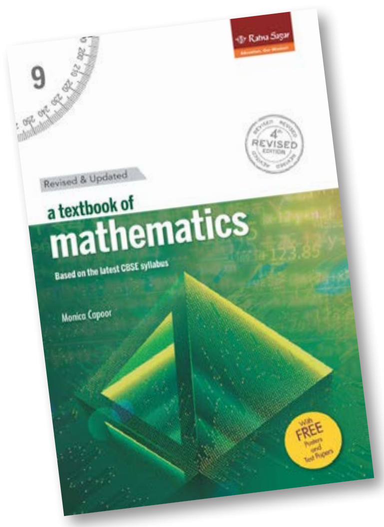
a textbook of

mathematics

COMPANION

a complete resource for teachers

Class
9



Ratna Sagar

CHAPTER 1 Number System

EXERCISE 1A

1. Rational Numbers:

Numbers which can be expressed in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$, are called rational numbers.

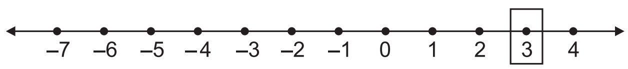
Examples: 1, 0, $\frac{2}{3}$, $\frac{-3}{4}$, $\frac{17}{9}$, etc.

2. Five rational numbers equivalent to $\frac{3}{7}$ are $\frac{3}{7} \times \frac{2}{2}$,

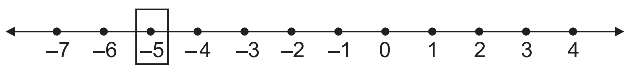
$\frac{3}{7} \times \frac{3}{3}$, $\frac{3}{7} \times \frac{4}{4}$, $\frac{3}{7} \times \frac{5}{5}$, $\frac{3}{7} \times \frac{6}{6}$

or $\mathbf{\frac{6}{14}}$, $\mathbf{\frac{9}{21}}$, $\mathbf{\frac{12}{28}}$, $\mathbf{\frac{15}{35}}$, $\mathbf{\frac{18}{42}}$.

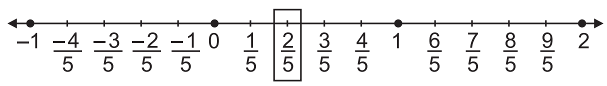
3. (*i*) 3



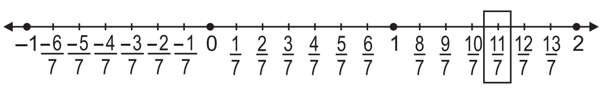
(*ii*) –5



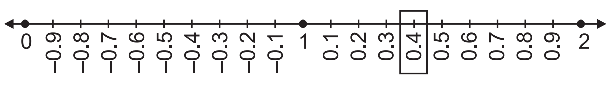
(*iii*) $\frac{2}{5}$



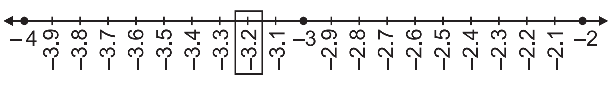
(*iv*) $\frac{11}{7}$ $\left(= 1\frac{4}{7}\right)$



(*v*) 1.4



(*vi*) –3.2



For Q4 to Q8: Answers will vary. Sample answers are given below:

4. (*i*) A rational number between 2 and 3 is $\frac{1}{2}$ (2 + 3) = $\mathbf{\frac{5}{2}}$.

(*ii*) A rational number between $\frac{5}{6}$ and $\frac{6}{7}$ is $\frac{1}{2}\left(\frac{5}{6} + \frac{6}{7}\right)$

$= \frac{1}{2}\left(\frac{35 + 36}{42}\right) = \frac{1}{2}\left(\frac{71}{42}\right) = \mathbf{\frac{71}{84}}$.

(*iii*) A rational number between $\frac{-2}{3}$ and $\frac{1}{4}$ is

$\frac{1}{2}\left(\frac{-2}{3} + \frac{1}{4}\right) = \frac{1}{2}\left(\frac{-8 + 3}{12}\right) = \frac{1}{2}\left(\frac{-5}{12}\right) = \mathbf{\frac{-5}{24}}$.

(*iv*) A rational number between –2 and $\frac{1}{2}$ is

$\frac{1}{2}\left(-2 + \frac{1}{2}\right) = \frac{1}{2}\left(\frac{-4 + 1}{2}\right) = \frac{1}{2}\left(\frac{-3}{2}\right) = \mathbf{\frac{-3}{4}}$.

(*v*) A rational number between 0.45 and 1.2 is

$\frac{1}{2}$ (0.45 + 1.2) = $\frac{1.65}{2}$ = **0.825.**

(*vi*) A rational number between 1.2 and 1.3 is

$\frac{1}{2}$ (1.2 + 1.3) = $\frac{1}{2}$ (2.5) = **1.25.**

5. (*i*) A rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$ is

$\frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{2}\left(\frac{2 + 3}{6}\right) = \frac{1}{2}\left(\frac{5}{6}\right) = \frac{5}{12}$

A rational number lying between $\frac{1}{3}$ and $\frac{5}{12}$ is

$\frac{1}{2}\left(\frac{1}{3} + \frac{5}{12}\right) = \frac{1}{2}\left(\frac{4 + 5}{12}\right) = \frac{1}{2}\left(\frac{9}{12}\right) = \frac{1}{2}\left(\frac{3}{4}\right) = \frac{3}{8}$

A rational number lying between $\frac{5}{12}$ and $\frac{1}{2}$ is

$\frac{1}{2}\left(\frac{5}{12} + \frac{1}{2}\right) = \frac{1}{2}\left(\frac{5 + 6}{12}\right) = \frac{1}{2}\left(\frac{11}{12}\right) = \frac{11}{24}$

Three rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$ are

$\mathbf{\frac{3}{8}}$, $\mathbf{\frac{5}{12}}$, $\mathbf{\frac{11}{24}}$.

(*ii*) $\frac{-2}{5} = \frac{-2}{5} \times \frac{20}{20} = \frac{-40}{100}$

and $\frac{-1}{5} = \frac{-1}{5} \times \frac{20}{20}$ $\frac{-20}{100}$

$-25 < -24 < -23$

$\frac{-25}{100} < \frac{-24}{100} < \frac{-23}{100}$

$-\frac{1}{4} < -\frac{6}{25} < \frac{-23}{100}$

Hence, three rational numbers between $\frac{-2}{5}$ and $\frac{-1}{5}$

are $\mathbf{\frac{-1}{4}}$, $\mathbf{\frac{-6}{25}}$, $\mathbf{\frac{-23}{100}}$.

6. Let $a = \frac{1}{2}$, $b = \frac{5}{7}$ and $n = 5$.

Then, $\frac{1}{2} < \frac{5}{7}$ $[\because\ 1 \times 7 < 5 \times 2]$

Then, $\frac{b-a}{n+1} = \frac{\frac{5}{7}-\frac{1}{2}}{5+1} = \frac{\frac{10-7}{14}}{6}$

$= \frac{\frac{3}{14}}{6} = \frac{3}{14} \times \frac{1}{6}$

$= \frac{1}{28}$

So, the five rational numbers between $\frac{1}{2}$ and $\frac{5}{7}$ are

$a + \frac{(b-a)}{(n+1)}, a + 2\frac{(b-a)}{(n+1)}, a + 3\frac{(b-a)}{n+1}, a + 4\frac{(b-a)}{(n+1)}$

and $a + 5\frac{(b-a)}{(n+1)}$

or $\frac{1}{2}+\frac{1}{28}, \frac{1}{2}+2\left(\frac{1}{28}\right), \frac{1}{2}+3\left(\frac{1}{28}\right), \frac{1}{2}+4\left(\frac{1}{28}\right)$

and $\frac{1}{2}+5\left(\frac{1}{28}\right)$

or $\frac{1}{2}+\frac{1}{28}, \frac{1}{2}+\frac{1}{14}, \frac{1}{2}+\frac{3}{28}, \frac{1}{2}+\frac{1}{7}$ and $\frac{1}{2}+\frac{5}{28}$

or $\frac{14+1}{28}, \frac{7+1}{14}, \frac{14+3}{28}, \frac{7+2}{14}$, and $\frac{14+5}{28}$

or $\frac{15}{28}, \frac{8}{14}, \frac{17}{28}, \frac{9}{14}$ and $\frac{19}{28}$

or $\frac{15}{28}, \frac{4}{7}, \frac{17}{28}, \frac{9}{14}$ and $\frac{19}{28}$

Hence, five rational numbers between $\frac{1}{2}$ and $\frac{5}{7}$ are

$\mathbf{\frac{15}{28}, \frac{4}{7}, \frac{17}{28}, \frac{9}{14}}$ and $\mathbf{\frac{19}{28}}$.

7. Let $a = 0$, $b = 0.1$ and $n = 9$.
Clearly $a < b$

Then, $\frac{b-a}{(n+1)} = \frac{0.1-0}{9+1} = \frac{0.1}{10} = 0.01 = x$ (Say)

So, the nine rational numbers between 0 and 0.1 are
$a + x, a + 2x, a + 3x, a + 4x, a + 5x, a + 6x, a + 7x,$
$a + 8x$ and $a + 9x$

or $0 + 0.01, 0 + 2(0.01), 0 + 3(0.01), 0 + 4(0.01),$
$0 + 5(0.01), 0 + 6(0.01), 0 + 7(0.01), 0 + 8(0.01)$
and $0 + 9(0.01)$

or 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, and 0.09

or $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \frac{4}{100}, \frac{5}{100}, \frac{6}{100}, \frac{7}{100}, \frac{8}{100}$

and $\frac{9}{100}$.

or $\frac{1}{100}, \frac{1}{50}, \frac{3}{100}, \frac{1}{25}, \frac{1}{20}, \frac{3}{50}, \frac{7}{100}, \frac{2}{25}$ and $\frac{9}{100}$.

8. $\frac{-5}{13} = \frac{-5}{13} \times \frac{10}{10} = \frac{-50}{130}$ and $\frac{11}{13} = \frac{11}{13} \times \frac{10}{10} = \frac{110}{130}$

$\because -50 < -49 < -48 < -47 \ldots < -1 < 0 < 1 < 2 < \ldots < 110$

$\therefore \frac{-50}{130} < \frac{-49}{130} < \frac{-48}{130} < \frac{-47}{130} \ldots < \frac{-1}{130} < \frac{0}{130}$

$< \frac{2}{130} < \ldots < \frac{110}{130}$.

Hence, rational numbers between $\frac{-5}{13}$ and $\frac{11}{13}$ are

$\frac{-49}{130}, \frac{-48}{130}, \frac{-47}{130} \ldots, \frac{-1}{130}, \frac{0}{130}, \frac{1}{130}, \frac{2}{130}, \ldots \frac{50}{130}$.

EXERCISE 1B

1. The numbers which cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called irrational numbers.
The decimal expansion of irrational numbers is non-terminating and non-recurring. Some examples of irrational numbers are $\sqrt{2}$, $\sqrt[3]{3}$, π, 0.1011011110123, …

2. No, the fifth root of a positive integer may be rational or irrational.
For example: $\sqrt[5]{13}$ is irrational whereas $\sqrt[5]{32} = 2$ is rational.

3. $\sqrt{2}$ on number line [Example 3, pg. 1.9].

4. $\sqrt{3}$ on number line [Example 4, pg. 1.9].
$\sqrt{5}$ on number line [Example 5, pg. 1.10].

5. $\sqrt{6}$, $\sqrt{7}$ on number line [Example 7, pg. 1.11]
For $\sqrt{8}$, continue the spiral and draw a right triangle with one side containing the right angle as $\sqrt{7}$ units and the other as 1 unit.

6. Square root spiral [Example 6, pg. 1.11].

7. $\sqrt{3.5}$ geometrically on the number line
[Example 8, pg. 1.12. Change the measurement of line segment AB to 3.5 units and follow the same steps as given in example 8.]

8. $\sqrt{4.2}$ geometrically on the number line.
[Example 8, pg. 1.12. Change the measurement of line segment AB to 4.2 units and follow the same steps as given in example 8.]

EXERCISE 1C

1. (*i*) Let $x = 0.\overline{3}$.

Then, $x = 0.3333\ldots$ … (1)

$\therefore$ $10x = 3.3333\ldots$ … (2)

On subtracting (1) from (2), we get

$9x = 3$

$\Rightarrow x = \frac{3}{9}$

$\Rightarrow x = \frac{1}{3}$

Hence, $0.\overline{3} = \mathbf{\frac{1}{3}}$

(*ii*) Let $x = 0.\overline{18}$

Then, $x = 0.181818\ldots$ … (1)

$\therefore$ $100x = 18.181818\ldots$ … (2)

On subtracting (1) from (2), we get

$99x = 18$

$\Rightarrow x = \frac{18}{99}$

$$\Rightarrow \quad x = \frac{2}{11}$$

Hence, $0.\overline{18} = \mathbf{\frac{2}{11}}$

(*iii*) Let $x = 1.\overline{27}$

Then, $x = 1.272727\ldots \quad \ldots (1)$

$\therefore \quad 100x = 127.272727\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$99x = 126$$

$$\Rightarrow \quad x = \frac{126}{99}$$

$$\Rightarrow \quad x = \frac{14}{11}$$

Hence, $1.\overline{27} = \mathbf{\frac{14}{11}}$.

(*iv*) Let $x = 0.\overline{001}$

Then, $x = 0.001001001\ldots \quad \ldots (1)$

$\therefore \quad 1000x = 1.001001001\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$999x = 1$$

$$\Rightarrow \quad x = \frac{1}{999}$$

Hence, $0.\overline{001} = \mathbf{\frac{1}{999}}$.

(*v*) Let $x = 0.\overline{324}$

Then, $x = 0.324324324\ldots \quad \ldots (1)$

$\therefore \quad 1000x = 324.324324324\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$999x = 324$$

$$\Rightarrow \quad x = \frac{324}{999}$$

$$\Rightarrow \quad x = \frac{12}{37}$$

Hence, $0.\overline{324} = \mathbf{\frac{12}{37}}$.

(*vi*) Let $x = 0.2\overline{45}$

Then, $x = 0.2454545\ldots$

$\therefore \quad 10x = 2.454545\ldots \quad \ldots (1)$

and $1000x = 245.454545\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$990x = 243$$

$$\Rightarrow \quad x = \frac{243}{990}$$

$$\Rightarrow \quad x = \frac{27}{110}$$

Hence, $0.2\overline{45} = \mathbf{\frac{27}{110}}$.

(*vii*) Let $x = 10.03\overline{6}$

Then, $x = 10.036666\ldots$

$\therefore \quad 100x = 1003.6666\ldots \quad \ldots (1)$

and $1000x = 10036.6666\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$900x = 9033$$

$$\Rightarrow \quad x = \frac{9033}{900}$$

$$\Rightarrow \quad x = \frac{3011}{300}$$

Hence, $10.03\overline{6} = \mathbf{\frac{3011}{300}}$.

