2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$
$= 2x(x - 5) + 3(x - 5)$
$= \mathbf{(x - 5)\ (2x + 3)}$

By using the factor theorem

$$2x^2 - 7x - 15 = 2\left(x^2 - \frac{7}{2}x - \frac{15}{2}\right)$$
$= 2p(x)$, say

If a and b are zeroes of the polynomial $p(x)$,

then $2x^2 - 7x - 15 = 2(x - a)\ (x - b)$. So, $ab = \dfrac{-15}{2}$

So, some possibilities of a and b could be $\pm 1, \pm\dfrac{15}{2}$,

$\pm 3, \pm\dfrac{5}{2}, \pm 5$ and $\pm\dfrac{3}{2}$

By trail, we find that

$$p(5) = (5)^2 - \left(\frac{7}{2}\right)(5) - \frac{15}{2}$$
$$= 25 - \frac{35}{2} - \frac{15}{2}$$
$$= \frac{50 - 35 - 15}{2}$$
$$= \frac{50 - 50}{2}$$
$$= 0$$

So, $(x - 5)$ is a factor of $p(x)$.

Similarly, by trail we find that

$$p\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 - \left(\frac{7}{2}\right)\left(-\frac{3}{2}\right) - \frac{15}{2}$$
$$= \frac{9}{4} + \frac{21}{4} - \frac{15}{2}$$
$$= \frac{9 + 21 - 30}{4}$$
$$= \frac{30 - 30}{4}$$
$$= 0$$

So, $x - \left(\dfrac{-3}{2}\right) = \left(x + \dfrac{3}{2}\right)$ is a factor of $p(x)$.

$$\therefore \quad 2x^2 - 7x - 15 = 2(x - 5)\left(x + \frac{3}{2}\right)$$
$$= 2(x - 5)\left(\frac{2x + 3}{2}\right)$$
$$= (x - 5)\ (2x + 3)$$

Hence, $\mathbf{2x^2 - 7x - 15 = (x - 5)\ (2x + 3)}$

3. Let $p(x) = x^3 + 6x^2 + 11x + 6$

The constant term in $p(x)$ is equal to 6 and the factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

Putting $x = -1$ in $p(x)$, we have

$p(x) = p(-1)$
$= (-1)^3 + 6(-1)^2 + 11(-1) + 6$
$= -1 + 6 - 11 + 6$
$= -12 + 12 = 0$

$\therefore \quad (x + 1)$ is a factor of $p(x)$.

Similarly, $(x + 2)$ and $(x + 3)$ are factors of $p(x)$.

Since, $p(x)$ is a polynomial of degree 3,

so, it cannot have more than three linear factors.

$\therefore \quad p(x) = k(x + 1)\ (x + 2)\ (x + 3)$

$\Rightarrow \quad x^3 + 6x^2 + 11x + 6 = k(x + 1)\ (x + 2)\ (x + 3)$

Putting $x = 0$ on both side, we get

$6 = k(0 + 1)\ (0 + 2)\ (0 + 3)$

$\Rightarrow \quad 6 = 6k$

$\Rightarrow \quad k = 1$

Substituting $k = 1$ in $p(x) = k(x + 1)\ (x + 2)\ (x + 3)$, we get

$p(x) = (x + 1)\ (x + 2)\ (x + 3)$

Hence, $x^3 + 6x^2 + 11x + 6 = \mathbf{(x + 1)\ (x + 2)\ (x + 3)}$

4. Let $p(x) = x^3 - 3x^2 - x + 3$

The constant term in $p(x)$ is equal to 3 and the factors of 3 are ± 1 and ± 3.

Putting $x = 1$ in $p(x)$, we have

$p(x) = p(1)$
$= (1)^3 - 3(1)^2 - (1) + 3$
$= 1 - 3 - 1 + 3$
$= 4 - 4$
$= 0$

$\therefore$ $(x - 1)$ is a factor of $p(x)$.

Similarly, $(x - 3)$ and $(x + 1)$ are factors of $p(x)$, since $p(x)$ is a polynomial of degree 3.

So, it cannot have more than three linear factors.

$\therefore \quad p(x) = k(x - 1)(x - 3)(x + 1)$

$\Rightarrow \quad x^3 - 3x^2 - x + 3 = k(x - 1)(x - 3)(x + 1)$

Putting $x = 0$ on both sides, we get

$3 = k(0 - 1)(0 - 3)(0 + 1)$

$\Rightarrow \quad 3 = 3k$

$\Rightarrow \quad k = 1$

Hence, $x^3 - 3x^2 - x + 3 =$ $\mathbf{(x - 1)(x - 3)(x + 1)}$.

5. Let $\quad p(x) = x^3 + 5x^2 - 4x - 20$

The constant term in $p(x)$ is equal to -20 and the factors of -20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$ and ± 20.

Putting $x = 2$, in $p(x)$, we have

$$\begin{aligned} p(x) &= p(2) \\ &= (2)^3 + 5(2)^2 - 4(2) - 20 \\ &= 8 + 20 - 8 - 20 \\ &= 0 \end{aligned}$$

$\therefore$ $(x - 2)$ is a factor of $p(x)$.

Similarly, $(x + 2)$ and $(x + 5)$ are factors of $p(x)$.

Since $p(x)$ is a polynomial of degree 3.

So, it cannot have more than three linear factors

$\therefore \quad p(x) = k(x - 2)(x + 2)(x + 5)$

$\Rightarrow \quad x^3 + 5x^2 - 4x - 20 = k(x - 2)(x + 2)(x + 5)$

Putting $x = 0$ on both sides, we get

$-20 = k(0 - 2)(0 + 2)(0 + 5)$

$\Rightarrow \quad -20 = -20k$

$\Rightarrow \quad k = 1$

Hence, $x^3 + 5x^2 - 4x - 20 =$ $\mathbf{(x - 2)(x + 2)(x + 5)}$.

6. Let $\quad p(x) = x^3 - 2x^2 - 5x + 6$

The constant term in $p(x)$ is 6 and the factors of 6 are $\pm 1, \pm 2, \pm 3$ and ± 6.

Putting $x = 1$ in $p(x)$, we have

$$\begin{aligned} p(x) &= p(1) \\ &= (1)^3 - 2(1)^2 - 5(1) + 6 \\ &= 1 - 2 - 5 + 6 \\ &= 7 - 7 \\ &= 0 \end{aligned}$$

$\therefore$ $(x - 1)$ is a factor of $p(x)$.

Similarly, $(x - 3)$ and $(x + 2)$ are factors of $p(x)$.

Since $p(x)$ is a polynomial of degree 3.

So, it cannot have more than three linear factors

$\therefore \quad p(x) = k(x - 1)(x - 3)(x + 2)$

Putting $x = 0$ on both sides, we get

$6 = k(0 - 1)(0 - 3)(0 + 2)$

$\Rightarrow \quad 6 = 6k$

$\Rightarrow \quad k = 1$

Hence, $x^3 - 2x^2 - 5x + 6 =$ $\mathbf{(x - 1)(x - 3)(x + 2)}$

7. Let $\quad p(x) = x^3 - 8x^2 + x + 42$

The constant term in $p(x)$ is 42 and the factors of 42 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 6, \pm 7, \pm 14$ ± 21 and ± 42.

Putting $x = -2$, in $p(x)$, we have

$$\begin{aligned} p(x) &= p(-2) \\ &= (-2)^3 - 8(-2)^2 + (-2) + 42 \\ &= -8 - 32 - 2 + 42 \\ &= -42 + 42 \\ &= 0 \end{aligned}$$

$\therefore$ $(x + 2)$ is a factor of $p(x)$.

Similarly, $(x - 3)$ and $(x - 7)$ are factors of $p(x)$.

Since $p(x)$ is a polynomial of degree 3.

So, it cannot have more than three linear factors.

$\therefore \quad p(x) = k(x + 2)(x - 3)(x - 7)$

$\Rightarrow \quad x^3 - 8x^2 + x + 42 = k(x + 2)(x - 3)(x - 7)$

Putting $x = 0$ on both sides, we get

$42 = k(0 + 2)(0 - 3)(0 - 7)$

$\Rightarrow \quad 42 = 42k$

$\Rightarrow \quad k = 1$

Hence, $x^3 - 8x^2 + x + 42 =$ $\mathbf{(x + 2)(x - 3)(x - 7)}$.

8. Let $\quad p(x) = 2x^3 - x^2 - 13x - 6$

Putting $x = 3$, we get

$$\begin{aligned} p(3) &= 2(3)^3 - (3)^2 - 13(3) - 6 \\ &= 54 - 9 - 39 - 6 \\ &= 54 - 54 \\ &= 0 \end{aligned}$$

By factor theorem, $(x - 3)$ is a factor of $2x^3 - x^2 - 13x - 6$.

On dividing $2x^3 - x^2 - 13x - 6$ by $x - 3$, we get

$$\begin{array}{l} x - 3 \overline{)\, 2x^3 - x^2 - 13x - 6 \,(} \, 2x^2 + 5x + 2 \\ \quad\;\; 2x^3 - 6x^2 \\ \quad\;\; - \quad + \\ \hline \qquad\quad 5x^2 - 13x - 6 \\ \qquad\quad 5x^2 - 15x \\ \qquad\quad - \quad + \\ \hline \qquad\qquad\quad 2x - 6 \\ \qquad\qquad\quad 2x - 6 \\ \qquad\qquad\quad - \quad + \\ \hline \qquad\qquad\qquad 0 \end{array}$$

$$\begin{aligned} \therefore \quad 2x^3 - x^2 - 13x - 6 &= (2x^2 + 5x + 2)(x - 3) \\ &= (2x^2 + 4x + x + 2)(x - 3) \\ &= [2x(x + 2) + 1(x + 2)](x - 3) \\ &= (x + 2)(2x + 1)(x - 3) \end{aligned}$$

$\therefore \quad 2x^3 - x^2 - 13x - 6 =$ $\mathbf{(x - 3)(x + 2)(2x + 1)}$

9. Let $\quad p(x) = 9x^3 - 27x^2 - 100x + 300$

Putting $x = 3$ in $p(x)$, we get

$$\begin{aligned} p(3) &= 9(3)^3 - 27(3)^2 - 100(3) + 300 \\ &= 243 - 243 - 300 + 300 \\ &= 0 \end{aligned}$$

By factor theorem, $(x - 3)$ is a factor of $p(x)$.

On dividing $9x^3 - 27x^2 - 100x + 300$ by $(x - 3)$, we get

$$\begin{array}{l} x - 3 \overline{)\, 9x^3 - 27x^2 - 100x + 300 \,(} \, 9x^2 - 100 \\ \quad\;\; 9x^3 - 27x^2 \\ \quad\;\; - \quad + \\ \hline \qquad\qquad - 100x + 300 \\ \qquad\qquad - 100x + 300 \\ \qquad\qquad + \qquad - \\ \hline \qquad\qquad\qquad 0 \end{array}$$

$$\begin{aligned} \therefore \quad 9x^3 - 27x^2 - 100x + 300 &= (9x^2 - 100)(x - 3) \\ &= [(3x^2) - (10)^2](x - 3) \\ &= (3x + 10)(3x - 10)(x - 3) \end{aligned}$$

$\therefore \quad 9x^3 - 27x^2 - 100x + 300 =$ $\mathbf{(3x + 10)(3x - 10)(x - 3)}$

10. Let $\quad p(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

The constant term in $p(x)$ is 24 and its factors are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12$ and ± 24.

Putting $x = 1$ in $p(x)$, we get

$$
\begin{aligned}
p(x) &= p(1) \\
&= (1)^4 - 10(1)^3 + 35(1)^2 - 50(1) + 24 \\
&= 1 - 10 + 35 - 50 + 24 \\
&= 60 - 60 \\
&= 0
\end{aligned}
$$

$\therefore$ $(x - 1)$ is a factor of $p(x)$.

Similarly, by trial we get $(x - 2)$, $(x - 3)$ and $(x - 4)$ as factors of $p(x)$. Since $p(x)$ is a polynomial of degree 4, therefore it cannot have more than 4 linear factors.

$\therefore$ $p(x) = k(x - 1)(x - 2)(x - 3)(x - 4)$

$\Rightarrow$ $x^4 - 10x^3 + 35x^2 - 50x + 24 = k(x - 1)(x - 2)(x - 3)(x - 4)$

Putting $x = 0$ on both sides, we get

$$
\begin{aligned}
24 &= k(0 - 1)(0 - 2)(0 - 3)(0 - 4) \\
\Rightarrow \quad 24 &= 24k \\
\Rightarrow \quad k &= 1
\end{aligned}
$$

Hence, $x^4 - 10x^3 + 35x^2 - 50x + 24 =$ $\mathbf{(x - 1)(x - 2)(x - 3)(x - 4)}$.

11. Let $p(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$

The constant term of $p(x)$ is -8 and some of its factors are ±1, ±2, ±4 and ±8.

Putting $x = 1$ in $p(x)$, we get

$$
\begin{aligned}
p(x) &= p(1) \\
&= (1)^4 - 6(1)^3 + 7(1)^2 + 6(1) - 8 \\
&= 1 - 6 + 7 + 6 - 8 \\
&= 14 - 14 \\
&= 0
\end{aligned}
$$

Similarly, by trial we get $(x + 1)$, $(x - 2)$ and $(x - 4)$ as factors of $p(x)$.

Since $p(x)$ is a polynomial of degree 4, therefore it cannot have more 4 linear factors.

$\therefore$ $p(x) = k(x - 1)(x + 1)(x - 2)(x - 4)$

$\Rightarrow$ $x^4 - 6x^3 + 7x^2 + 6x - 8 = k(x + 1)(x + 1)(x - 2)(x - 4)$

Putting $x = 0$ on both sides, we get

$$
\begin{aligned}
-8 &= k(0 - 1)(0 + 1)(0 - 2)(0 - 4) \\
\Rightarrow \quad -8 &= k(-8) \\
\Rightarrow \quad k &= 1
\end{aligned}
$$

Hence, $x^4 - 6x^3 + 7x^2 + 6x - 8 =$ $\mathbf{(x - 1)(x + 1)(x - 2)(x - 4)}$.