(*viii*) Let $x = 0.3\overline{178}$

Then, $x = 0.3178178178\ldots$

$\therefore \quad 10x = 3.178178178\ldots \quad \ldots (1)$

and $10000x = 3178.178178178\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$9990 = 3175$$

$$\Rightarrow \quad x = \frac{3175}{9990}$$

$$\Rightarrow \quad x = \frac{635}{1998}$$

Hence, $0.3\overline{178} = \mathbf{\frac{635}{1998}}$.

(*ix*) Let $x = 2.5434343\ldots$

$\therefore \quad 10x = 25.434343\ldots \quad \ldots (1)$

and $1000x = 2543.37343\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$990x = 2518$$

$$\Rightarrow \quad x = \frac{2518}{990}$$

$$\Rightarrow \quad x = \frac{1259}{495}$$

Hence, $2.5434343\ldots = \mathbf{\frac{1259}{495}}$.

2. Let $x = 2.\overline{6}$

Then, $x = 2.6666\ldots \quad \ldots (1)$

$\therefore \quad 10x = 26.6666\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$$9x = 24$$

$$\Rightarrow \quad x = \frac{24}{9}$$

$$\Rightarrow \quad x = \frac{8}{3}$$

$$\Rightarrow \quad 2.\overline{6} = \frac{8}{3} \quad \ldots (3)$$

Let $y = 0.\overline{9}$

Then, $y = 0.9999\ldots \quad \ldots (4)$

$\therefore \quad 10y = 9.9999\ldots \quad \ldots (5)$

On subtracting (4) from (5), we get

$$9y = 9$$

$$\Rightarrow \quad y = \frac{9}{9}$$

$$\Rightarrow \quad y = 1$$

$$\Rightarrow \quad 0.\overline{9} = 1 \quad \ldots (6)$$

Now, $2.\overline{6} + 0.\overline{9} = \frac{8}{3} + 1$ [Using (3) and (6)]

$$= \frac{8+3}{3}$$

$$= \frac{11}{3}$$

Hence, $2.\overline{6} + 0.\overline{9} = \mathbf{\frac{11}{3}}$.

3. Let $x = 0.\overline{12}$

Then, $x = 0.121212\ldots$ … (1)

$\therefore$ $100x = 12.121212\ldots$ … (2)

On subtracting (1) from (2), we get

$$99x = 12$$

$$\Rightarrow \quad x = \frac{12}{99}$$

$$\Rightarrow \quad x = \frac{4}{33}$$

$$\Rightarrow \quad 0.\overline{12} = \frac{4}{33} \quad \ldots (3)$$

Let $y = 0.\overline{1}$

Then, $y = 0.1111\ldots$ … (4)

$\therefore$ $10y = 1.1111\ldots$ … (5)

On subtracting (4) from (5), we get

$$9y = 1$$

$$\Rightarrow \quad y = \frac{1}{9}$$

$$\Rightarrow \quad 0.\overline{1} = \frac{1}{9} \quad \ldots (6)$$

Let $z = 0.\overline{23}$

Then, $z = 0.232323\ldots$ … (*i*)

$\therefore$ $100z = 23.232323\ldots$ … (*ii*)

On subtracting (*i*) from (*ii*), we get

$$99z = 23$$

$$\Rightarrow \quad z = \frac{23}{99}$$

$$\Rightarrow \quad 0.\overline{23} = \frac{23}{99} \quad \ldots (7)$$

$$\text{LHS} = 0.\overline{12} + 0.\overline{1}$$

$$= \frac{4}{33} + \frac{1}{9} \quad \text{[Using (3) and (6)]}$$

$$= \frac{12+11}{99}$$

$$= \frac{23}{99}$$

$$= \text{RHS} \quad \text{[Using (7)]}$$

Hence, $0.\overline{12} + 0.\overline{1} = 0.\overline{23}$.

4. (*i*) Unlike rational numbers, irrational numbers cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(*ii*) The decimal expansion of irrational numbers is non-terminating, non-recurring whereas the decimal expansion of rational numbers is either terminating or non-terminating recurring.

5. (*i*) $\frac{27}{40}$ is a **rational number**. Since denominator $40 = 2^3 \times 5$ is of form 2^n5^m where m, n are non-negative integers,

$\therefore$ the given rational number has a terminating decimal expansion.

$$\frac{27}{40} = \frac{27}{2^3 \times 5} = \frac{27 \times 5^2}{2^3 \times 5^3}$$

$$= \frac{27 \times 25}{(2 \times 5)^3} = \frac{675}{1000} = 0.675$$

(*ii*) $\frac{3}{11}$ is a **rational number** in its simplest form.

Since denominator 11 is not of the form 2^n5^m where m, n are non-negative integers and it has a prime factor 11 which is other than 2 or 5,

$\therefore$ the given rational number has non-terminating recurring decimal expansion.

```
11 ) 30 ( 0.2727…
    – 22
       80
     – 77
        30
      – 22
         80
       – 77
          3
```

Hence, $\frac{3}{11} = \mathbf{0.\overline{27}}$.

(*iii*) $\sqrt{1.44} = \mathbf{1.2} = \frac{12}{10} = \frac{6}{5}$ is a **rational number**.

(*iv*) $\sqrt{7}$ is an **irrational number**.

(*v*) $-\sqrt{81} = \mathbf{-9}$ is a **rational number**.

(*vi*) $\sqrt{0.04} = \mathbf{0.2}$ is a **rational number**.

(*vii*) π is an **irrational number**.

(*viii*) $\frac{22}{7}$ is a **rational number**.

Since denominator 7 is not of the form 2^m5^n where m, n are non-negative integers and it has a prime factor 7 which is other than 2 or 5,

$\therefore$ the given rational number has non-terminating recurring decimal expansion.

```
7 ) 22 ( 3.142857
  – 21
     10
    – 7
     30
   – 28
      20
    – 14
       60
     – 56
        40
      – 35
         50
       – 49
          1
```

Hence, $\frac{22}{7} = 3.\overline{142857}$.

6. (*i*) $\frac{11}{80}$ is a rational number in its simplest form, *i.e.* the numerator 11 and denominator 80 have no common factors other than 1.

Denominator $80 = 2^4 \times 5$ is of the form 2^m5^n where m and n are non-negative integers and it has no prime factor other than 2 and 5.

$\therefore$ the given rational number has a **terminating decimal**

(*ii*) $\frac{48}{455}$ is a rational number in its simplest form, *i.e.* the numerator 48 and denominator 455 have no common factors other than 1.

Denominator $455 = 5 \times 7 \times 13$ is not of the form 2^m5^n where m and n are non-negative integers and it has prime factors 7 and 13 which are other than 2 or 5.

$\therefore$ the given rational number has a **non-terminating repeating decimal.**

(*iii*) $\frac{7}{12}$ is a rational number in its simplest form, *i.e.* the numerator 7 and denominator 12 have no common factors other than 1.

Denominator $12 = 2^2 \times 3$ is not of the form 2^m5^n where m and n are non-negative integers and has a prime factor 3 which is other than 2 or 5.

$\therefore$ the given rational number has a **non-terminating repeating decimal.**

(*iv*) $\frac{6}{35}$ is a rational number in its simplest form.

Denominator $35 = 5 \times 7$ is not of the form 2^m5^n where m and n are non-negative integers and it has a prime factor 7 which is other than 2 or 5.

$\therefore$ the given rational number has a **non-terminating repeating decimal.**

(*v*) $\frac{13}{125}$ is a rational number in its simplest form.

Denominator $125 = 5^3 = 2^0 \times 5^3$ is of the form 2^m5^n where m and n are non-negative integers and it has no prime factor other than 5.

$\therefore$ the given rational number has a **terminating decimal**.

7.

```
13 ) 100 ( 0.076923076923
    − 91
      90
    − 78
     120
   − 117
      30
    − 26
      40
    − 39
     100
    − 91
      90
    − 78
     120
   − 117
      30
    − 26
      40
    − 39
       1
```

$$\frac{1}{13} = 0.\overline{076923}$$

$$\therefore \quad \frac{2}{13} = 2(0.\overline{076923}) = 0.\overline{153846}$$

$$\frac{5}{13} = 5(0.\overline{076923}) = 0.\overline{384615}$$

$$\frac{7}{13} = 7(0.\overline{076923}) = 0.\overline{538461}$$

Hence, $\mathbf{\frac{1}{13} = 0.\overline{076923}}$, $\mathbf{\frac{2}{13} = 0.\overline{153846}}$,

$\mathbf{\frac{5}{13} = 0.\overline{384615}}$ and $\mathbf{\frac{7}{13} = 0.\overline{538461}}$.

Q8 – Q11: Answers will vary. Sample answers are given below.

8. Let $a = 0.10$ and $b = 0.12$.

Clearly a and b rational numbers such that $a < b$. In the first place of decimal, rational numbers a and b have 1, but in the second place of decimal a has a 0 and b has 2.

So, we can consider the numbers

c = **0.1010010001 …**

and d = **0.1101001000100001 …**

Clearly, irrational numbers c and d lie between a and b as $a < c < d < b$.

9. Let $a = 0.2101$ and $b = 0.\overline{2} = 0.2222 \ldots$

Clearly, a and b are rational numbers such that $a < b$.

In the first place of decimal, a has 2 and b has 2.

In the second place of decimal, a has 1 and b has 2.

In the third place of decimal, a has 0 and b has 2.

In the fourth place of decimal, a has 1 and b has 2.

So, we consider the number

c = **0.22010010001 …**

Clearly, irrational number c lies between a and b as $a < c < b$.

10. Let $a = \sqrt{2} = 1.414213562 \ldots$

and $\sqrt{3} = 1.732050808 \ldots$

Since the decimal expansion of $\sqrt{2}$ and $\sqrt{3}$ are non-terminating, non-repeating,

$\therefore$ $\sqrt{2}$ and $\sqrt{3}$ are irrational numbers.

Also, $\sqrt{3} > \sqrt{2}$.

In the first decimal place, a has 4 and b has 7.

So, we consider the numbers

c = **1.501001000100001…**

and d = **1.601001000100001…**

Clearly, irrational numbers c and d lie between a and b as $a < c < d < b$.

11. Let $a = \frac{3}{11}$

$= 0.272727\ldots$

$= 0.\overline{27}$

```
11 ) 30 ( 0.2727…
   − 22
     80
   − 77
     30
   − 22
     80
   − 77
      3
```

Let $b = \frac{5}{7}$

$= 0.714285714285 \ldots$

$= 0.\overline{714285}$

$$\begin{array}{l} 7\,)\,50\,(\,.714285 \\ -49 \\ \quad 10 \\ \quad -7 \\ \quad\; 30 \\ \quad -28 \\ \qquad 20 \\ \qquad -14 \\ \qquad\; 60 \\ \qquad -56 \\ \qquad\;\; 40 \\ \qquad -35 \\ \qquad\quad 5 \end{array}$$

In the first place of decimal, a has 2 and b has 7,

$\therefore$ $a < b$.

In the second place of decimal, a has 7 and b has 1.

So, we consider rational numbers.

c = **0.2802800028000…**

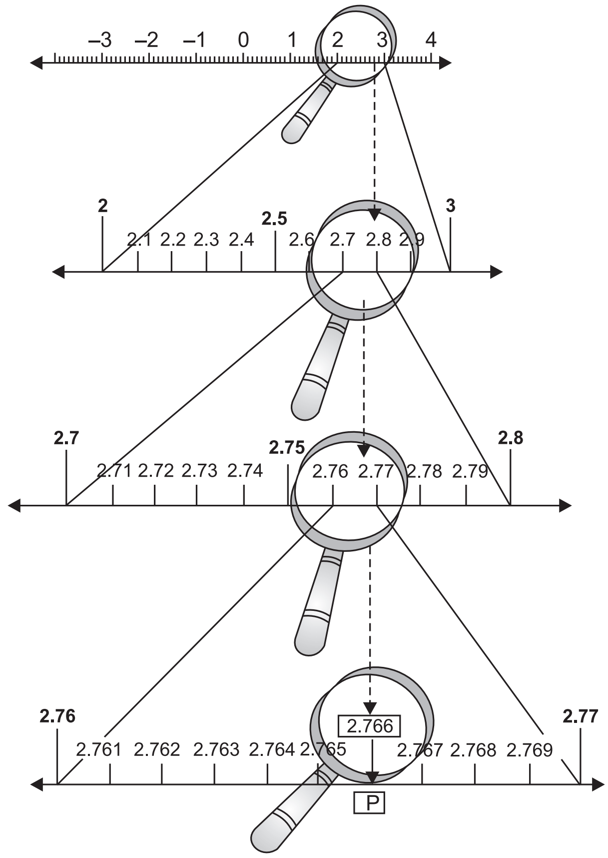
and d = **0.5878394426…**

Clearly, c and d are irrational numbers between a and b as $a < c < d < b$.

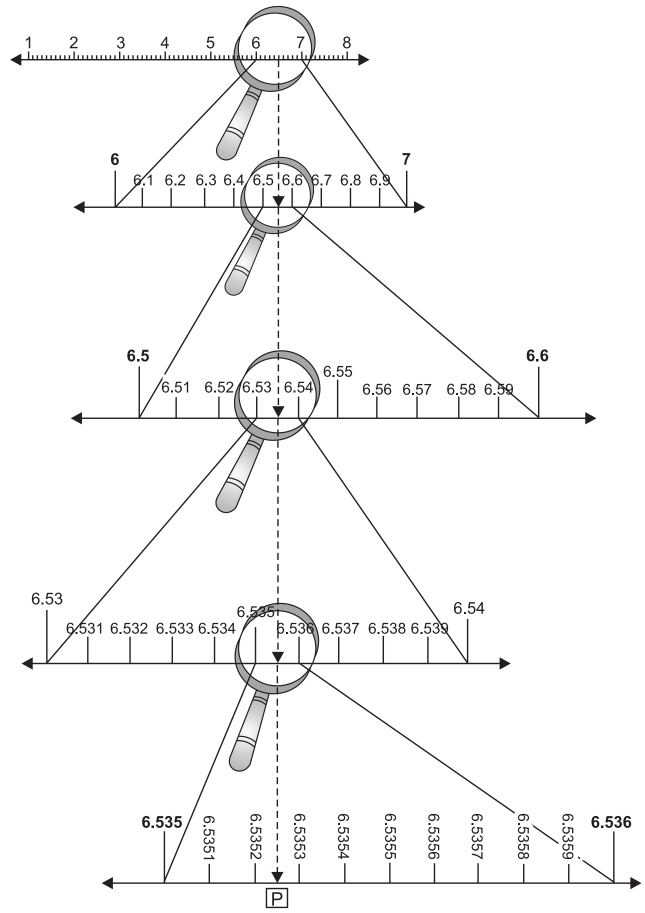
12. For terminating decimal representation of rational number $\frac{p}{q}$ $(q \neq 0)$, the prime factorization of q is of the form 2^n5^m, where n and m are non-negative integers.

EXERCISE 1D

1. Representation of 2.766 on the number line using successive magnification.



2. Representation of $6.\overline{53}$ on the number line up to 4 decimal places using successive magnification.



EXERCISE 1E

1. Add

(*i*) $(4\sqrt{3} + 7\sqrt{2})$ and $(\sqrt{3} - 5\sqrt{2})$

$= 4\sqrt{3} + 7\sqrt{2} + \sqrt{3} - 5\sqrt{2}$

$= 4\sqrt{3} + \sqrt{3} + 7\sqrt{2} - 5\sqrt{2}$

$= \mathbf{5\sqrt{3} + 2\sqrt{2}}$

(*ii*) $(\sqrt{5} + 2\sqrt{3})$ and $(2\sqrt{5} - 5\sqrt{3})$

$= \sqrt{5} + 2\sqrt{3} + 2\sqrt{5} - 5\sqrt{3}$

$= \sqrt{5} + 2\sqrt{5} + 2\sqrt{3} - 5\sqrt{3}$

$= \mathbf{3\sqrt{5} - 3\sqrt{3}}$

(*iii*) $\left(6\sqrt{7} - \frac{1}{2}\sqrt{2} + \frac{2}{3}\sqrt{5}\right)$ and $\left(\frac{1}{3}\sqrt{5} + \frac{3}{2}\sqrt{2} - \sqrt{7}\right)$

$= 6\sqrt{7} - \frac{\sqrt{2}}{2} + \frac{2\sqrt{5}}{3} + \frac{\sqrt{5}}{3} + \frac{3\sqrt{2}}{2} - \sqrt{7}$

$= 6\sqrt{7} - \sqrt{7} + \frac{2\sqrt{5}}{3} + \frac{\sqrt{5}}{3} + \frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$

$= 5\sqrt{7} + \frac{3\sqrt{5}}{3} + \frac{2\sqrt{2}}{2}$

$= \mathbf{5\sqrt{7} + \sqrt{5} + \sqrt{2}}$

2. (*i*) Multiply $2\sqrt{3}$ by $5\sqrt{27}$

$$2\sqrt{3} \times 5\sqrt{27} = 2\sqrt{3} \times 5\sqrt{9\times 3}$$
$$= 2\sqrt{3} \times 5\times 3\sqrt{3}$$
$$= 30\sqrt{3\times 3}$$
$$= 30 \times 3$$
$$= \mathbf{90}$$

(*ii*) $3\sqrt{28}$ by $2\sqrt{7}$

$$3\sqrt{28} \times 2\sqrt{7} = 3\sqrt{4\times 7} \times 2\sqrt{7}$$
$$= 3\times 2\sqrt{7} \times 2\sqrt{7}$$
$$= 12 \times 7$$
$$= \mathbf{84}$$

(*iii*) $3\sqrt{8}$ by $3\sqrt{2}$

$$3\sqrt{8} \times 3\sqrt{2} = 9\sqrt{8\times 2}$$
$$= 9\sqrt{16}$$
$$= 9 \times 4$$
$$= \mathbf{36}$$

(*iv*) $4\sqrt{12}$ by $7\sqrt{6}$

$$4\sqrt{12} \times 7\sqrt{6} = 28\sqrt{12\times 6}$$
$$= 28\sqrt{2\times 6\times 6}$$
$$= 28 \times 6\sqrt{2}$$
$$= \mathbf{168\sqrt{2}}$$

(*v*) $\sqrt[3]{2}$ by $\sqrt[4]{3}$

$$\sqrt[3]{2} \times \sqrt[4]{3} = \sqrt[12]{2^4} \times \sqrt[12]{3^3}$$
$$= \sqrt[12]{2^4 \times 3^3}$$
$$= \sqrt[12]{16\times 27}$$
$$= \mathbf{\sqrt[12]{432}}$$

(*vi*) $2\sqrt[4]{3}$ by $5\sqrt[4]{81}$

$$2\sqrt[4]{3} \times 5\sqrt[4]{81} = 2\sqrt[4]{3} \times 5\sqrt[4]{3^4}$$
$$= 2\sqrt[4]{3} \times 5 \times 3$$
$$= \mathbf{30\sqrt[4]{3}}$$

3. Simplify $\sqrt{2}\times\sqrt[3]{3}\times\sqrt[4]{4}$

(*i*)
$$\sqrt{2}\times\sqrt[3]{3}\times\sqrt[4]{4} = \sqrt{2}\times\sqrt[3]{3}\times\sqrt[4]{2^2}$$
$$= \sqrt[12]{2^6}\times\sqrt[12]{3^4}\times\sqrt[12]{2^6}$$
$$= \sqrt[12]{2^6\times 3^4\times 2^6}$$
$$= \sqrt[12]{2^{12}\times 3^4}$$
$$= 2\sqrt[12]{3^4}$$
$$= \mathbf{2\sqrt[3]{3}}$$

(*ii*)
$$\sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{32} = \sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{2^5}$$
$$= \sqrt[12]{2^4}\times\sqrt[12]{2^3}\times\sqrt[12]{2^5}$$
$$= \sqrt[12]{2^4\times 2^3\times 2^5}$$
$$= \sqrt[12]{2^{12}}$$
$$= \mathbf{2}$$

4. (*i*) Divide $12\sqrt{15}$ by $4\sqrt{3}$

$$12\sqrt{15} \div 4\sqrt{3} = \frac{12\sqrt{15}}{4\sqrt{3}}$$
$$= 3\sqrt{\frac{15}{3}}$$
$$= \mathbf{3\sqrt{5}}$$

(*ii*)
$$4\sqrt{28} \div 3\sqrt{7} = \frac{4\sqrt{28}}{3\sqrt{7}}$$
$$= \frac{4\sqrt{4\times 7}}{3\sqrt{7}}$$
$$= \frac{4\times 2\sqrt{7}}{3\sqrt{7}}$$
$$= \mathbf{\frac{8}{3}}$$

(*iii*)
$$21\sqrt{384} \div 8\sqrt{96} = \frac{21\sqrt{384}}{8\sqrt{96}}$$
$$= \frac{21\sqrt{2^7\times 3}}{8\sqrt{2^5\times 3}}$$
$$= \frac{21}{8}\sqrt{\frac{2^7\times 3}{2^5\times 3}}$$
$$= \frac{21}{8}\sqrt{2^2}$$
$$= \frac{21}{8}\times 2$$
$$= \mathbf{\frac{21}{4}}$$

(*iv*)
$$\sqrt[6]{12} \div \sqrt{3}\,\sqrt[3]{2} = \frac{\sqrt[6]{2^2\times 3}}{\sqrt{3}\,\sqrt[3]{2}}$$
$$= \frac{\sqrt[6]{2^2\times 3}}{\sqrt[6]{3^3}\times\sqrt[6]{2^2}}$$
$$= \sqrt[6]{\frac{2^2\times 3}{3^3\times 3}}$$
$$= \sqrt[6]{\frac{1}{3^2}}$$
$$= \mathbf{\sqrt[3]{\frac{1}{3}}}$$

5. (*i*) $\sqrt{147} - \sqrt{108} - \sqrt{3}$

$$= \sqrt{7\times 7\times 3} - \sqrt{2\times 2\times 3\times 3\times 3} - \sqrt{3}$$
$$= 7\sqrt{3} - 2\times 3\sqrt{3} - \sqrt{3}$$
$$= 7\sqrt{3} - 6\sqrt{3} - \sqrt{3}$$
$$= 7\sqrt{3} - 7\sqrt{3}$$
$$= \mathbf{0}$$

(*ii*) $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$

$= 4\sqrt{3} - 3\sqrt{2\times2\times3} + 2\sqrt{3\times5\times5}$

$= 4\sqrt{3} - 3\times2\sqrt{3} + 2\times 5\sqrt{3}$

$= 4\sqrt{3} - 6\sqrt{3} + 10\sqrt{3}$

$= 14\sqrt{3} - 6\sqrt{3}$

$= \mathbf{8\sqrt{3}}$

(*iii*) $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

$= 3\sqrt{3\times3\times5} - \sqrt{5\times5\times5} + \sqrt{2\times2\times2\times5\times5} - \sqrt{5\times5\times2}$

$= 3\times 3\sqrt{5} - 5\sqrt{5} + 2\times5\sqrt{2} - 5\sqrt{2}$

$= 9\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2}$

$= \mathbf{4\sqrt{5} + 5\sqrt{2}}$

(*iv*) $3\sqrt{48} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$

$= 3\sqrt{2\times2\times2\times2\times3} - \frac{5}{2}\times\frac{\sqrt{1}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} + 4\sqrt{3}$

$= 3\times2\times 2\sqrt{3} - \frac{5\sqrt{3}}{6} + 4\sqrt{3}$

$= 12\sqrt{3} - \frac{5}{6}\sqrt{3} + 4\sqrt{3}$

$= 16\sqrt{3} - \frac{5}{6}\sqrt{3}$

$= \sqrt{3}\left(16 - \frac{5}{6}\right)$

$= \sqrt{3}\left(\frac{96-5}{6}\right)$

$= \sqrt{3}\left(\frac{91}{6}\right)$

$= \mathbf{\frac{91}{6}\sqrt{3}}$

(*v*) $2\sqrt[3]{4} + 7\sqrt[3]{32} - \sqrt[3]{500}$

$= 2\sqrt[3]{4} + 7\sqrt[3]{2^5} - \sqrt[3]{2\times2\times5\times5\times5}$

$= 2\sqrt[3]{4} + 7\times2\sqrt[3]{2^2} - 5\sqrt[3]{2^2}$

$= 2\sqrt[3]{4} + 14\sqrt[3]{4} - 5\sqrt[3]{4}$

$= 16\sqrt[3]{4} - 5\sqrt[3]{4}$

$= \mathbf{11\sqrt[3]{4}}$

(*vi*) $2\sqrt[3]{54} + 3\sqrt[3]{16} + 5\sqrt[3]{128}$

$= 2\sqrt[3]{2\times3^3} + 3\sqrt[3]{2^4} + 5\sqrt[3]{2^7}$

$= 2\times3\sqrt[3]{2} + 3\times2\sqrt[3]{2} + 5\times2\times2\sqrt[3]{2}$

$= 6\sqrt[3]{2} + 6\sqrt[3]{2} + 20\sqrt[3]{2}$

$= \mathbf{32\sqrt[3]{2}}$

(*vii*) $\sqrt{125} - 4\sqrt{6} + \sqrt{294} - 2\sqrt{\frac{1}{6}}$

$= \sqrt{5^3} - 4\sqrt{6} + \sqrt{2\times3\times7^2} - \frac{2}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$

$= 5\sqrt{5} - 4\sqrt{6} + 7\sqrt{6} - \frac{2\sqrt{6}}{6}$

$= 5\sqrt{5} + 7\sqrt{6} - 4\sqrt{6} - \frac{\sqrt{6}}{3}$

$= 5\sqrt{5} + \sqrt{6}\left(7 - 4 - \frac{1}{3}\right)$

$= 5\sqrt{5} + \sqrt{6}\left(3 - \frac{1}{3}\right)$

$= 5\sqrt{5} + \sqrt{6}\left(\frac{9-1}{3}\right)$

$= \mathbf{5\sqrt{5} + \frac{8}{3}\sqrt{6}}$

(*viii*) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

$= \sqrt[4]{3^4} - 8\sqrt[3]{6^3} + 15\sqrt[5]{2^5} + \sqrt{15^2}$

$= 3 - 8\times6 + 15\times2 + 15$

$= 3 - 48 + 30 + 15$

$= 48 - 48$

$= \mathbf{0}$

6. (*i*) $(5+\sqrt{3})(7+\sqrt{5}) = 7(5+\sqrt{3}) + \sqrt{5}(5+\sqrt{3})$

$= \mathbf{35 + 7\sqrt{3} + 5\sqrt{5} + \sqrt{15}}$

(*ii*) $(3+\sqrt{2})(4+\sqrt{3}) = 4(3+\sqrt{2}) + \sqrt{3}(3+\sqrt{2})$

$= \mathbf{12 + 4\sqrt{2} + 3\sqrt{3} + \sqrt{6}}$

(*iii*) $(\sqrt{5}+\sqrt{2})(\sqrt{3}+\sqrt{2})$

$= \sqrt{5}(\sqrt{3}+\sqrt{2}) + \sqrt{2}(\sqrt{3}+\sqrt{2})$

$= \mathbf{\sqrt{15} + \sqrt{10} + \sqrt{6} + 2}$

(*iv*) $(\sqrt{13}+\sqrt{11})(\sqrt{13}-\sqrt{11})$

$= \left[(\sqrt{13})^2 - (\sqrt{11})^2\right]$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= 13 - 11$

$= \mathbf{2}$

(*v*) $(4+\sqrt{3})(4-\sqrt{3})$

$= [(4)^2 - (\sqrt{3})^2]$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= 16 - 3$

$= \mathbf{13}$

(*vi*) $(\sqrt{13}-\sqrt{6})(\sqrt{13}+\sqrt{6})$

$= [(\sqrt{13})^2 - (\sqrt{6})^2]$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= 13 - 6$

$= \mathbf{7}$

(*vii*) $(3\sqrt{5}+2\sqrt{7})(3\sqrt{5}-2\sqrt{7})$

$= \left[(3\sqrt{5})^2 - (2\sqrt{7})^2\right]$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= [9(5) - 4(7)]$

$= (45 - 28)$

$= \mathbf{17}$

(viii) $(\sqrt{5}+\sqrt{7})^2$

$= (\sqrt{5})^2 + (\sqrt{7})^2 + 2\sqrt{5}\times\sqrt{7}$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$= 5 + 7 + 2\sqrt{35}$

$= \mathbf{12 + 2\sqrt{35}}$

(ix) $(3\sqrt{5}+5\sqrt{2})^2$

$= (3\sqrt{5})^2 + (5\sqrt{2})^2 + 2\times 3\sqrt{5}\times 5\sqrt{2}$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$= 9(5) + 25(2) + 30\sqrt{10}$

$= 45 + 50 + 30\sqrt{10}$

$= \mathbf{95 + 30\sqrt{10}}$

(x) $(4\sqrt{3}-3\sqrt{5})^2$

$= (4\sqrt{3})^2 + (3\sqrt{5})^2 - 2\times 4\sqrt{3}\times 3\sqrt{5}$

$= 16(3) + 9(5) - 24\sqrt{15}$

$= 48 + 45 - 24\sqrt{15}$

$= \mathbf{93 - 24\sqrt{15}}$

EXERCISE 1F

1. (i) $2\sqrt{2}\times\sqrt{2} = 2\times\sqrt{2\times 2} = 2\times 2 = 4$

$\therefore$ $\mathbf{\sqrt{2}}$ is the simplest rationalisation factor of $2\sqrt{2}$.