12. Let $p(x) = 2x^4 - 3x^3 - 7x^2 + 12x - 4$

Putting $x = 1$ in $p(x)$, we get

$$
\begin{aligned}
p(1) &= 2(1)^4 - 3(1)^3 - 7(1)^2 + 12(1) - 4 \\
&= 2 - 3 - 7 + 12 - 4 \\
&= 14 - 14 \\
&= 0
\end{aligned}
$$

$\therefore$ $x - 1$ is a factor of $p(x)$. ...(1)

Putting $x = 2$ in $p(x)$, we get

$$
\begin{aligned}
p(2) &= 2(2)^4 - 3(2)^3 - 7(2)^2 + 12(2) - 4 \\
&= 32 - 24 - 28 + 24 - 4 \\
&= 0
\end{aligned}
$$

$\therefore$ $(x - 2)$ is a factor of $p(x)$. ... (2)

Since $(x - 1)(x - 2)$ are both factors of $p(x)$, [From (1) and (2)]

$\therefore$ $(x - 1)(x - 2)$, i.e. $x^2 - 3x + 2$ is a factor of $p(x)$.

On dividing $p(x)$ by $x^2 - 3x + 2$, we get

$$
\begin{array}{r|l|l}
x^2 - 3x + 2 & 2x^4 - 3x^3 - 7x^2 + 12x - 4 & 2x^2 + 3x - 2 \\
 & \underset{-}{2x^4} \underset{+}{- 6x^3} \underset{-}{+ 4x^2} & \\
\hline
 & 3x^3 - 11x^2 + 12x - 4 & \\
 & \underset{+}{3x^3} \underset{-}{- 9x^2} + 6x & \\
\hline
 & -2x^2 + 6x - 4 & \\
 & \underset{+}{-2x^2} \underset{-}{+ 6x} \underset{+}{- 4} & \\
\hline
 & 0 &
\end{array}
$$

$\therefore$ $2x^4 - 3x^3 - 7x^2 + 12x - 4$

$$
\begin{aligned}
&= (2x^2 + 3x - 2)(x^2 - 3x + 2) \\
&= [2x^2 + 4x - x - 2](x - 1)(x - 2) \\
&= [2x(x + 2) - 1(x + 2)](x - 1)(x - 2) \\
&= (x + 2)(2x - 1)(x - 1)(x - 2)
\end{aligned}
$$

Hence, $2x^4 - 3x^3 - 7x^2 + 12x - 4 =$ $\mathbf{(x - 1)(x + 2)(x - 2)(2x - 1)}$.

13. Let $p(x) = x^4 + x^3 - 7x^2 - x + 6$

Putting $x = 1$ in $p(x)$, we get

$$
\begin{aligned}
p(1) &= (1)^4 + (1)^3 - 7(1)^2 - (1) + 6 \\
&= 1 + 1 - 7 - 1 + 6 \\
&= 8 - 8 \\
&= 0
\end{aligned}
$$

$\therefore$ $(x - 1)$ in a factor of $p(x)$. ... (1)

Putting $x = 2$ in $p(x)$, we get

$$
\begin{aligned}
p(2) &= (2)^4 + (2)^3 - 7(2)^2 - 2 + 6 \\
&= 16 + 8 - 28 - 2 + 6 \\
&= 30 - 30 = 0
\end{aligned}
$$

$\therefore$ $(x - 2)$ is a factor of $p(x)$ ... (2)

Since $(x - 1)$ and $(x - 2)$ both are factors of $p(x)$ [From (1) and (2)]

$\therefore$ $(x - 1)(x - 2)$, i.e. $x^2 - 3x + 2$ is a factor of $p(x)$,

On dividing $p(x)$ by $x^2 - 3x + 2$, we get

$$
\begin{array}{r|l|l}
x^2 - 3x + 2 & x^4 + x^3 - 7x^2 - x + 6 & x^2 + 4x + 3 \\
 & x^4 \underset{+}{- 3x^3} \underset{-}{+ 2x^2} & \\
\hline
 & 4x^3 - 9x^2 - x + 6 & \\
 & \underset{-}{4x^3} \underset{+}{- 12x^2} \underset{-}{+ 8x} & \\
\hline
 & 3x^2 - 9x + 6 & \\
 & \underset{-}{3x^2} \underset{+}{- 9x} \underset{-}{+ 6} & \\
\hline
 & 0 &
\end{array}
$$

$\therefore$ $x^4 + x^3 - 7x^2 - x + 6$

$$
\begin{aligned}
&= (x^2 + 4x + 3)(x^2 - 3x + 2) \\
&= [x^2 + x + 3x + 3](x - 1)(x - 2) \\
&= [x(x + 1) + 3x(x + 1)](x - 1)(x - 2) \\
&= (x + 1)(x + 3)(x - 1)(x - 2)
\end{aligned}
$$

Hence, $x^4 + x^3 - 7x^2 - x + 6 =$ $\mathbf{(x + 1)(x + 3)(x - 1)(x - 2)}$

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (*b*) $x^3 + \dfrac{4x^{\frac{3}{2}}}{\sqrt{x}}$

$x^3 + \dfrac{4x^{\frac{3}{2}}}{x^{\frac{1}{2}}} = x^3 + 4x^{\frac{3}{2}-\frac{1}{2}} = x^3 + 4x$ has only non-negative integral powers of x so, it is a polynomial.

2. (*d*) **–7**

$$(2x^2 - 5)(4 + 3x^2) = 8x^2 - 20 + 6x^4 - 15x^2$$
$$= 6x^4 - 7x^2 - 20.$$

3. (*b*) **0**

$$\sqrt{2} = \sqrt{2}\,x^0$$

∴ It is a polynomial of degree zero.

4. (*b*) **5**

$$(x^3 - 2)(x^2 + 11) = x^5 - 2x^2 + 11x^3 - 22$$
$$= x^5 + 11x^3 - 2x^2 - 22$$

5. (*d*) **not defined**
The degree of zero polynomial is not defined because $p(x) = c$, $f(x) = 0x$, $g(x) = 0x^2$, $b(x) = 0x^3$, $d(x) = 0x^7$ are all equal to zero polynomial (constant polynomial 0)

6. (*b*) $6x^5 + x^3 + \dfrac{x}{8} + \dfrac{\sqrt{3}}{5}$
(as per the definition of a polynomial)

7. (*a*) quadratic polynomial in x
The given polynomial $x^2 + 5x - \dfrac{1}{2}$ is a polynomial of degree 2 in variable x. Therefore, it is a quadratic polynomial in x.