(ii) $\sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2}$

Now, $3\sqrt{2}\times\sqrt{2} = 3\times\sqrt{2\times 2} = 3\times 2 = 6$

$\therefore$ $\mathbf{\sqrt{2}}$ is the simplest rationalisation factor of $\sqrt{18}$.

(iii) $\sqrt{75} = \sqrt{3\times 5\times 5} = 5\sqrt{3}$

Now, $5\sqrt{3}\times\sqrt{3} = 5\times\sqrt{3\times 3} = 5\times 3 = 15$

$\therefore$ $\mathbf{\sqrt{3}}$ is the simplest rationalisation factor of $\sqrt{75}$.

(iv) $\sqrt{112} = \sqrt{2\times 2\times 2\times 2\times 7} = 2\times 2\sqrt{7} = 4\sqrt{7}$

Now, $4\sqrt{7}\times\sqrt{7} = 4\times\sqrt{7\times 7} = 4\times 7 = 28$

$\therefore$ $\mathbf{\sqrt{7}}$ is the simplest rationalisation factor of $\sqrt{112}$.

(v) $\sqrt[3]{36} = \sqrt[3]{2\times 2\times 3\times 3}$

Now,

$\sqrt[3]{2\times 2\times 3\times 3}\times\sqrt[3]{2\times 3} = \sqrt[3]{2\times 2\times 2\times 3\times 3\times 3}$

$= 2\times 3 = 6$

$\therefore$ $\sqrt[3]{2\times 3}$, i.e $\mathbf{\sqrt[3]{6}}$ is the simplest rationalisation factor of $\sqrt[3]{36}$.

(vi) $\sqrt[3]{72} = \sqrt[3]{2\times 2\times 2\times 3\times 3} = 2\sqrt[3]{3\times 3}$

Now, $2\sqrt[3]{3\times 3}\times\sqrt[3]{3} = 2\times\sqrt[3]{3\times 3\times 3} = 2\times 3 = 6$

$\therefore$ $\mathbf{\sqrt[3]{3}}$ is the simplest rationalisation factor of $\sqrt[3]{72}$.

(vii) $2\sqrt[3]{5}$

$2\sqrt[3]{5}\times\sqrt[3]{5\times 5} = 2\sqrt[3]{5\times 5\times 5} = 2\times 5 = 10$

$\therefore$ $\sqrt[3]{5\times 5}$, i.e. $\mathbf{\sqrt[3]{25}}$ is the simplest rationalisation factor of $2\sqrt[3]{5}$.

(viii) $\sqrt[4]{768} = \sqrt[4]{2^8\times 3} = 2\times 2\sqrt[4]{3} = 4\sqrt[4]{3}$

Now, $4\sqrt[4]{3}\times\sqrt[4]{3\times 3\times 3} = 4\sqrt[4]{3\times 3\times 3\times 3}$

$= 4\times 3 = 12$

$\therefore$ $\sqrt[4]{3\times 3\times 3}$, i.e. $\mathbf{\sqrt[4]{27}}$ is the simplest rationalising factor of $\sqrt[4]{768}$.

(ix) $\sqrt[6]{192} = \sqrt[6]{2^6\times 3} = 6\sqrt[6]{3}$

Now, $6\sqrt[6]{3}\times\sqrt[6]{3\times 3\times 3\times 3\times 3}$

$= 6\sqrt[6]{3\times 3\times 3\times 3\times 3\times 3} = 6\times 3 = 18$

$\therefore$ $\sqrt[6]{3\times 3\times 3\times 3\times 3}$, i.e. $\mathbf{\sqrt[6]{243}}$ is the simplest ratrionalisation factor of $\sqrt[6]{192}$.

(x) $(\sqrt{3}+\sqrt{5})\times(\sqrt{3}-\sqrt{5}) = (\sqrt{3})^2 - (\sqrt{5})^2$

$= 3 - 5 = -2$

$\therefore$ $\mathbf{\sqrt{3}-\sqrt{5}}$ is the simplest rationalisation factor of $\sqrt{3}+\sqrt{5}$.

(xi) $(5-\sqrt{6})\times(5+\sqrt{6}) = (5)^2 - (\sqrt{6})^2 = 25 - 6 = 19$

$\therefore$ $\mathbf{5+\sqrt{6}}$ is the simplest rationalisation factor of $5-\sqrt{6}$.

(xii) $(2\sqrt{2}+3\sqrt{3})\times(2\sqrt{2}-3\sqrt{3}) = (2\sqrt{2})^2 - (3\sqrt{3})^2$

$= 4(2) - 9(3)$

$= 16 - 27$

$= -11$

$\therefore$ $\mathbf{2\sqrt{2}-3\sqrt{3}}$ is the simplest rationalisation factor of $2\sqrt{2}+3\sqrt{3}$.

2. (i) $\dfrac{4}{\sqrt{5}} = \dfrac{4}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}} = \mathbf{\dfrac{4\sqrt{5}}{5}}$

(ii) $\dfrac{4\sqrt{3}}{3\sqrt{7}} = \dfrac{4\sqrt{3}}{3\sqrt{7}}\times\dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{4\sqrt{21}}{3\times 7} = \mathbf{\dfrac{4\sqrt{21}}{21}}$

(iii) $\dfrac{1}{\sqrt{12}} = \dfrac{1}{\sqrt{2\times 2\times 3}} = \dfrac{1}{2\sqrt{3}} = \dfrac{1}{2\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{3}}{2\times 3} = \mathbf{\dfrac{\sqrt{3}}{6}}$

(iv) $\dfrac{4}{\sqrt[3]{16}} = \dfrac{4}{\sqrt[3]{2\times 2\times 2\times 2}} = \dfrac{4}{2\sqrt[3]{2}} = \dfrac{2}{\sqrt[3]{2}}\times\dfrac{\sqrt[3]{2\times 2}}{\sqrt[3]{2\times 2}}$

$= \dfrac{2\sqrt[3]{4}}{\sqrt[3]{2\times 2\times 2}} = \dfrac{2\sqrt[3]{4}}{2} = \mathbf{\sqrt[3]{4}}$

(v) $\dfrac{3\sqrt[3]{5}}{\sqrt[3]{9}} = \dfrac{3\sqrt[3]{5}}{\sqrt[3]{3\times 3}}\times\dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{3\sqrt[3]{5\times 3}}{\sqrt[3]{3\times 3\times 3}}$

$= \dfrac{3\sqrt[3]{15}}{3} = \mathbf{\sqrt[3]{15}}$

(vi) $\dfrac{2\sqrt[3]{5}}{\sqrt[3]{7}} = \dfrac{2\sqrt[3]{5}}{\sqrt[3]{7}}\times\dfrac{\sqrt[3]{7\times 7}}{\sqrt[3]{7\times 7}} = \dfrac{2\sqrt[3]{5\times 7\times 7}}{\sqrt[3]{7\times 7\times 7}} = \mathbf{\dfrac{2\sqrt[3]{245}}{7}}$

(vii) $\dfrac{2\sqrt[5]{4}}{\sqrt[5]{16}} = \dfrac{2\sqrt[5]{4}}{\sqrt[5]{2^4}}\times\dfrac{\sqrt[5]{2}}{\sqrt[5]{2}} = \dfrac{2\sqrt[5]{4\times 2}}{\sqrt[5]{2^5}} = \dfrac{2\sqrt[5]{8}}{2} = \mathbf{\sqrt[5]{8}}$

(*viii*) $\dfrac{\sqrt{2}+1}{\sqrt{3}} = \dfrac{(\sqrt{2}+1)}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \mathbf{\dfrac{\sqrt{6}+\sqrt{3}}{3}}$

3. (*i*) $\dfrac{1}{1+\sqrt{2}} = \dfrac{1}{1+\sqrt{2}} \times \dfrac{1-\sqrt{2}}{1-\sqrt{2}} = \dfrac{1-\sqrt{2}}{(1)^2-(\sqrt{2})^2}$

$= \dfrac{1-\sqrt{2}}{-1} = \mathbf{\sqrt{2}-1}$

(*ii*) $\dfrac{\sqrt{7}}{\sqrt{5}+1} = \dfrac{\sqrt{7}}{\sqrt{5}+1} \times \dfrac{\sqrt{5}-1}{\sqrt{5}-1} = \dfrac{\sqrt{35}-\sqrt{7}}{(\sqrt{5})^2-(1)^2}$

$= \dfrac{\sqrt{35}-\sqrt{7}}{5-1} = \mathbf{\dfrac{\sqrt{35}-\sqrt{7}}{4}}$

(*iii*) $\dfrac{2\sqrt{3}}{\sqrt{6}+2} = \dfrac{2\sqrt{3}}{\sqrt{6}+2} \times \dfrac{\sqrt{6}-2}{\sqrt{6}-2} = \dfrac{2\sqrt{18}-4\sqrt{3}}{(\sqrt{6})^2-(2)^2}$

$= \dfrac{2\times 3\sqrt{2}-4\sqrt{3}}{6-4} = \dfrac{6\sqrt{2}-4\sqrt{3}}{2}$

$= \mathbf{3\sqrt{2}-2\sqrt{3}}$

(*iv*) $\dfrac{3}{\sqrt{5}+\sqrt{2}} \times \dfrac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}-\sqrt{2})} = \dfrac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \dfrac{3(\sqrt{5}-\sqrt{2})}{5-2}$

$= \dfrac{3(\sqrt{5}-\sqrt{2})}{3}$

$= \mathbf{\sqrt{5}-\sqrt{2}}$

(*v*) $\dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} = \dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} \times \dfrac{(\sqrt{6}-\sqrt{3})}{\sqrt{6}-\sqrt{3}}$

$= \dfrac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6})^2-(\sqrt{3})^2} = \dfrac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3}$

$= \dfrac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{3} = \sqrt{2}(\sqrt{6}-\sqrt{3})$

$= \sqrt{12}-\sqrt{6} = \mathbf{2\sqrt{3}-\sqrt{6}}$

(*vi*) $\dfrac{1}{\sqrt{7}-\sqrt{6}} = \dfrac{1}{\sqrt{7}-\sqrt{6}} \times \dfrac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$= \dfrac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \dfrac{\sqrt{7}+\sqrt{6}}{7-6}$

$= \mathbf{\sqrt{7}+\sqrt{6}}$

(*vii*) $\dfrac{2}{\sqrt{3}-\sqrt{5}} = \dfrac{2}{(\sqrt{3}-\sqrt{5})} \times \dfrac{(\sqrt{3}+\sqrt{5})}{(\sqrt{3}+\sqrt{5})}$

$= \dfrac{2(\sqrt{3}+\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2} = \dfrac{2(\sqrt{3}+\sqrt{5})}{3-5}$

$= \dfrac{2(\sqrt{3}+\sqrt{5})}{-2} = \mathbf{-\sqrt{3}-\sqrt{5}}$

(*viii*) $\dfrac{2}{\sqrt{7}-\sqrt{5}} = \dfrac{2}{\sqrt{7}-\sqrt{5}} \times \dfrac{(\sqrt{7}+\sqrt{5})}{(\sqrt{7}+\sqrt{5})}$

$= \dfrac{2(\sqrt{7}+\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2} = \dfrac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$= \dfrac{2(\sqrt{7}+\sqrt{5})}{2} = \mathbf{\sqrt{7}+\sqrt{5}}$

(*ix*) $\dfrac{1}{5+2\sqrt{5}} = \dfrac{1}{5+2\sqrt{5}} \times \dfrac{5-2\sqrt{5}}{5-2\sqrt{5}} = \dfrac{5-2\sqrt{5}}{(5)^2-(2\sqrt{5})^2}$

$= \dfrac{5-2\sqrt{5}}{25-20} = \mathbf{\dfrac{5-2\sqrt{5}}{5}}$

(*x*) $\dfrac{6}{3\sqrt{2}+2\sqrt{3}} = \dfrac{6}{(3\sqrt{2}+2\sqrt{3})} \times \dfrac{(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})}$

$= \dfrac{6(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2} = \dfrac{6(3\sqrt{2}-2\sqrt{3})}{9(2)-4(3)}$

$= \dfrac{6(3\sqrt{2}-2\sqrt{3})}{18-12} = \dfrac{6(3\sqrt{2}-2\sqrt{3})}{6}$

$= \mathbf{3\sqrt{2}-2\sqrt{3}}$

(*xi*) $\dfrac{1}{7-2\sqrt{3}} = \dfrac{1}{7-2\sqrt{3}} \times \dfrac{7+2\sqrt{3}}{7+2\sqrt{3}} = \dfrac{7+2\sqrt{3}}{(7)^2-(2\sqrt{3})^2}$

$= \dfrac{7+2\sqrt{3}}{49-12} = \mathbf{\dfrac{7+2\sqrt{3}}{37}}$

(*xii*) $\dfrac{6}{2\sqrt{3}-\sqrt{6}} = \dfrac{6}{(2\sqrt{3}-\sqrt{6})} \times \dfrac{(2\sqrt{3}+\sqrt{6})}{(2\sqrt{3}+\sqrt{6})}$

$= \dfrac{6(2\sqrt{3}+\sqrt{6})}{(2\sqrt{3})^2-(\sqrt{6})^2} = \dfrac{6(2\sqrt{3}+\sqrt{6})}{12-6}$

$= \dfrac{6(2\sqrt{3}+\sqrt{6})}{6} = \mathbf{2\sqrt{3}+\sqrt{6}}$

4. (*i*) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} = \dfrac{(2+\sqrt{3})}{(2-\sqrt{3})} \times \dfrac{(2+\sqrt{3})}{(2+\sqrt{3})} = \dfrac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$

$= \dfrac{(2)^2+(\sqrt{3})^2+2\times 2\times\sqrt{3}}{4-3}$

$= \dfrac{4+3+4\sqrt{3}}{1}$

$= \mathbf{7+4\sqrt{3}}$

(*ii*) $\dfrac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}} = \dfrac{(\sqrt{7}+\sqrt{2})}{(9+2\sqrt{14})} \times \dfrac{(9-2\sqrt{14})}{(9-2\sqrt{14})}$

$= \dfrac{9\sqrt{7}+9\sqrt{2}-2\sqrt{7\times 14}-2\sqrt{2\times 14}}{(9)^2-(2\sqrt{14})^2}$