8. (*b*) $\dfrac{5}{16}$

$$p(z) = z^4 - z^2 + z$$

Putting $z = \dfrac{1}{2}$, we get

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)$$
$$= \frac{1}{16} - \frac{1}{4} + \frac{1}{2}$$
$$= \frac{1 - 4 + 8}{16}$$
$$= \frac{9 - 4}{16}$$
$$= \frac{5}{16}$$

9. (*c*) $\dfrac{9}{10}$

$$p(x) = 2x^2 - 3x + 5$$

⇒ $p(0) = 2(0)^2 - 3(0) + 5 = 5$
$p(1) = 2(1)^2 - 3(1) + 5 = 4$

and $p(-1) = 2(-1)^2 - 3(-1) + 5$
$= 2 + 3 + 5 = 10$

∴ $\dfrac{p(0) + p(1)}{p(-1)} = \dfrac{9}{10}$

10. (*c*) **6 terms**
A polynomial of degree 5 with maximum number of terms is of the form $a_nx^5 + a_{n-1}x^4 + a_{n-2}x^3 + a_{n-3}x^2 + a_{n-4}x^1 + a_{n-5}x^0$ so it can have at most 6 terms.

11. (*d*) $-\dfrac{1}{a}$
A real number k is called a zero of the polynomial $p(x)$ if $p(k) = 0$.
∴ Zero of polynomial $p(x) = ax + 1$ is given by $p(x) = 0$
⇒ $ax + 1 = 0$
⇒ $ax = -1$
⇒ $x = -\dfrac{1}{a}$

12. (*b*) **–2, –5**
Zeroes of polynomial $p(x) = (x + 2)(x + 5)$ are given by $p(x) = 0$
⇒ $(x + 2)(x + 5) = 0$
⇒ Either $x + 2 = 0$ or $(x + 5) = 0$
⇒ $x = -2$ or $x = -5$

13. (*d*) **0, 1, 2**
Zeroes of polynomial $p(x) = x(x - 1)(x - 2)$ are given by $p(x) = 0$
⇒ $x(x - 1)(x - 2) = 0$
⇒ Either $x = 0$ or $x - 1 = 0$ or $x - 2 = 0$
⇒ $x = 0$ or $x = 1$ or $x = 2$

14. (*c*) **1**
By trial, we find

$$p(1) = (1)^3 + 3(1)^2 - 3(1) - 1$$
$$= 1 + 3 - 3 - 1$$
$$= 0$$

∴ 1 is a zero of the given polynomial.

15. (*d*) **2**
Let $p(x) = x^2 - 5x + 4$
Putting $x = 3$ in $p(x)$, we get

$$p(3) = 3^2 - 5(3) + 4$$
$$= 9 - 15 + 4$$
$$= 13 - 15$$
$$= -2$$

For 3 to become a zero of the given polynomial 2 has to the added so that, $p(3)$ becomes equal to 0.

16. (*a*) **15**
Let $p(x) = x^2 - 16x + 30.$
Putting $x = 15$ in $p(x)$, we get

$$p(15) = 15^2 - 16 \times 15 + 30$$
$$= 225 - 240 + 30$$
$$= 255 - 240$$
$$= 15$$

∴ For 15 to become a zero of the given polynomial, 15 has to be subtracted from it so that, $p(15)$ becomes equal to 0.

17. (*c*) $x^2 - 2$
If $\sqrt{2}$ and $\sqrt{2}$ are zeroes of the given polynomial, then $(x - \sqrt{2})$ and $(x + \sqrt{2})$ are linear factors of the polynomial so, the required polynomial is $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$

18. (*c*) **3**

Let $p(x) = x^2 - 2k + 2$

Since $x = 2$ is zero of $p(x)$

$\therefore \quad p(2) = 0$

$\Rightarrow \quad (2)^2 - 2k + 2 = 0$

$\Rightarrow \quad 4 - 2k + 2 = 0$

$\Rightarrow \quad 6 - 2k = 0$

$\Rightarrow \quad 6 = 2k$

$\Rightarrow \quad k = 3$

19. (*a*) **–9**

Let $p(x) = x^3 + 3x^2 - 3x + k$

For –3 to be a zero of $p(x)$, $p(-3)$ has to be equal to zero.

$\therefore \quad (-3)^3 + 3(-3)^2 - 3(-3) + k = 0$

$\Rightarrow \quad -27 + 27 + 9 + k = 0$

$\Rightarrow \quad k = -9$

20. (*b*) **0**

By the remainder theorem, when $p(x)$ is divided by $x + 1$

i.e. $x - (-1)$ the remainder is equal to $p(-1)$.

$p(x) = x^3 + 1$

$\therefore \quad \text{Remainder} = p(-1)$

$= (-1)^3 + 1$

$= -1 + 1$

$= 0$

21. (*b*) **50**

Let $p(x) = x^{51} + 51$

By the remainder theorem, when $p(x) = x^{51} + 51$ is divided by $x + 1$, the remainder is equal to $p(-1)$

$\therefore \quad \text{Remainder} = (-1)^{51} + 51$

$= -1 + 51$

$= 50$

22. (*b*) **0**

Let $p(x) = x^2 + 2x + 1$

By the remainder theorem, when $p(x)$ is divided by $x + 1$, the remainder is equal to $p(-1)$

$\therefore \quad \text{Remainder} = (-1)^2 + 2(-1) + 1$

$= 1 - 2 + 1$

$= 2 - 2$

$= 0$

23. (*d*) **19**

By the remainder theorem, when $f(x)$ is divided by $x - 2$, the remainder is equal to $f(2)$.

$\therefore \quad \text{Remainder} = f(2)$

$= (2)^3 + 4(2)^2 - 3(2) + 1$

$= 8 + 16 - 6 + 1$

$= \mathbf{19}$

24. (*d*) **2**

Let $p(x) = 2x^2 + kx.$

Since $x + 1 = x - (-1)$ is a factor of

$p(x) = 2x^2 + kx$

therefore by factor theorem, we have $p(-1) = 0$

$\Rightarrow \quad 2(-1)^2 + k(-1) = 0$

$\Rightarrow \quad 2 - k = 0$

$\Rightarrow \quad k = 2$

25. (*a*) **0**

Let $p(x) = x^4 - a^2x^2 + 3x - 6a.$

Since $x + a = x - (-a)$ is a factor $p(x) = x^4 - a^2x^2 + 3x - 6a$, therefore by factor theorem, we have

$(-a)^4 - a^2(-a)^2 + 3(-a) - 6a = 0$

$\Rightarrow \quad a^4 - a^4 - 3a - 6a = 0$

$\Rightarrow \quad -9a = 0$

$\Rightarrow \quad a = 0$

26. (*b*) $\mathbf{x^3 + x^2 + x + 1}$

Let $p(x) = x^3 + x^2 + x + 1$

By trial, we get

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0$

By factor theorem $x - (-1)$

i.e. $x + 1$ is a factor of $x^3 + x^2 + x + 1$.