$= \dfrac{9\sqrt{7}+9\sqrt{2}-2\times 7\sqrt{2}-2\times 2\sqrt{7}}{81-4(14)}$

$= \dfrac{9\sqrt{7}+9\sqrt{2}-14\sqrt{2}-4\sqrt{7}}{81-56}$

$$= \frac{5\sqrt{7} - 5\sqrt{2}}{25} = \frac{5(\sqrt{7} - \sqrt{2})}{25}$$

$$= \mathbf{\frac{\sqrt{7} - \sqrt{2}}{5}}$$

(*iii*) $$\frac{6 - 4\sqrt{3}}{6 + 4\sqrt{3}} = \frac{(6 - 4\sqrt{3})}{(6 + 4\sqrt{3})} \times \frac{(6 - 4\sqrt{3})}{6 - 4\sqrt{3}}$$

$$= \frac{(6 - 4\sqrt{3})^2}{(6)^2 - (4\sqrt{3})^2}$$

$$= \frac{(6)^2 + (4\sqrt{3})^2 - 2 \times 6 \times 4\sqrt{3}}{36 - 16(3)}$$

$$= \frac{36 + 16(3) - 48\sqrt{3}}{36 - 48}$$

$$= \frac{36 + 48 - 48\sqrt{3}}{-12}$$

$$= \frac{84 - 48\sqrt{3}}{-12} = \frac{48\sqrt{3} - 84}{12}$$

$$= \frac{12(4\sqrt{3} - 7)}{12} = \mathbf{4\sqrt{3} - 7}$$

(*iv*) $$\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{(\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3})} \times \frac{(\sqrt{2} + \sqrt{3})}{(\sqrt{2} + \sqrt{3})}$$

$$= \frac{(\sqrt{2} + \sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2}$$

$$= \frac{(\sqrt{2})^2 + (\sqrt{3})^2 + 2 \times \sqrt{2} \times \sqrt{3}}{2 - 3}$$

$$= \frac{2 + 3 + 2\sqrt{6}}{-1}$$

$$= \frac{5 + 2\sqrt{6}}{-1}$$

$$= \mathbf{-5 - 2\sqrt{6}}$$

(*v*) $$\frac{3\sqrt{2} + 1}{2\sqrt{5} - 3} = \frac{(3\sqrt{2} + 1)}{(2\sqrt{5} - 3)} \times \frac{(2\sqrt{5} + 3)}{(2\sqrt{5} + 3)}$$

$$= \frac{3\sqrt{2}(2\sqrt{5} + 3) + 1(2\sqrt{5} + 3)}{(2\sqrt{5})^2 - (3)^2}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{4(5) - 9}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{20 - 9}$$

$$= \mathbf{\frac{3 + 9\sqrt{2} + 2\sqrt{5} + 6\sqrt{10}}{11}}$$

(*vi*) $$\frac{13 + 3\sqrt{5}}{13 - 3\sqrt{5}} = \frac{(13 + 3\sqrt{5})}{(13 - 3\sqrt{5})} \times \frac{(13 + 3\sqrt{5})}{(13 + 3\sqrt{5})}$$

$$= \frac{(13 + 3\sqrt{5})^2}{(13)^2 - (3\sqrt{5})^2}$$

$$= \frac{(13)^2 + (3\sqrt{5})^2 + 2 \times 13 \times 3\sqrt{5}}{169 - 9(5)}$$

$$= \frac{169 + 9(5) + 78\sqrt{5}}{169 - 45}$$

$$= \frac{169 + 45 + 78\sqrt{5}}{124} = \frac{214 + 78\sqrt{5}}{124}$$

$$= \frac{2(107 + 39\sqrt{5})}{124} = \mathbf{\frac{107 + 39\sqrt{5}}{62}}$$

(*vii*) $$\frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} = \frac{(\sqrt{7} - \sqrt{5})}{(\sqrt{7} + \sqrt{5})} \times \frac{(\sqrt{7} - \sqrt{5})}{(\sqrt{7} - \sqrt{5})}$$

$$= \frac{(\sqrt{7} - \sqrt{5})^2}{(\sqrt{7})^2 - (\sqrt{5})^2}$$

$$= \frac{(\sqrt{7})^2 + (\sqrt{5})^2 - 2 \times \sqrt{7} \times \sqrt{5}}{(\sqrt{7})^2 - (\sqrt{5})^2}$$

$$= \frac{7 + 5 - 2\sqrt{35}}{7 - 5} = \frac{12 - 2\sqrt{35}}{2}$$

$$= \frac{2(6 - \sqrt{35})}{2} = \mathbf{6 - \sqrt{35}}$$

(*viii*) $$\frac{\sqrt{11} - \sqrt{7}}{\sqrt{11} + \sqrt{7}} = \frac{(\sqrt{11} - \sqrt{7})}{(\sqrt{11} + \sqrt{7})} \times \frac{(\sqrt{11} - \sqrt{7})}{(\sqrt{11} - \sqrt{7})}$$

$$= \frac{(\sqrt{11} - \sqrt{7})^2}{(\sqrt{11})^2 - (\sqrt{7})^2}$$

$$= \frac{(\sqrt{11})^2 + (\sqrt{7})^2 - 2 \times \sqrt{11} \times \sqrt{7}}{11 - 7}$$

$$= \frac{11 + 7 - 2\sqrt{77}}{4} = \frac{18 - 2\sqrt{77}}{4}$$

$$= \frac{2(9 - \sqrt{77})}{4} = \mathbf{\frac{9 - \sqrt{77}}{2}}$$

(*ix*) $$\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} = \frac{(4\sqrt{3} + 5\sqrt{2})}{(4\sqrt{3} + 3\sqrt{2})} \times \frac{(4\sqrt{3} - 3\sqrt{2})}{(4\sqrt{3} - 3\sqrt{2})}$$

$$= \frac{16(3) - 12\sqrt{6} + 20\sqrt{6} - 15(2)}{(4\sqrt{3})^2 - (3\sqrt{2})^2}$$

$$= \frac{48 + 8\sqrt{6} - 30}{16(3) - 9(2)} = \frac{18 + 8\sqrt{6}}{48 - 18}$$

$$= \frac{2(9 + 4\sqrt{6})}{30} = \mathbf{\frac{9 + 4\sqrt{6}}{15}}$$

5. (*i*) $$\frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{5}}$$

$$= \frac{1}{(\sqrt{2} + \sqrt{3}) + (\sqrt{5})} \times \frac{(\sqrt{2} + \sqrt{3}) - (\sqrt{5})}{(\sqrt{2} + \sqrt{3}) - (\sqrt{5})}$$

$$= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{(\sqrt{2} + \sqrt{3})^2 - (\sqrt{5})^2}$$

$$= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{2 + 3 + 2\sqrt{6} - 5}$$

$$= \frac{\sqrt{2} + \sqrt{3} - \sqrt{5}}{2\sqrt{6}}$$

$$= \frac{(\sqrt{2}+\sqrt{3}-\sqrt{5})}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{12}+\sqrt{18}-\sqrt{30}}{2\times 6}$$

$$= \mathbf{\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}}$$

(*ii*) $\frac{1}{3+\sqrt{5}-2\sqrt{2}}$

$$= \frac{1}{(3+\sqrt{5})-(2\sqrt{2})} \times \frac{(3+\sqrt{5})+(2\sqrt{2})}{(3+\sqrt{5})+(2\sqrt{2})}$$

$$= \frac{3+\sqrt{5}+2\sqrt{2}}{(3+\sqrt{5})^2-(2\sqrt{2})^2}$$

$$= \frac{3+2\sqrt{2}+\sqrt{5}}{9+5+6\sqrt{5}-4(2)}$$

$$= \frac{3+2\sqrt{2}+\sqrt{5}}{14-8+6\sqrt{5}}$$

$$= \frac{3+2\sqrt{2}+\sqrt{5}}{6+6\sqrt{5}}$$

$$= \frac{3+2\sqrt{2}+\sqrt{5}}{6(1+\sqrt{5})} \times \frac{(1-\sqrt{5})}{(1-\sqrt{5})}$$

$$= \frac{3+2\sqrt{2}+\sqrt{5}-3\sqrt{5}-2\sqrt{10}-5}{6[(1)^2-(\sqrt{5})^2]}$$

$$= \frac{-2+2\sqrt{2}-2\sqrt{5}-2\sqrt{10}}{6(1-5)}$$

$$= \frac{-2(1-\sqrt{2}+\sqrt{5}+\sqrt{10})}{6(-4)}$$

$$= \mathbf{\frac{1-\sqrt{2}+\sqrt{5}+\sqrt{10}}{12}}$$

(*iii*) $\frac{6}{3+\sqrt{2}-\sqrt{5}} = \frac{6}{(3+\sqrt{2})-(\sqrt{5})} \times \frac{(3+\sqrt{2})+(\sqrt{5})}{(3+\sqrt{2})+(\sqrt{5})}$

$$= \frac{6(3+\sqrt{2}+\sqrt{5})}{(3+\sqrt{2})^2-(\sqrt{5})^2}$$

$$= \frac{6(3+\sqrt{2}+\sqrt{5})}{9+2+6\sqrt{2}-5}$$

$$= \frac{6(3+\sqrt{2}+\sqrt{5})}{6+6\sqrt{2}}$$

$$= \frac{6(3+\sqrt{2}+\sqrt{5})}{6(1+\sqrt{2})} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})}$$

$$= \frac{(3+\sqrt{2}+\sqrt{5})\times(1-\sqrt{2})}{(1)^2-(\sqrt{2})^2}$$

$$= \frac{3-3\sqrt{2}+\sqrt{2}-2+\sqrt{5}-\sqrt{10}}{1-2}$$

$$= \frac{-\sqrt{10}+\sqrt{5}-2\sqrt{2}+1}{-1}$$

$$= \mathbf{\sqrt{10}-\sqrt{5}+2\sqrt{2}-1}$$

(*iv*) $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$

$$= \frac{1}{(\sqrt{7}+\sqrt{6}-\sqrt{13})} \times \frac{(\sqrt{7}+\sqrt{6})+(\sqrt{13})}{(\sqrt{7}+\sqrt{6})+(\sqrt{13})}$$

$$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^2-(\sqrt{13})^2} = \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{7+6+2\sqrt{42}-13}$$

$$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$$

$$= \frac{\sqrt{7\times 42}+\sqrt{6\times 42}+\sqrt{13\times 42}}{2\sqrt{42\times 42}}$$

$$= \frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{2\times 42}$$

$$= \mathbf{\frac{1}{84}(7\sqrt{6}+6\sqrt{7}+\sqrt{546})}$$

(*v*) $\frac{1}{1+\sqrt{2}-\sqrt{3}} = \frac{1}{(1+\sqrt{2})-(\sqrt{3})} \times \frac{(1+\sqrt{2})+(\sqrt{3})}{(1+\sqrt{2})+(\sqrt{3})}$

$$= \frac{1+\sqrt{2}+\sqrt{3}}{(1+\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{1+\sqrt{2}+\sqrt{3}}{1+2+2\sqrt{2}-3}$$

$$= \frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+2+\sqrt{6}}{2\times 2}$$

$$= \mathbf{\frac{2+\sqrt{2}+\sqrt{6}}{4}}$$

6. (*i*) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{3+1-2\sqrt{3}}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

$$= 2+(-1)\sqrt{3} \quad \dots (1)$$

$\therefore$ $2-\sqrt{3} = a+b\sqrt{3}$

$= 2+(-1)\sqrt{3}$ [Using (1)]

$\Rightarrow$ $\boldsymbol{a} = 2$ and $\boldsymbol{b} = -1$

(*ii*) $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{(3-\sqrt{5})}{(3+2\sqrt{5})} \times \frac{(3-2\sqrt{5})}{(3-2\sqrt{5})}$

$$= \frac{9-6\sqrt{5}-3\sqrt{5}+2\times 5}{(3)^2-(2\sqrt{5})^2}$$

$$= \frac{19-9\sqrt{5}}{9-20}$$

$$= \frac{19 - 9\sqrt{5}}{-11}$$

$$= \frac{-9\sqrt{5}}{-11} + \frac{19}{-11}$$

$$= \frac{9}{11}\sqrt{5} - \frac{9}{11} \qquad \ldots (1)$$

$$\frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} = a\sqrt{5} - b$$

$$= \frac{9}{11}\sqrt{5} - \frac{19}{11} \qquad [\text{Using } (1)]$$

$$\Rightarrow \quad \boldsymbol{a} = \frac{\mathbf{9}}{\mathbf{11}} \text{ and } \boldsymbol{b} = \frac{\mathbf{19}}{\mathbf{11}}$$

(*iii*)
$$\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{(\sqrt{5} + \sqrt{3})}{\sqrt{5} - \sqrt{3}} \times \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})}$$

$$= \frac{(\sqrt{5} + \sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{5 + 3 + 2\sqrt{15}}{5 - 3}$$

$$= \frac{8 + 2\sqrt{15}}{2}$$

$$= 4 + \sqrt{15} \qquad \ldots (1)$$

$$\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = a + b\sqrt{15}$$

$$= 4 + \sqrt{15} \qquad [\text{Using } (1)]$$

$$= 4 + (1)\sqrt{15}$$

$$\Rightarrow \quad \boldsymbol{a} = 4 \text{ and } \boldsymbol{b} = 1$$

(*iv*)
$$\frac{\sqrt{2} - \sqrt{5}}{\sqrt{2} + \sqrt{5}} = \frac{(\sqrt{2} - \sqrt{5})}{(\sqrt{2} + \sqrt{5})} \times \frac{(\sqrt{2} - \sqrt{5})}{(\sqrt{2} - \sqrt{5})}$$

$$= \frac{(\sqrt{2} - \sqrt{5})^2}{(\sqrt{2})^2 - (\sqrt{5})^2}$$

$$= \frac{2 + 5 - 2\sqrt{10}}{2 - 5}$$

$$= \frac{7 - 2\sqrt{10}}{-3}$$

$$= -\frac{7}{3} + \frac{2}{3}\sqrt{10} \qquad \ldots (1)$$

$$\frac{\sqrt{2} - \sqrt{5}}{\sqrt{2} + \sqrt{5}} = a + b\sqrt{10}$$

$$= -\frac{7}{3} + \frac{2}{3}\sqrt{10} \qquad [\text{Using } (1)]$$

$$\Rightarrow \quad \boldsymbol{a} = -\frac{\mathbf{7}}{\mathbf{3}} \text{ and } \boldsymbol{b} = \frac{\mathbf{2}}{\mathbf{3}}$$

(*v*)
$$\frac{3 + \sqrt{7}}{3 - \sqrt{7}} = \frac{(3 + \sqrt{7})}{(3 - \sqrt{7})} \times \frac{(3 + \sqrt{7})}{(3 - \sqrt{7})}$$

$$= \frac{(3 + \sqrt{7})^2}{(3)^2 - (\sqrt{7})^2}$$

$$= \frac{9 + 7 + 6\sqrt{7}}{9 - 7}$$

$$= \frac{16 + 6\sqrt{7}}{2}$$

$$= 8 + 3\sqrt{7} \qquad \ldots (1)$$

$$\frac{3 + \sqrt{7}}{3 - \sqrt{7}} = a + b\sqrt{7}$$

$$= 8 + 3\sqrt{7} \qquad [\text{Using } (1)]$$

$$\Rightarrow \quad \boldsymbol{a} = 8 \text{ and } \boldsymbol{b} = 3$$

(*vi*)
$$\frac{5 + \sqrt{6}}{5 - \sqrt{6}} = \frac{(5 + \sqrt{6})}{(5 - \sqrt{6})} \times \frac{(5 + \sqrt{6})}{(5 + \sqrt{6})}$$

$$= \frac{(5 + \sqrt{6})^2}{(5)^2 - (\sqrt{6})^2}$$

$$= \frac{25 + 6 + 10\sqrt{6}}{25 - 6}$$

$$= \frac{31 + 10\sqrt{6}}{19}$$

$$= \frac{31}{19} + \frac{10}{19}\sqrt{6} \qquad \ldots (1)$$

$$\frac{5 + \sqrt{6}}{5 - \sqrt{6}} = a + b\sqrt{6}$$

$$= \frac{31}{19} + \frac{10}{19}\sqrt{6}$$

$$\Rightarrow \quad \boldsymbol{a} = \frac{\mathbf{31}}{\mathbf{19}} \text{ and } \boldsymbol{b} = \frac{\mathbf{10}}{\mathbf{19}}$$

(*vii*)
$$\frac{1 + \sqrt{6}}{3 + \sqrt{6}} = \frac{(1 + \sqrt{6})}{(3 + \sqrt{6})} \times \frac{(3 - \sqrt{6})}{(3 - \sqrt{6})}$$

$$= \frac{3 - \sqrt{6} + 3\sqrt{6} - 6}{(3)^2 - (\sqrt{6})^2}$$

$$= \frac{-3 + 2\sqrt{6}}{9 - 6}$$

$$= \frac{-3 + 2\sqrt{6}}{3}$$

$$= -1 + \frac{2}{3}\sqrt{6} \qquad \ldots (1)$$

$$\frac{1 + \sqrt{6}}{3 + \sqrt{6}} = a + b\sqrt{6}$$

$$= -1 + \frac{2}{3}\sqrt{6} \qquad [\text{Using } (1)]$$

$$\Rightarrow \quad \boldsymbol{a} = -1 \text{ and } \boldsymbol{b} = \frac{\mathbf{2}}{\mathbf{3}}$$

(*viii*)
$$\frac{3 + 2\sqrt{2}}{2 - \sqrt{2}} = \frac{(3 + 2\sqrt{2})}{(2 - \sqrt{2})} \times \frac{(2 + \sqrt{2})}{(2 + \sqrt{2})}$$

$$= \frac{6 + 3\sqrt{2} + 4\sqrt{2} + 4}{(2)^2 - (\sqrt{2})^2}$$

$$= \frac{10 + 7\sqrt{2}}{4 - 2}$$

$$= \frac{10 + 7\sqrt{2}}{2}$$

$$= 5 + \frac{7}{2}\sqrt{2} \qquad \ldots (1)$$

$$\frac{3+2\sqrt{2}}{2-\sqrt{2}} = a + b\sqrt{2}$$

$$= 5 + \frac{7}{2}\sqrt{2} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \boldsymbol{a} = 5 \text{ and } \boldsymbol{b} = \frac{\mathbf{7}}{\mathbf{2}}$$

(*ix*)
$$\frac{3+\sqrt{7}}{3-4\sqrt{7}} = \frac{(3+\sqrt{7})}{(3-4\sqrt{7})} \times \frac{(3+4\sqrt{7})}{(3+4\sqrt{7})}$$

$$= \frac{9+12\sqrt{7}+3\sqrt{7}+28}{(3)^2-(4\sqrt{7})^2}$$

$$= \frac{37+15\sqrt{7}}{9-112}$$

$$= \frac{37+15\sqrt{7}}{-103}$$

$$= \frac{-37}{103} - \frac{15}{103}\sqrt{7}$$

$$= \frac{-37}{103} + \left(\frac{-15}{103}\right)\sqrt{7} \qquad \ldots (1)$$

$$\frac{3+\sqrt{7}}{3-4\sqrt{7}} = a + b\sqrt{7}$$

$$= \frac{-37}{103} + \left(\frac{-15}{103}\right)\sqrt{7}$$

[Using (1)]

$$\Rightarrow \quad \boldsymbol{a} = \frac{\mathbf{-37}}{\mathbf{103}} \text{ and } b = \frac{\mathbf{-15}}{\mathbf{103}}$$

(*x*)
$$\frac{5+\sqrt{3}}{7-4\sqrt{3}} = \frac{(5+\sqrt{3})}{(7-4\sqrt{3})} \times \frac{(7+4\sqrt{3})}{(7+4\sqrt{3})}$$

$$= \frac{35+20\sqrt{3}+7\sqrt{3}+12}{(7)^2-(4\sqrt{3})^2}$$

$$= \frac{47+27\sqrt{3}}{49-48}$$

$$= 47 + 27\sqrt{3} \qquad \ldots (1)$$

$$\frac{5+\sqrt{3}}{7-4\sqrt{3}} = a + b\sqrt{3} = 47 + 27\sqrt{3}$$

[Using (1)]

$$\Rightarrow \quad \boldsymbol{a} = 47 \text{ and } \boldsymbol{b} = 27$$

(*xi*)
$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})} \times \frac{(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})}$$

$$= \frac{6+2\sqrt{6}+3\sqrt{6}+6}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$= \frac{12+5\sqrt{6}}{9(2)-4(3)}$$

$$= \frac{12+5\sqrt{6}}{18-12}$$

$$= \frac{12+5\sqrt{6}}{6}$$

$$= 2 + \frac{5}{6}\sqrt{6} \qquad \ldots (1)$$

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a + b\sqrt{6}$$

$$= 2 + \frac{5}{6}\sqrt{6} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \boldsymbol{a} = 2 \text{ and } \boldsymbol{b} = \frac{\mathbf{5}}{\mathbf{6}}$$

(*xii*)
$$\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1}$$

$$= \frac{(\sqrt{7}-1)(\sqrt{7}-1)}{(\sqrt{7}+1)(\sqrt{7}-1)} - \frac{(\sqrt{7}+1)(\sqrt{7}+1)}{(\sqrt{7}-1)(\sqrt{7}+1)}$$

$$= \frac{(\sqrt{7}-1)^2}{(\sqrt{7})^2-(1)^2} - \frac{(\sqrt{7}+1)^2}{(\sqrt{7})^2-(1)^2}$$

$$= \frac{7+1-2\sqrt{7}}{7-1} - \frac{7+1+2\sqrt{7}}{7-1}$$

$$= \frac{8-2\sqrt{7}}{6} - \frac{8+2\sqrt{7}}{6}$$

$$= \frac{8-2\sqrt{7}-8-2\sqrt{7}}{6}$$

$$= \frac{-4\sqrt{7}}{6}$$

$$= \frac{-2\sqrt{7}}{3}$$

$$= 0 + \left(\frac{-2}{3}\right)\sqrt{7} \qquad \ldots (1)$$

$$\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7} = 0 + \left(\frac{-2}{3}\right)\sqrt{7}$$

[Using (1)]

$$\Rightarrow \quad \boldsymbol{a = 0 \text{ and } b} = \frac{\mathbf{-2}}{\mathbf{3}}$$

7. (*i*)
$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} + \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})} \times \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{5+3+2\sqrt{15}}{5-3} + \frac{5+3-2\sqrt{15}}{5-3}$$

$$= \frac{8+2\sqrt{15}}{2} + \frac{8-2\sqrt{15}}{2}$$

$$= 4 + \sqrt{15} + 4 - \sqrt{15} = \mathbf{8}$$

(*ii*)
$$\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$$

$$= \frac{(\sqrt{5}-2)\times(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)} - \frac{(\sqrt{5}+2)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$$

$$= \frac{(\sqrt{5}-2)^2}{(\sqrt{5})^2-(2)^2} - \frac{(\sqrt{5}+2)^2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{5+4-4\sqrt{5}}{5-4} - \frac{5+4+4\sqrt{5}}{5-4}$$

$$= (9-4\sqrt{5}) - (9+4\sqrt{5})$$

$$= 9-4\sqrt{5}-9-4\sqrt{5}$$

$$= \mathbf{-8\sqrt{5}}$$

(*iii*) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

$$= \frac{(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})} \times \frac{(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})} + \frac{2\sqrt{3}}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$

$$= \frac{(3\sqrt{2}-2\sqrt{3})^2}{(3\sqrt{2})^2-(2\sqrt{3})^2} + \frac{6+2\sqrt{6}}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{9(2)+4(3)-12\sqrt{6}}{9(2)-4(3)} + \frac{6+2\sqrt{6}}{3-2}$$

$$= \frac{18+12-12\sqrt{6}}{18-12} + (6+2\sqrt{6})$$

$$= \frac{30-12\sqrt{6}}{6} + (6+2\sqrt{6})$$

$$= 5-2\sqrt{6}+6+2\sqrt{6}$$

$$= \mathbf{11}$$

(*iv*) $\frac{1}{1+\sqrt{2}+\sqrt{3}} + \frac{1}{1-\sqrt{2}+\sqrt{3}}$

$$= \frac{1}{(1+\sqrt{2})+(\sqrt{3})} \times \frac{(1+\sqrt{2})-(\sqrt{3})}{(1+\sqrt{2})-(\sqrt{3})} + \frac{1}{(1-\sqrt{2})+(\sqrt{3})} \times \frac{(1-\sqrt{2})-(\sqrt{3})}{(1-\sqrt{2})-(\sqrt{3})}$$

$$= \frac{(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2})^2-(\sqrt{3})^2} + \frac{(1-\sqrt{2}-\sqrt{3})}{(1-\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{(1+\sqrt{2}-\sqrt{3})}{1+2+2\sqrt{2}-3} + \frac{(1-\sqrt{2}-\sqrt{3})}{1+2-2\sqrt{2}-3}$$

$$= \frac{(1+\sqrt{2}-\sqrt{3})}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{(1-\sqrt{2}-\sqrt{3})}{-2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+2-\sqrt{6}}{4} + \frac{(\sqrt{2}-2-\sqrt{6})}{-4}$$

$$= \frac{\sqrt{2}+2-\sqrt{6}}{4} - \frac{(\sqrt{2}-2-\sqrt{6})}{4}$$

$$= \frac{\sqrt{2}+2-\sqrt{6}-\sqrt{2}+2+\sqrt{6}}{4}$$

$$= \frac{4}{4} = \mathbf{1}$$

8. (*i*) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2+\sqrt{5}}$

$$= \frac{1}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} + \frac{2}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} + \frac{1}{(2+\sqrt{5})} \times \frac{(2-\sqrt{5})}{(2-\sqrt{5})}$$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} + \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{2-\sqrt{5}}{(2)^2-(\sqrt{5})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} + \frac{2(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2-\sqrt{5}}{4-5}$$

$$= 2-\sqrt{3} + \frac{2(\sqrt{5}+\sqrt{3})}{2} + \frac{2-\sqrt{5}}{-1}$$

$$= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$

$$= \mathbf{2\sqrt{5}}$$

(*ii*) $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} + \frac{2\sqrt{3}}{\sqrt{6}+2} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

$$= \frac{3\sqrt{2}}{(\sqrt{6}-\sqrt{3})} \times \frac{(\sqrt{6}+\sqrt{3})}{(\sqrt{6}+\sqrt{3})} + \frac{2\sqrt{3}}{(\sqrt{6}+2)} \times \frac{(\sqrt{6}-2)}{(\sqrt{6}-2)} - \frac{4\sqrt{3}}{(\sqrt{6}-\sqrt{2})} \times \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})}$$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{(\sqrt{6})^2-(\sqrt{3})^2} + \frac{2\sqrt{3}(\sqrt{6}-2)}{(\sqrt{6})^2-(2)^2} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{6-3} + \frac{2\sqrt{3}(\sqrt{6}-2)}{6-4} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{6-2}$$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{3} + \frac{2\sqrt{3}(\sqrt{6}-2)}{2} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{4}$$

$$= \sqrt{12}+\sqrt{6}+\sqrt{18}-2\sqrt{3}-\sqrt{18}-\sqrt{6}$$

$$= 2\sqrt{3}+\sqrt{6}+3\sqrt{2}-2\sqrt{3}-3\sqrt{2}-\sqrt{6}$$

$$= \mathbf{0}$$

(*iii*) $\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-3\sqrt{2}} - \frac{3\sqrt{2}}{3+2\sqrt{3}}$

$$= \frac{4\sqrt{3}}{(2-\sqrt{2})} \times \frac{(2+\sqrt{2})}{(2+\sqrt{2})} - \frac{30}{(4\sqrt{3}-3\sqrt{2})} \times \frac{(4\sqrt{3}+3\sqrt{2})}{(4\sqrt{3}+3\sqrt{2})} - \frac{3\sqrt{2}}{(3+2\sqrt{3})} \times \frac{(3-2\sqrt{3})}{(3-2\sqrt{3})}$$

$$= \frac{4\sqrt{3}(2+\sqrt{2})}{(2)^2-(\sqrt{2})^2} - \frac{30(4\sqrt{3}+3\sqrt{2})}{(4\sqrt{3})^2-(3\sqrt{2})^2} - \frac{3\sqrt{2}(3-2\sqrt{3})}{(3)^2-(2\sqrt{3})^2}$$

$$= \frac{4\sqrt{3}(2+\sqrt{2})}{4-2} - \frac{30(4\sqrt{3}+3\sqrt{2})}{48-18} - \frac{3\sqrt{2}(3-2\sqrt{3})}{9-12}$$

$$= \frac{4\sqrt{3}(2+\sqrt{2})}{2} - \frac{30(4\sqrt{3}+3\sqrt{2})}{30} - \frac{3\sqrt{2}(3-2\sqrt{3})}{-3}$$

$$= 2\sqrt{3}(2+\sqrt{2}) - (4\sqrt{3}+3\sqrt{2}) + \sqrt{2}(3-2\sqrt{3})$$

$$= 4\sqrt{3}+2\sqrt{6} - 4\sqrt{3}-3\sqrt{2} + 3\sqrt{2}-2\sqrt{6}$$

$$= \mathbf{0}$$

9. (*i*) $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1}$

$= \frac{1}{(3+\sqrt{7})} \times \frac{(3-\sqrt{7})}{(3-\sqrt{7})} + \frac{1}{(\sqrt{7}+\sqrt{5})} \times \frac{(\sqrt{7}-\sqrt{5})}{(\sqrt{7}-\sqrt{5})}$

$+ \frac{1}{(\sqrt{5}+\sqrt{3})} \times \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} + \frac{1}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= \frac{3-\sqrt{7}}{(3)^2-(\sqrt{7})^2} + \frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7})^2-(\sqrt{5})^2} + \frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$+ \frac{\sqrt{3}-1}{(\sqrt{3})^2-(1)^2}$

$= \frac{3-\sqrt{7}}{9-7} + \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{\sqrt{3}-1}{3-1}$