27. (*a*) $\mathbf{x - 1}$

$x^2 - 1 = (x + 1)(x - 1)$

$x^4 - 1 = (x^2 + 1)(x^2 - 1)$

$= (x^2 + 1)(x + 1)(x - 1)$

$(x - 1)^2 = (x - 1)(x - 1)$

Common factor is $(x - 1)$

28. (*b*) $\mathbf{-(2 - x)(3 - x)}$

$-x^2 + 5x - 6 = -x^2 + 2x + 3x - 6$

$= x(-x + 2) - 3(-x + 2)$

$= (-x + 2)(x - 3)$

$= (2 - x)(x - 3)$

$= -(2 - x)(3 - x)$

29. (*c*) **695**

$(348)^2 - (347)^2 = (348 + 347)(348 - 347)$

$= 695 \times 1$

$= 695$

$\therefore \quad a^2 - b^2 = (a + b)(a - b)$

30. (*b*) $\mathbf{x^2 + y^2 + z^2 + 2xy + 2yz + 2zx}$

Algebraic identity:

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

31. (*c*) $\mathbf{x^3 - y^3 - 3x^2y + 3xy^2}$

Algebraic identity:

$(x - y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

32. (*d*) $\mathbf{\dfrac{x^4}{16} - 81y^4}$

$\left(\frac{x}{2} - 3y\right)\left(3y + \frac{x}{2}\right)\left(\frac{x^2}{4} + 9y^2\right)$

$= \left(\frac{x^2}{4} - 9y^2\right)\left(\frac{x^2}{4} + 9y^2\right)$

$= \frac{x^4}{16} - 81y^4 \qquad [\because a - b)(a + b) = a^2 - b^2]$

33. (*a*) **10000**

$75 \times 75 + 2 \times 75 \times 25 + 25 \times 25$

$= (75)^2 + 2 \times 75 \times 25 + (25)$

$= (75 + 25)^2 \qquad [\because x^2 + 2xy + y^2 = (x + y)^2]$

$= (100)^2 = 10000$

34. (*c*) **11**

$\frac{8.83 \times 8.83 - 2.17 \times 2.17}{6.66}$

$= \frac{(8.83)^2 - (2.17)^2}{6.66}$

$= \frac{(8.83 + 2.17) - (8.83 - 2.17)}{6.66}$

$[\because (a + b)(a - b) = a^2 - b^2]$

$= \frac{(11)(6.66)}{(6.66)} = 11$

35. (*b*) **3*xyz***

Algebraic Identity:

If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

36. (*b*) $\mathbf{\frac{1}{4}}$

$$49x^2 - y = (7x + \frac{1}{2})(7x - \frac{1}{2})$$

$$\Rightarrow \quad 49x^2 - y = (49x^2 - \frac{1}{4})$$

$$[\because (a+b)(a-b) = a^2 - b^2]$$

$$\Rightarrow \quad y = \frac{1}{4}$$

37. (*b*) **2*x* − 1, 2*x* − 3**

$$4x^2 + 4x - 3 = 4x^2 - 2x + 6x - 3$$
$$= 2x(2x - 1) + 3(2x - 1)$$
$$= (2x - 1)(2x + 3)$$

38. (*d*) **(3*y* + 2) (4*y* − 3)**

$$12y^2 - y - 6 = 12y^2 + 8y - 9y - 6$$
$$= 4y(3y + 2) - 3(3y + 2)$$
$$= (3y + 2)(4y - 3)$$

39. (*c*) $\frac{1}{2}\left(1+\frac{x}{5}\right)\left(1-\frac{x}{5}\right)$

$$\frac{1}{2} - \frac{x^2}{50} = \frac{1}{2}\left(1 - \frac{x^2}{25}\right)$$
$$= \frac{1}{2}\left[(1)^2 - \left(\frac{x^2}{5}\right)\right]$$
$$= \frac{1}{2}\left(1+\frac{x}{5}\right)\left(1-\frac{x}{5}\right)$$

$$\because \quad a^2 - b^2 = (a+b)(a-b)$$

40. (*b*) **(*a* + 3) (*a*² − 3*a* + 9)**

$$a^3 + 27 = (a)^3 + (3)^3$$
$$= (a + 3)(a^2 - 3a + 3^2)$$
$$= (a + 3)(a^2 - 3a + 9)$$
$$[\because x^3 + y^3 = (x + y)(x^2 - xy + y^2)]$$

41. (*b*) $\pm(\sqrt{2}a + \sqrt{3}b)$

$$\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$$
$$= \sqrt{\left(\sqrt{2}a\right)^2 + 2\left(\sqrt{2}a\right)\left(\sqrt{3}b\right) + \left(\sqrt{3}b\right)^2}$$
$$= \sqrt{(\sqrt{2}a + \sqrt{3}b)^2} \qquad [\because x^2 + 2xy + y^2 = (x + y)^2]$$
$$= \pm(\sqrt{2}a + \sqrt{3}b)$$

42. (*c*) **−4, −3, 0**

Let $p(x) = (x + 2)(x - 2)$
$= x^2 - 4$

Then, $p(0) = (0)^2 - 4 = -4$
$p(1) = (1)^2 - 4 = -3$
$p(-2) = (-2)^2 - 4 = 0$

43. (*b*) $\mathbf{\frac{-31}{4}}$

$$p(x) = x^2 - 4x + 3$$
$$\Rightarrow \quad p(2) = (2)^2 - 4(2) + 3$$
$$= 4 - 8 + 3$$
$$= -1$$
$$p(-1) = (-1)^2 - 4(-1) + 3$$
$$= 1 + 4 + 3 = 8$$

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 3$$
$$= \frac{1}{4} - 2 + 3$$
$$= \frac{5}{4}$$

$$\because \quad p(2) - p(-1) + p\left(\frac{1}{2}\right) = -1 - 8 + \frac{5}{4}$$
$$= \frac{-4 - 32 + 5}{4}$$
$$= \frac{-36 + 5}{4}$$
$$= \frac{-31}{4}$$

44. (*c*) **1**

Let $p(x) = x^3 - 2mx^2 + 16$

By the factor theorem $p(x)$ will be exactly divisible by $x + 2$ i.e $x - (-2)$

If $p(-2) = 0$

$\therefore \quad (-2)^3 - 2m(-2)^2 + 16 = 0$

$\Rightarrow \quad -8 - 8m + 16 = 0$

$\Rightarrow \quad 8 = 8m$

$\Rightarrow \quad m = 1$

45. (*a*) **−2**

Since $2x - 1 = 2\left(x - \frac{1}{2}\right)$ is a factor of $8x^4 + 4x^3 - 16x^2 + 10x + a$

$\therefore$ By the factor theorem, we have

$$8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + a = 0$$
$$\Rightarrow \quad \frac{8}{16} + \frac{4}{8} - \frac{16}{4} + 5 + a = 0$$
$$\Rightarrow \quad \frac{1}{2} + \frac{1}{2} - 4 + 5 + a = 0$$
$$\Rightarrow \quad 1 - 4 + 5 + a = 0$$
$$\Rightarrow \quad 6 - 4 + a = 0$$
$$\Rightarrow \quad 2 + a = 0$$
$$\Rightarrow \quad a = -2$$