$= \frac{3-\sqrt{7}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2}$

$= \frac{1}{2}(3-\sqrt{7}+\sqrt{7}-\sqrt{5}+\sqrt{5}-\sqrt{3}+\sqrt{3}-1)$

$= \frac{1}{2}(2)$

$= \mathbf{1}$

(*ii*) $\frac{1}{2+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}}$

$= \frac{1}{(2+\sqrt{5})} \times \frac{(2-\sqrt{5})}{(2-\sqrt{5})} + \frac{1}{(\sqrt{5}+\sqrt{6})} \times \frac{(\sqrt{5}-\sqrt{6})}{(\sqrt{5}-\sqrt{6})}$

$+ \frac{1}{(\sqrt{6}+\sqrt{7})} \times \frac{(\sqrt{6}-\sqrt{7})}{(\sqrt{6}-\sqrt{7})} + \frac{1}{(\sqrt{7}+2\sqrt{2})}$

$\times \frac{(\sqrt{7}-2\sqrt{2})}{(\sqrt{7}-2\sqrt{2})}$

$= \frac{2-\sqrt{5}}{(2)^2-(\sqrt{5})^2} + \frac{\sqrt{5}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{6})^2} + \frac{\sqrt{6}-\sqrt{7}}{(\sqrt{6})^2-(\sqrt{7})^2}$

$+ \frac{\sqrt{7}-2\sqrt{2}}{(\sqrt{7})^2-(2\sqrt{2})^2}$

$= \frac{2-\sqrt{5}}{4-5} + \frac{\sqrt{5}-\sqrt{6}}{5-6} + \frac{\sqrt{6}-\sqrt{7}}{6-7} + \frac{\sqrt{7}-2\sqrt{2}}{7-8}$

$= \frac{2-\sqrt{5}}{-1} + \frac{\sqrt{5}-\sqrt{6}}{-1} + \frac{\sqrt{6}-\sqrt{7}}{-1} + \frac{\sqrt{7}-2\sqrt{2}}{-1}$

$= -2+\sqrt{5} - \sqrt{5}+\sqrt{6} - \sqrt{6}+\sqrt{7} - \sqrt{7}+2\sqrt{2}$

$= -2+2\sqrt{2}$

$= 2\sqrt{2}-2$

$= \mathbf{2(\sqrt{2}-1)}$

10. Let $x = \frac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}}$

Then, $x^2 = \frac{4(2+6+2\sqrt{12})}{9(2+\sqrt{3})}$

$\Rightarrow x^2 = \frac{4(8+2\sqrt{12})}{9(2+\sqrt{3})}$

$= \frac{4(8+2\times 2\sqrt{3})}{9(2+\sqrt{3})}$

$= \frac{4(8+4\sqrt{3})}{9(2+\sqrt{3})}$

$= \frac{4\times 4(2+\sqrt{3})}{9(2+\sqrt{3})} = \frac{16}{9}$

$\Rightarrow x = \frac{4}{3}$ (rejecting the –ve value)

Hence, $\frac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}} = \mathbf{\frac{4}{3}}$

11. $\left\{\sqrt{5+2\sqrt{6}}\right\} + \left\{\sqrt{8-2\sqrt{15}}\right\}$

$= \left\{\sqrt{3+2+2\sqrt{6}}\right\} + \left\{\sqrt{5+3-2\sqrt{15}}\right\}$

$= \left\{\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2(\sqrt{3})(\sqrt{2})}\right\}$

$+ \left\{\sqrt{(\sqrt{5})^2+(\sqrt{3})^2-2(\sqrt{5})(\sqrt{3})}\right\}$

$= \left\{\sqrt{(\sqrt{3}+\sqrt{2})^2}\right\} + \left\{\sqrt{(\sqrt{5}-\sqrt{3})^2}\right\}$

$= (\sqrt{3}+\sqrt{2}) + (\sqrt{5}-\sqrt{3})$

$= \mathbf{\sqrt{2}+\sqrt{5}}$

12. Let $x = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

Then, $x^2 = \frac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{(\sqrt{5}+2)(\sqrt{5}-2)}}{(\sqrt{5}+1)}$

$= \frac{2\sqrt{5}+2\sqrt{(\sqrt{5})^2-(2)^2}}{(\sqrt{5}+1)}$

$= \frac{2\sqrt{5}+2\sqrt{5-4}}{(\sqrt{5}+1)}$

$= \frac{2\sqrt{5}+2\sqrt{1}}{(\sqrt{5}+1)}$

$= \frac{2(\sqrt{5}+1)}{(\sqrt{5}+1)} = 2$

$\Rightarrow x = \mathbf{\sqrt{2}}$ (rejecting the –ve value)

13. (*i*) $\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{4(1.732)}{3} = \frac{6.928}{3}$

$= \mathbf{2.309}$

(*ii*) $\frac{5-\sqrt{2}}{\sqrt{2}} = \frac{(5-\sqrt{2})}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}-2}{2}$

$= \frac{5(1.4142)-2}{2} = \frac{7.071-2}{2}$

$= \frac{5.071}{2}$ $\mathbf{= 2.535}$

(*iii*) $\dfrac{1}{\sqrt{2}+1} = \dfrac{1}{(\sqrt{2}+1)} \times \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \dfrac{\sqrt{2}-1}{2-1}$

$= \dfrac{1.4142-1}{1} = 0.4142 \approx \mathbf{0.414}$

(*iv*) $\dfrac{1}{\sqrt{2}+\sqrt{3}} = \dfrac{1}{\sqrt{2}+\sqrt{3}} \times \dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$= \dfrac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2} = \dfrac{\sqrt{2}-\sqrt{3}}{2-3}$

$= \dfrac{1.4142-1.732}{-1} = \mathbf{0.317}$

(*v*) $\dfrac{87}{7-2\sqrt{5}} = \dfrac{87}{(7-2\sqrt{5})} \times \dfrac{(7+2\sqrt{5})}{(7+2\sqrt{5})}$

$= \dfrac{87(7+2\sqrt{5})}{(7)^2-(2\sqrt{5})^2}$

$= \dfrac{87(7+2\sqrt{5})}{49-20}$

$= \dfrac{87(7+2\sqrt{5})}{29}$

$= 3[7+2(2.236)]$
$= 3(7+4.472)$
$= 3(11.472)$
$= \mathbf{34.416}$

(*vi*) $\dfrac{25}{\sqrt{40}-\sqrt{80}} = \dfrac{25}{2\sqrt{10}-4\sqrt{5}}$

$= \dfrac{25}{(2\sqrt{10}-4\sqrt{5})} \times \dfrac{(2\sqrt{10}+4\sqrt{5})}{(2\sqrt{10}+4\sqrt{5})}$

$= \dfrac{25(2\sqrt{10}+4\sqrt{5})}{(2\sqrt{10})^2-(4\sqrt{5})^2}$

$= \dfrac{25(2\sqrt{10}+4\sqrt{5})}{40-80}$

$= \dfrac{25(2\sqrt{10}+4\sqrt{5})}{-40}$

$= \dfrac{-5(2\sqrt{10}+4\sqrt{5})}{8}$

$= \dfrac{-5[2(3.162)+4(2.236)]}{8}$

$= \dfrac{-5(6.324+8.944)}{8}$

$= \dfrac{-5(15.268)}{8}$

$= \dfrac{-76.340}{8}$

$= \mathbf{-9.542}$

(*vii*) $\dfrac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-\sqrt{6}} = \dfrac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-\sqrt{6})(3\sqrt{5}+2\sqrt{6})}$

$= \dfrac{6\sqrt{30}+24-15-2\sqrt{30}}{(3\sqrt{5})^2-(2\sqrt{6})^2}$

$= \dfrac{4\sqrt{30}+9}{45-24}$

$= \dfrac{4\sqrt{30}+9}{21}$

$= \dfrac{4\sqrt{10}\sqrt{3}+9}{21}$

$= \dfrac{4(3.162)\,(1.732)+9}{21}$

$= \dfrac{30.906336}{21}$

$= \mathbf{1.471}$

(*viii*) $\dfrac{1}{\sqrt{3}-\sqrt{2}-1} = \dfrac{1}{(\sqrt{3}-\sqrt{2})-1} \times \dfrac{(\sqrt{3}-\sqrt{2})+1}{(\sqrt{3}-\sqrt{2})+1}$

$= \dfrac{\sqrt{3}-\sqrt{2}+1}{(\sqrt{3}-\sqrt{2})^2-(1)^2}$

$= \dfrac{\sqrt{3}-\sqrt{2}+1}{3+2-2\sqrt{6}-1}$

$= \dfrac{\sqrt{3}-\sqrt{2}+1}{4-2\sqrt{6}}$

$= \dfrac{\sqrt{3}-\sqrt{2}+1}{2(2-\sqrt{6})} \times \dfrac{(2+\sqrt{6})}{(2+\sqrt{6})}$

$= \dfrac{2\sqrt{3}+\sqrt{18}-2\sqrt{2}-\sqrt{12}+2+\sqrt{6}}{2(4-6)}$

$= \dfrac{2\sqrt{3}+3\sqrt{2}-2\sqrt{2}-2\sqrt{3}+2+\sqrt{6}}{2(-2)}$

$= \dfrac{\sqrt{2}+\sqrt{6}+2}{-4}$

$= \dfrac{\sqrt{2}+(\sqrt{2}\sqrt{3})+2}{-4}$

$= \dfrac{\sqrt{2}(1+\sqrt{3}+\sqrt{2})}{-4}$

$= \dfrac{1.4142\,(1+1.732+1.4142)}{-4}$

$= \dfrac{1.4142(4.1462)}{-4}$

$= \dfrac{5.863556}{-4}$

$= \mathbf{-1.465}$

(*ix*) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} + \dfrac{2-\sqrt{3}}{2+\sqrt{3}} + \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \dfrac{(2+\sqrt{3})}{(2-\sqrt{3})} \times \dfrac{(2+\sqrt{3})}{(2+\sqrt{3})} + \dfrac{(2-\sqrt{3})}{(2+\sqrt{3})} \times \dfrac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$+ \dfrac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \dfrac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= \dfrac{(2+\sqrt{3})^2}{4-3} + \dfrac{(2-\sqrt{3})^2}{4-3} + \dfrac{(\sqrt{3}-1)^2}{3-1}$

$= \dfrac{4+3+4\sqrt{3}}{1} + \dfrac{4+3-4\sqrt{3}}{1} + \dfrac{3+1-2\sqrt{3}}{2}$

$= 7 + 4\sqrt{3} + 7 - 4\sqrt{3} + \frac{4 - 2\sqrt{3}}{2}$

$= 14 + 2 - \sqrt{3}$

$= 16 - \sqrt{3}$

$= 16 - 1.732$

$=$ **14.268**

14. $x = 7 + \sqrt{40} = 7 + 2\sqrt{10}$

$\Rightarrow \frac{1}{x} = \frac{1}{7 + 2\sqrt{10}}$

$= \frac{1}{7 + 2\sqrt{10}} \times \frac{7 - 2\sqrt{10}}{7 - 2\sqrt{10}}$

$= \frac{7 - 2\sqrt{10}}{(7)^2 - (2\sqrt{10})^2}$

$= \frac{7 - 2\sqrt{10}}{49 - 40}$

$= \frac{7 - 2\sqrt{10}}{9}$

$x + \frac{1}{x} = 7 + 2\sqrt{10} + \frac{7 - 2\sqrt{10}}{9}$

$= \frac{63 + 18\sqrt{10} + 7 - 2\sqrt{10}}{9}$

$= \frac{70 + 16\sqrt{10}}{9}$ … (1)

Now, $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 + 2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)$

$= x + \frac{1}{x} + 2$

$= \frac{70 + 16\sqrt{10}}{9} + 2$ [Using (1)]

$= \frac{70 + 16\sqrt{10} + 18}{9}$

$= \frac{88 + 16\sqrt{10}}{9}$

$\therefore \sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{\frac{88 + 16\sqrt{10}}{9}}$

$= \sqrt{\frac{4(22 + 4\sqrt{10})}{9}}$

$= \frac{2}{3}\sqrt{22 + 4\sqrt{10}}$

$= \frac{2}{3}\sqrt{20 + 2 + 4\sqrt{10}}$

$= \frac{2}{3}\sqrt{(\sqrt{20})^2 + (\sqrt{2})^2 + 2\sqrt{20}\sqrt{2}}$

$= \frac{2}{3}\sqrt{(\sqrt{20} + \sqrt{2})^2}$

$= \frac{2}{3}\sqrt{(2\sqrt{5} + \sqrt{2})^2}$

$= \mathbf{\frac{2}{3}(2\sqrt{5} + \sqrt{2})}$

15. $x = 9 - 4\sqrt{5}$

$\Rightarrow \frac{1}{x} = \frac{1}{9 - 4\sqrt{5}}$

$= \frac{1}{(9 - 4\sqrt{5})} \times \frac{(9 + 4\sqrt{5})}{(9 + 4\sqrt{5})}$

$= \frac{9 + 4\sqrt{5}}{(9)^2 - (4\sqrt{5})^2}$

$= \frac{9 + 4\sqrt{5}}{81 - 80}$

$= 9 + 4\sqrt{5}$

$\therefore x + \frac{1}{x} = 9 - 4\sqrt{5} + 9 + 4\sqrt{5} = 18$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 18^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 324$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 324$

$\Rightarrow x^2 + \frac{1}{x^2} =$ **322**

16. (*i*) $x = \frac{5 - \sqrt{21}}{2}$

$\Rightarrow \frac{1}{x} = \frac{1}{\frac{5 - \sqrt{21}}{2}}$

$= \frac{2}{(5 - \sqrt{21})} \times \frac{(5 + \sqrt{21})}{(5 + \sqrt{21})}$

$= \frac{2(5 + \sqrt{21})}{25 - 21}$

$= \frac{5 + \sqrt{21}}{2}$

$x + \frac{1}{x} = \frac{5 - \sqrt{21}}{2} + \frac{5 + \sqrt{21}}{2}$

$= \frac{5 - \sqrt{21} + 5 + \sqrt{21}}{2}$

$= \frac{10}{2} = 5$

(*ii*) $\left(x + \frac{1}{x}\right)^2 = 5^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 25$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = 25 - 2$

= 23

17.

$$x = \frac{1}{2-\sqrt{3}}$$
$$= \frac{1}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$
$$= \frac{(2+\sqrt{3})}{(2)^2-(\sqrt{3})^2}$$
$$= \frac{2+\sqrt{3}}{4-3}$$
$$= 2+\sqrt{3}$$
$$x = 2+\sqrt{3}$$
$$\Rightarrow \quad x-2 = \sqrt{3}$$
$$\Rightarrow \quad (x-2)^2 = (\sqrt{3})^2$$
$$\Rightarrow \quad x^2-4x+4 = 3$$
$$\Rightarrow \quad x^2-4x+4-3 = 0$$
$$\Rightarrow \quad x^2-4x+1 = 0$$