46. (*d*) $\left(\mathbf{3x - \frac{1}{6}}\right)\left(\mathbf{x + \frac{5}{6}}\right)$

$$\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$$
$$= \left(2x + \frac{1}{3} + x - \frac{1}{2}\right)\left(2x + \frac{1}{3} - x + \frac{1}{2}\right)$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= \left(3x - \frac{1}{6}\right)\left(x + \frac{5}{6}\right)$$

47. (*a*) $\mathbf{9a^2 + 25b^2 + c^2 - 30ab + 10bc - 6ac}$

$(3a - 5b - c)^2$

$= [(3a) + (-5b) + (-c)]^2$

$= (3a)^2 + (-5b)^2 + (-c)^2 + 2(3a)(-5b) + 2(-5b)(-c) + 2(-c)(3a)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx]$

$= 9a^2 + 25b^2 + c^2 - 30ab + 10bc - 6ac$

48. (*b*) $\frac{x^3}{8} + 8y^3$

$$\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$$

$$= \left(\frac{x}{2} + 2y\right)\left[\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)(2y) + (2y)^2\right]$$

$$= \left(\frac{x}{2}\right)^3 + (2y)^3 \qquad [\because \ (x + y)(x^2 - xy + y^2) = x^3 + y^3]$$

$$= \frac{x^3}{8} + 8y^3$$

49. (*a*) $(a - \sqrt{2}b)(a^2 + \sqrt{2}ab + 2b^2)$

$$a^3 - 2\sqrt{2}\,b^3 = (a)^3 - (\sqrt{2}b)^3$$
$$= (a - \sqrt{2}\,b)\,[(a)^2 + (a)(\sqrt{2}\,b)(\sqrt{2}\,b)^2]$$
$$[\because x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$$
$$= (a - \sqrt{2}\,b)(a^2 + \sqrt{2}\,ab + 2b^2)$$

50. (*b*) $x^3 + \frac{1}{27} + x^2 + \frac{1}{3}x$

$$\left(x + \frac{1}{3}\right)^3 = (x)^3 + \left(\frac{1}{3}\right)^3 + 3(x)^2\left(\frac{1}{3}\right) + 3(x)\left(\frac{1}{3}\right)^2$$
$$[\because (x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2]$$
$$= x^3 + \frac{1}{27} + x^2 + \frac{1}{3}x$$

51. (*a*) **750**

$$10^3 - (5)^3 - (5)^3 = (10)^3 + (-5)^3 + (-5)^3$$
$$= 3 \times 10 \times (-5) \times (-5)$$
$$= 750$$
[If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$]

52. (*a*) **62**

$$x + \frac{1}{x} = 8$$
$$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 = (8)^2$$
$$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = 64$$
$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 62$$

53. (*c*) **–224**

$$p^3 - q^3 = (p - q)(p^2 + pq + q^2)$$
$$= (p - q)(p^2 + q^2 + pq)$$
$$= (p - q)(p^2 + q^2 - 2pq + 2pq + pq)$$
[Adding and subtracting $2pq$ in the second bracket]
$$= (p - q)[(p - q)^2 + 3pq]$$
$$= -8[(-8)^2 + 3(-12)] = -8(64 - 36)$$
$$= -8(28)$$
$$= -224$$

54. (*a*) **25**

$$(3x)^2 - 2(3x)(5) + (5)^2 = 9x^2 - 30x + 25$$
$$= (3x - 5)^2 \qquad \ldots (1)$$
Given polynomial $= 9x^2 - 30x + k$ … (2)

So, $9x^2 - 30x + k$ is a perfect square when $k = 25$

[Using (1) and (2)]

55. (*c*) **115**

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
$$\Rightarrow \quad (13)^2 = a^2 + b^2 + c^2 + 2(27)$$
$$\Rightarrow \quad a^2 + b^2 + c^2 = 169 - 54$$
$$\Rightarrow \quad a^2 + b^2 + c^2 = 115$$

SHORT ANSWER QUESTIONS

1. Let $p(x) = 3x^3 + x^2 - 20x + 12$

Now, $3x - 2 = 3\left(x - \frac{2}{3}\right)$

By the factor theorem $3x - 2$ will be a factor of $p(x)$,

if $p\left(\frac{2}{3}\right) = 0$

$$p\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 20\left(\frac{2}{3}\right) + 12$$
$$= 3\left(\frac{8}{27}\right) + \frac{4}{9} - \frac{40}{3} + 12$$
$$= \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$$
$$= \frac{8 + 4 - 120 + 108}{9}$$
$$= \frac{120 - 120}{9} = 0$$

Hence, $3\left(x - \frac{2}{3}\right)$ i.e $(3x - 2)$ is a factor of the given polynomial.

2.
$$2x + 1 = 2\left(x + \frac{1}{2}\right)$$
$$= 2\left[x - \left(\frac{-1}{2}\right)\right]$$

Let $p(x) = 2x^2 - x + 1$

By the remainder theorem, when $p(x)$ is divided by $2x + 1$, the remainder is given by $p\left(\frac{-1}{2}\right)$

Now, $p(x) = 2x^2 - x + 1$

$$\Rightarrow \quad p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right) + 1$$
$$= 2\left(\frac{1}{4}\right) + \frac{1}{2} + 1$$
$$= \frac{1}{2} + \frac{1}{2} + 1 = 2$$

Hence, the remainder = **2**.

3.
$$101 \times 102 = (100 + 1)(100 + 2)$$
$$= (100)^2 + (1 + 2)\,100 + 1 \times 2$$
$$[\because (x + a)(x + b) = x^2 + (a + b)x + ab]$$
$$= 10000 + 3 \times 100 + 2$$
$$= 10000 + 300 + 2$$
$$= \mathbf{10302}$$

4.
$$(2a + b)^2 = 4a^2 + 2(2a)(b) + b^2$$
$$[\because (x + y)^2 = x^2 + 2xy + y^2]$$
$$\Rightarrow \quad (2a + b)^2 = 4a^2 + 4ab + b^2$$
$$\Rightarrow \quad (2a + b)^2 = (4a^2 + b^2) + 4ab$$

$= 40 + 4(6)$
$= 40 + 24$
$= 64$
$2a + b = \sqrt{64} = \pm 8$

5. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$= (x + y)(x^2 + y^2 + 2xy - 2xy - xy)$

[Adding and subtracting $2xy$ in the second bracket]

$= (x + y)[(x + y)^2 - 3xy]$ $\quad [\because x^2 + y^2 + 2xy = (x + y)^2]$

$= 3[(3)^2 - 3(2)]$
$= 3(9 - 6)$
$= 3(3)$
$= \mathbf{9}$

6. $x - \dfrac{1}{x} = 6$ [given]

$\Rightarrow \left(x - \dfrac{1}{x}\right)^2 = (6)^2$

$\Rightarrow x^2 + \dfrac{1}{x^2} - 2(x)\left(\dfrac{1}{x}\right) = 36$

$\Rightarrow x^2 + \dfrac{1}{x^2} - 2 = 36$

$\Rightarrow x^2 + \dfrac{1}{x^2} = 36 + 2 = 38$ ... (1)

Now, $x^3 - \dfrac{1}{x^3} = \left(x - \dfrac{1}{x}\right)\left[x^2 + (x)\left(\dfrac{1}{x}\right) + \left(\dfrac{1}{x}\right)^2\right]$

$[\because x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$= \left(x - \dfrac{1}{x}\right)\left[\left(x^2 + \dfrac{1}{x^2}\right) + 1\right]$

$= 6(38 + 1)$ [Using (1)]
$= 6(39)$
$= \mathbf{234}$

7. $(0.645) \times (0.645) + 2(0.645) \times (0.355) + (0.355) \times (0.355)$

$= (0.645)^2 + 2(0.645) \times (0.355) + (0.355)^2$

$= (0.645 + 0.355)^2$ $\quad [\because x^2 - 2xy^2 + y^2 = (x + y)^2]$

$= (1.000)^2$

$= \mathbf{1}$

8. $(x + 2y - 5z)^2 - (x - 2y + 5z)^2$

$= (x + 2y - 5z + x - 2y + 5z)(x + 2y - 5z - x + 2y - 5z)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

$= (2x)(4y - 10z)$

$= \mathbf{8xy - 20xz}$

9. $\left(2a - \dfrac{3}{a} + 1\right)\left(2a + \dfrac{3}{a} + 1\right)$

$= \left[(2a + 1) - \dfrac{3}{a}\right]\left[(2a + 1) + \dfrac{3}{a}\right]$

$= (2a + 1)^2 - \left(\dfrac{3}{a}\right)^2$

$= \mathbf{4a^2 + 4a + 1 - \dfrac{9}{a^2}}$ $\quad [\because (x - y)(x + y) = x^2 - y^2]$

10. $(3x + 5y)(9x^2 - 15xy + 25y^2)$

$= (3x + 5y)[(3x)^2 - (3x)(5y) + (5y)^2]$

$= (3x)^3 + (5y)^3$ $\quad [\because (a + b)(a^2 - ab + b^2) = a^3 + b^3]$

$= \mathbf{27x^3 + 125y^3}$

11. $\left(x - \dfrac{2}{x}\right)\left(x + 2 + \dfrac{4}{x^2}\right)$

$= \left(x - \dfrac{2}{x}\right)\left[(x)^2 + (x)\left(\dfrac{2}{x}\right) + \left(\dfrac{2}{x}\right)^2\right]$

$= (x)^3 - \left(\dfrac{2}{x}\right)^3$ $\quad [\because (a - b)(a^2 + ab + b^2) = a^3 - b^3]$

$= \mathbf{x^3 - \dfrac{8}{x^3}}$

12. $4(x + y)^2 - 9(x - y)^2 = [2(x + y)]^2 - [3(x - y)]^2$

$= (2x + 2y)^2 - (3x - 3y)^2 = (2x + 2y + 3x - 3y)(2x + 2y - 3x + 3y)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

$= (5x - y)(-x + 5y)$

$= \mathbf{(5x - y)(5y - x)}$

13. Let $p(x) = x^3 - 3x^2 + 3x + 7$

Putting $x = -1$ in $p(x)$, we get

$p(-1) = (-1)^3 - 3(-1)^2 + 3(-1) + 7$
$= -1 - 3 - 3 + 7$
$= 7 - 7$
$= 0$

$\therefore$ By the factor theorem $x - (-1)$ i.e $(x + 1)$ is a factor of $p(x)$

On dividing $p(x)$ by $(x + 1)$, we get

```
x + 1 ) x³ − 3x² + 3x + 7 ( x² − 4x + 7
        x³ +  x²
        −    −
        ----------
            − 4x² + 3x
            − 4x² − 4x
            +      +
            ----------
                  7x + 7
                  7x + 7
                −    −
                ------
                     0
```

$\therefore x^3 - 3x^2 + 3x + 7 = \mathbf{(x + 1)(x^2 - 4x + 7)}$

14. $\dfrac{x^2 + 5x + 4}{x^2 + 2x + 1} = \dfrac{x^2 + x + 4x + 4}{x^2 + x + x + 1}$

$= \dfrac{x(x + 1) + 4(x + 1)}{x(x + 1) + 1(x + 1)}$

$= \dfrac{(x + 1)(x + 4)}{(x + 1)(x + 1)} = \mathbf{\dfrac{x + 4}{x + 1}}$

15. Since, 0, 4 and –4 are three zeroes of the required cubic polynomial.

$\therefore$ $(x - 0)$, $(x - 4)$ and $[x - (-4)]$ are the three linear factors of the required cubic polynomial.

Since a cubic polynomial cannot have more than three linear factors, so the required cubic polynomial is

$(x - 0)(x - 4)[x - (-4)] = x(x - 4)(x + 4)$
$= x(x^2 - 16)$
$= \mathbf{x^3 - 16x}$

UNIT TEST

1. (*b*) **15**

Degree of a constant polynomial is zero.

2. (*b*) **24**

$$f(x) = 2x^3 + 9x^2 + 10x + 3$$
$$\Rightarrow f(1) = 2(1)^3 + 9(1)^2 + 10(1) + 3 = 2 + 9 + 10 + 3 = 24$$

3. (*a*) **3**

Let $p(x) = x^2 - 5x + 6$

$$p(3) = (3)^2 - 5(3) + 6 = 9 - 15 + 6 = 15 - 15 = 0$$

Since, $p(3) = 0$

$\therefore$ 3 is zero of the given polynomial

4. (*a*) **28**

Let $p(x) = x^2 + 11x + k$

Since −4 is a zero of $p(x)$

$\therefore$ $p(-4) = 0$

$\Rightarrow (-4)^2 + 11(-4) + k = 0$

$\Rightarrow 16 - 44 + k = 0$

$\Rightarrow -28 + k = 0$

$\Rightarrow k = 28$

5. (*b*) **0**

$$x + 1 = x - (-1)$$

By the remainder theorem when $p(x)$ is divided by $x + 1 = x - (-1)$, the remainder is equal to $p(-1)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

6. (*c*) ±10

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$
$$\Rightarrow (x + y + z)^2 = 40 + 2(30) = 40 + 60 = 100$$
$$\Rightarrow x + y + z = \sqrt{100} = \pm 10$$

7.
$$p(x) = (x - 2)^2 - (x + 2)^2 = (x - 2 + x + 2)\ (x - 2 - x - 2)$$
$[\because a^2 - b^2 = (a + b)\ (a - b)]$
$$= 2x(-4) = -8x$$

Zero of the polynomial $p(x) = -8x$ is given by

$p(x) = 0$

$\Rightarrow -8x = 0$

$\Rightarrow x = 0$

Hence, zero of the given polynomial is **0**.

8. Let $p(x) = x^{10} - 1$

and $g(x) = x^{11} - 1$

By the factor theorem $x - 1$ will be a factor of both $p(x)$ and $g(x)$ if

$p(1) = 0$ and $g(1) = 0$

Now, $p(x) = x^{10} - 1$

$\Rightarrow p(x) = (1)^{10} - 1 = 1 - 1 = 0$

and $g(x) = x^{11} - 1$

$\Rightarrow g(1) = (1)^{11} - 1 = 1 - 1 = 0$

Since both $p(1)$ and $g(1)$ are equal to zero, therefore $(x - 1)$ is a factor of both the given polynomials.

9.
$$(0.99)^2 = (1 - 0.01)^2 = (1)^2 - 2(1)\ (0.01) + (0.01)^2$$
[Using $(x - y)^2 = x^2 - 2xy + y^2$]
$$= 1 - 0.02 + 0.0001 = 1.0001 - 0.02 = \mathbf{0.9801}$$

10.
$$(2)^3 - \left(\frac{1}{5}\right)^3 = \left(2 - \frac{1}{5}\right)\left[(2)^2 + (2)\left(\frac{1}{5}\right) + \left(\frac{1}{5}\right)^2\right]$$
[Using $x^3 - y^3 = (x - y)\ (x^2 + xy + y^2)$]
$$= \left(\frac{10-1}{5}\right)\left(4 + \frac{2}{5} + \frac{1}{25}\right) = \left(\frac{9}{5}\right)\left(\frac{100+10+1}{25}\right) = \left(\frac{9}{5}\right)\left(\frac{111}{25}\right) = \mathbf{\frac{999}{125}}$$

11. $55^3 - (25)^3 - (30)^3 = (55)^3 + (-25)^3 + (-30)^3$

Here $55 + (-25) + (-30) = 0$

and we know that if $x + y + z = 0$,

then $x^3 + y^3 + z^3 = 3xyz$

$\therefore$ $(55)^3 + (-25)^3 + (-30)^3 = 3(55)\ (-25)\ (-30) = \mathbf{123750}$

12.
$$\frac{3.59 \times 3.59 - 2.41 \times 2.41}{3.59 + 2.41}$$
$$= \frac{(3.59)^2 - (2.41)^2}{(2.59 + 2.41)}$$
$$= \frac{(3.59 + 2.41)(3.59 - 2.41)}{(3.59 + 2.41)}$$
[Using $x^2 - y^2 = (x + y)\ (x - y)$]
$$= 3.59 - 2.41 = \mathbf{1.18}$$

13.
$$a(a - 3) - b\ (b - 3) = a^2 - 3a - b^2 + 3b = a^2 - b^2 - 3a + 3b = (a + b)\ (a - b) - 3(a - b) = \mathbf{(a - b)\ (a + b - 3)}$$

14. Let $p(x) = 2x^3 + ax^2 + 11x + a + 3$

By the factor theorem $(2x - 1) = 2\left(x - \frac{1}{2}\right)$ will be a factor of $p(x)$ if $p\left(\frac{1}{2}\right) = 0$

$$\Rightarrow 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) + a + 3 = 0$$
$$\Rightarrow \frac{2}{8} + \frac{a}{4} + \frac{11}{2} + a + 3 = 0$$
$$\Rightarrow a + \frac{a}{4} = -\frac{2}{8} - \frac{11}{2} - 3$$
$$\Rightarrow \frac{4a + a}{4} = -\frac{1}{4} - \frac{11}{2} - 3$$

$\Rightarrow \quad \frac{5a}{4} = \frac{-1-22-12}{4} = \frac{-35}{4}$

$\Rightarrow \quad 5a = -35$

$\Rightarrow \quad \mathbf{a = -7}$

15. $3x + 2y = 12$ [given]

$\Rightarrow \quad (3x+2y)^2 = (12)^2$ [squaring both sides]

$\Rightarrow \quad 9x^2 + 4y^2 + 12xy = 144$

$\Rightarrow \quad 9x^2 + 4y^2 = 144 - 12xy$

$= 144 - 12(6)$

$= 144 - 72$

$= \mathbf{72}$

16. $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)$

$= x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 = 7 + 2 = 9$

$\Rightarrow \quad x + \frac{1}{x^2} = \sqrt{9}$

$\Rightarrow \quad x + \frac{1}{x} = \mathbf{\pm 3}$

17. $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right)$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = \left(x^2 + \frac{1}{x^2}\right) - 2$

$= 83 - 2$

$= 81$

$\Rightarrow \quad \left(x - \frac{1}{x}\right) = \sqrt{81}$

$= \pm 9$

Now, $\left(x - \frac{1}{x}\right) = 9$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^3 = (9)^3$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3x\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3(9) = 729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 729 + 27$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = +756$ … (1)

and $\left(x - \frac{1}{x}\right) = -9$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^3 = (-9)^3$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = -729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3(-9) = -729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = -729 - 27$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = -756$ … (2)

Hence, $x^3 - \frac{1}{x^3} = \mathbf{\pm 756}$ [Using (1) and (2)]

18. $1 - 18x - 63x^2 = 1 - 21x + 3x - 63x^2$

$= (1 - 21x) + 3x(1 - 21x)$

$= \mathbf{(1 - 21x)\ (1 + 3x)}$

19. $x^2 + \frac{1}{6}x - \frac{1}{6} = \frac{1}{6}(6x^2 + x - 1)$

$= \frac{1}{6}(6x^2 + 3x - 2x - 1)$

$= \frac{1}{6}[3x(2x + 1) - 1(2x + 1)]$

$= \mathbf{\frac{1}{6}(2x + 1)\ (3x - 1)}$

20. $27x^4 - 8x = x(27x^3 - 8) = x[(3x)^3 - (2)^3]$

$= x(3x - 2)\ [(3x)^2 + (3x)(2) + (2)^2]$

[Using $x^3 - y^3 = (x - y)\ (x^2 + xy + y^2)$]

$= \mathbf{x(3x - 2)\ (9x^2 + 6x + 4)}$

21. $(x + y + 2z)\ (x^2 + y^2 + 4z^2 - xy - 2yz - 2zx)$

$= [(x) + (y) + (2z)]\ [(x)^2 + (y)^2 + (2z)^2 - (x)(y) - (y)(2z) - (2z)x]$

$= (x)^3 + (y)^3 + (2z)^3 - 3(x)(y)(2z)$

[Using $(a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$]

$= \mathbf{x^3 + y^3 + 8z^3 - 6xyz}$

22. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow \quad a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$\Rightarrow \quad a^2 + b^2 + c^2 = (6)^2 - 2(11)$

$= 36 - 22$

$= 14$ … (1)

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)\ [(a^2 + b^2 + c^2) - (ab + bc + ca)]$

$= (6)\ [(14) - (11)]$ [Using (1)]

$= 6(3)$

$= \mathbf{18}$

23. $(2x - 5y)^3 - (2x + 5y)^3$

$= (2x - 5y - 2x - 5y)\ [(2x - 5y)^2 + (2x - 5y)\ (2x + 5y) + (2x + 5y)^2]$

[Using $a^3 - b^3 = (a - b)\ (a^2 + ab + b^2)$]

$= (-10y)\ [4x^2 + 25y^2 - 20xy + 4x^2 - 25y^2 + 4x^2 + 25y^2 + 20xy]$

$= (-10y)\ (12x^2 + 25y^2)$

$= \mathbf{-120x^2y - 250y^3}$