Now, $x^3-2x^2-7x+4 = x(x^2-4x+1)+2(x^2-4x+1)+2$

$$= x \times 0 + 2 \times 0 + 2 = \mathbf{2}$$

18. $x = \dfrac{\sqrt{2}+1}{\sqrt{2}-1} = \dfrac{(\sqrt{2}+1)}{(\sqrt{2}-1)} \times \dfrac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \dfrac{(\sqrt{2}+1)^2}{(\sqrt{2})^2-(1)^2}$

$$= \frac{2+1+2\sqrt{2}}{2-1} = 3+2\sqrt{2}$$

$y = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \dfrac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2}$

$$= \frac{2+1-2\sqrt{2}}{2-1} = 3-2\sqrt{2}$$

x^2+y^2+xy

$$= (3+2\sqrt{2})^2 + (3-2\sqrt{2})^2 + (3+2\sqrt{2})(3-2\sqrt{2})$$
$$= 9+4(2)+12\sqrt{2}+9+4(2)-12\sqrt{2}+9-8$$
$$= 27+8 = \mathbf{35}$$

19. $x = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})} \times \dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} = \frac{3+2-2\sqrt{6}}{1} = 5-2\sqrt{6}$$

$y = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \times \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} = \frac{3+2-2\sqrt{6}}{1} = 5+2\sqrt{6}$$

x^2+xy+y^2

$$= (5-2\sqrt{6})^2 + (5-2\sqrt{6})(5+2\sqrt{6}) + (5+2\sqrt{6})^2$$
$$= 25+24-20\sqrt{6}+25-24+25+24+20\sqrt{6}$$
$$= \mathbf{99}$$

20. $a = \dfrac{(\sqrt{5}+\sqrt{10})}{(\sqrt{10}-\sqrt{5})} \times \dfrac{(\sqrt{10}+\sqrt{5})}{(\sqrt{10}+\sqrt{5})}$

$$= \frac{(\sqrt{5}+\sqrt{10})^2}{10-5}$$
$$= \frac{(\sqrt{5}+\sqrt{10})^2}{5}$$
$$\Rightarrow \quad \sqrt{a} = \frac{(\sqrt{5}+\sqrt{10})}{\sqrt{5}}$$

$b = \dfrac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}} = \dfrac{(\sqrt{10}-\sqrt{5})}{(\sqrt{10}+\sqrt{5})} \times \dfrac{(\sqrt{10}-\sqrt{5})}{(\sqrt{10}-\sqrt{5})}$

$$= \frac{(\sqrt{10}-\sqrt{5})^2}{10-5} = \frac{(\sqrt{10}-\sqrt{5})^2}{5}$$
$$\Rightarrow \quad \sqrt{b} = \frac{(\sqrt{10}-\sqrt{5})}{\sqrt{5}}$$

LHS $= \sqrt{a}-\sqrt{b}-2\sqrt{ab} = \sqrt{a}-\sqrt{b}-2\sqrt{a}\sqrt{b}$

$$= \frac{(\sqrt{5}+\sqrt{10})}{\sqrt{5}} - \frac{(\sqrt{10}-\sqrt{5})}{\sqrt{5}} - 2\left(\frac{\sqrt{5}+\sqrt{10}}{\sqrt{5}}\right)\left(\frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}\right)$$
$$= \frac{(\sqrt{5}+\sqrt{10})}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} - \frac{(\sqrt{10}-\sqrt{5})}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} - \frac{2(10-5)}{5}$$
$$= \frac{(5+\sqrt{50})}{5} - \frac{(\sqrt{50}-5)}{5} - \frac{2(5)}{5}$$
$$= \frac{5+5\sqrt{2}-5\sqrt{2}+5}{5} - 2 = \frac{10}{5} - 2 = 2-2 = 0 = \text{RHS}$$

EXERCISE 1G

1. (*i*) $2^{\frac{2}{3}} \times 2^{\frac{1}{3}} = 2^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 2^{\frac{3}{3}} = 2^1 = \mathbf{2}$

(*ii*) $3^{\frac{5}{6}} \times 3^{\frac{2}{3}} = 3^{\left(\frac{5}{6}+\frac{2}{3}\right)} = 3^{\left(\frac{5+4}{6}\right)} = 3^{\frac{9}{6}} = \mathbf{3^{\frac{3}{2}}}$

(*iii*) $4^{\frac{2}{5}} \times 4^{\frac{-2}{5}} = 4^{\frac{2}{5}-\frac{2}{5}} = 4^0 = \mathbf{1}$

2. (*i*) $\dfrac{3^{\frac{1}{3}}}{3^{\frac{1}{6}}} = 3^{\frac{1}{3}-\frac{1}{6}} = 3^{\frac{2-1}{6}} = \mathbf{3^{\frac{1}{6}}}$

(*ii*) $\dfrac{4^{\frac{6}{7}}}{4^{\frac{2}{3}}} = 4^{\frac{6}{7}-\frac{2}{3}} = 4^{\frac{18-14}{21}} = \mathbf{4^{\frac{4}{21}}}$

(*iii*) $\dfrac{7^{\frac{1}{4}}}{7^{\frac{1}{5}}} = 7^{\frac{1}{4}-\frac{1}{5}} = 7^{\frac{5-4}{20}} = \mathbf{7^{\frac{1}{20}}}$

3. (*i*) $2^{\frac{1}{4}} \times 3^{\frac{1}{4}} = (2 \times 3)^{\frac{1}{4}} = \mathbf{6^{\frac{1}{4}}}$

(*ii*) $3^{\frac{5}{8}} \times 5^{\frac{5}{8}} = (3 \times 5)^{\frac{5}{8}} = \mathbf{15^{\frac{5}{8}}}$

(*iii*) $7^{\frac{2}{3}} \times 3^{\frac{2}{3}} = (7 \times 3)^{\frac{2}{3}} = \mathbf{21^{\frac{2}{3}}}$

4. (*i*) $(5^3)^{\frac{1}{3}} = 5^{3 \times \frac{1}{3}} = 5^1 = \mathbf{5}$

(*ii*) $\left(7^{\frac{1}{3}}\right)^4 = 7^{\frac{1}{3} \times 4} = \mathbf{7^{\frac{4}{3}}}$

(*iii*) $\left(3^{\frac{1}{5}}\right)^{10} = 3^{\frac{1}{5} \times 10} = \mathbf{3^2}$

5. (*i*) $(36)^{\frac{1}{2}} = (6^2)^{\frac{1}{2}} = 6^{2\times\frac{1}{2}} = 6^1 = \mathbf{6}$

(*ii*) $(27)^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^{3\times\frac{1}{3}} = 3^1 = \mathbf{3}$

(*iii*) $(32)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2^1 = \mathbf{2}$

6. (*i*) $(81)^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^{4\times\frac{3}{4}} = 3^3 = \mathbf{27}$

(*ii*) $(243)^{\frac{3}{5}} = (3^5)^{\frac{3}{5}} = 3^{5\times\frac{3}{5}} = 3^3 = \mathbf{27}$

(*iii*) $(16)^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 2^{4\times\frac{5}{4}} = 2^5 = \mathbf{32}$

7. (*i*) $(27)^{\frac{-1}{3}} = (3^3)^{\frac{-1}{3}} = 3^{3\times\left(-\frac{1}{3}\right)} = 3^{-1} = \mathbf{\frac{1}{3}}$

(*ii*) $(512)^{\frac{-1}{9}} = (8^3)^{\frac{-1}{9}} = 8^{3\times\left(\frac{-1}{9}\right)} = 8^{\frac{-3}{9}} = 8^{\frac{-1}{3}}$

$= (2^3)^{\frac{-1}{3}} = 2^{3\left(\frac{-1}{3}\right)} = 2^{-1} = \mathbf{\frac{1}{2}}$

(*iii*) $(625)^{\frac{-3}{4}} = (5^4)^{\frac{-3}{4}} = 5^{4\left(\frac{-3}{4}\right)} = 5^{-3} = \frac{1}{5^3} = \mathbf{\frac{1}{125}}$

(*iv*) $\left(\frac{27}{64}\right)^{\frac{-2}{3}} = \left[\left(\frac{3}{4}\right)^3\right]^{\frac{-2}{3}} = \left(\frac{3}{4}\right)^{3\left(\frac{-2}{3}\right)} = \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$

$= \mathbf{\frac{16}{9}}$

(*v*) $\left(\frac{64}{125}\right)^{\frac{-2}{3}} = \left[\left(\frac{4}{5}\right)^3\right]^{\frac{-2}{3}} = \left(\frac{4}{5}\right)^{3\left(\frac{-2}{3}\right)} = \left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{4}\right)^2$

$= \mathbf{\frac{25}{16}}$

(*vi*) $\left(\frac{81}{16}\right)^{\frac{-3}{4}} = \left[\left(\frac{3}{2}\right)^4\right]^{\frac{-3}{4}} = \left(\frac{3}{2}\right)^{4\left(\frac{-3}{4}\right)} = \left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3$

$= \mathbf{\frac{8}{27}}$

8. (*i*) $(a^b + b^a)^{-1} = (2^3 + 3^2)^{-1} = (8 + 9)^{-1} = (17)^{-1} = \mathbf{\frac{1}{17}}$

(*ii*) $(a^a + b^b)^{-1} = (2^2 + 3^3)^{-1} = (4 + 27)^{-1} = (31)^{-1} = \mathbf{\frac{1}{31}}$

9. (*i*) $\frac{(243)^{\frac{3}{5}}\times 25^{\frac{3}{2}}}{(625)^{\frac{1}{2}}\times(8)^{\frac{4}{3}}\times(16)^{\frac{5}{4}}} = \frac{(3^5)^{\frac{3}{5}}\times(5^2)^{\frac{3}{2}}}{(25^2)^{\frac{1}{2}}\times(2^3)^{\frac{4}{3}}\times(2^4)^{\frac{5}{4}}}$

$= \frac{3^{5\times\frac{3}{5}}\times 5^{2\times\frac{3}{2}}}{25^{2\times\frac{1}{2}}\times 2^{3\times\frac{4}{3}}\times 2^{4\times\frac{5}{4}}}$

$= \frac{3^3\times 5^3}{25\times 2^4\times 2^5}$

$= \frac{3^3\times 5^3}{5^2\times 2^4\times 2^5}$

$= \frac{3^3\times 5^{3-2}}{2^{4+5}}$

$= \frac{3^3\times 5}{2^9}$

$= \frac{27\times 5}{512}$

$= \mathbf{\frac{135}{512}}$

(*ii*) $\frac{3^{40}+3^{39}}{3^{41}-3^{40}} = \frac{3^{39}(3+1)}{3^{40}(3-1)} = \frac{3^{39}(4)}{3^{40}(2)} = \frac{2}{3^{40-39}} = \mathbf{\frac{2}{3}}$

(*iii*) $27^{-\frac{1}{3}}\left[27^{\frac{1}{3}} - 27^{\frac{2}{3}}\right] = (3^3)^{\frac{-1}{3}}\left[(3^3)^{\frac{1}{3}} - (3^3)^{\frac{2}{3}}\right]$

$= 3^{3\left(\frac{-1}{3}\right)}\left[3^{3\times\frac{1}{3}} - 3^{3\times\frac{2}{3}}\right]$

$= 3^{-1}\left[3^1 - 3^2\right]$

$= \frac{1}{3}(3-9)$

$= \frac{1}{3}(-6)$

$= \mathbf{-2}$

(*iv*) $\frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 4^{\frac{1}{4}}}{10^{\frac{-1}{5}}\times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}}\times 5^{\frac{-7}{5}}}{4^{\frac{-3}{5}}\times 6}$

$= \frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times(2^2)^{\frac{1}{4}}}{(10)^{\frac{-1}{5}}\times 5^{\frac{3}{5}}} \times \frac{(2^2)^{\frac{-3}{5}}\times 6}{3^{\frac{4}{3}}\times 5^{\frac{-7}{5}}}$

$= \frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 2^{\frac{1}{2}}\times 2^{\frac{-6}{5}}\times 2^1\times 3^1}{2^{\frac{-1}{5}}\times 5^{\frac{-1}{5}}\times 5^{\frac{3}{5}}\times 3^{\frac{4}{3}}\times 5^{\frac{-7}{5}}}$

$= \frac{2^{\frac{1}{2}+\frac{1}{2}-\frac{6}{5}+1}\times 3^{\frac{1}{3}+1}}{2^{\frac{-1}{5}}\times 5^{\frac{-1}{5}+\frac{3}{5}-\frac{7}{5}}\times 3^{\frac{4}{3}}}$

$= \frac{2^{\frac{4}{5}}\times 3^{\frac{4}{3}}}{2^{\frac{-1}{5}}\times 5^{\frac{-5}{5}}\times 3^{\frac{4}{3}}}$

$= \frac{2^{\frac{4}{5}+\frac{1}{5}}}{5^{-1}}$

$= 2^{\frac{5}{5}}\times 5^1$

$= 2\times 5$

$= \mathbf{10}$

(*v*) $\frac{3}{(625)^{\frac{-1}{4}}} + \frac{2}{(343)^{\frac{-2}{3}}} + \frac{4}{(243)^{\frac{-1}{5}}}$

$= \frac{3}{(5^4)^{\frac{-1}{4}}} + \frac{2}{(7^3)^{\frac{-2}{3}}} + \frac{4}{(3^5)^{-\frac{1}{5}}}$

$= \frac{3}{5^{4\left(-\frac{1}{4}\right)}} + \frac{2}{7^{3\left(-\frac{2}{3}\right)}} + \frac{4}{3^{5\left(\frac{-1}{5}\right)}}$

$$= \frac{3}{5^{-1}} + \frac{2}{7^{-2}} + \frac{4}{3^{-1}}$$
$$= 3 \times 5 + 2 \times 7^2 + 4 \times 3$$
$$= 15 + 98 + 12$$
$$= \mathbf{125}$$

(*vi*) $\left(\frac{64}{125}\right)^{\frac{-2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{64}}$

$$= \left[\left(\frac{4}{5}\right)^3\right]^{\frac{-2}{3}} + \frac{1}{\left[\left(\frac{4}{5}\right)^4\right]^{\frac{1}{4}}} + \frac{5}{4}$$
$$= \left(\frac{4}{5}\right)^{3\left(\frac{-2}{3}\right)} + \frac{1}{\left(\frac{4}{5}\right)^{4\times\frac{1}{4}}} + \frac{5}{4}$$
$$= \left(\frac{4}{5}\right)^{-2} + \frac{1}{\left(\frac{4}{5}\right)} + \frac{5}{4}$$
$$= \left(\frac{5}{4}\right)^{2} + \frac{5}{4} + \frac{5}{4}$$
$$= \frac{5}{4}\left(\frac{5}{4} + 1 + 1\right)$$
$$= \frac{5}{4}\left(\frac{5+4+4}{4}\right)$$
$$= \frac{5}{4}\left(\frac{13}{4}\right)$$
$$= \mathbf{\frac{65}{16}}$$

10. (*i*) $(2^3)^4 = (2^2)^p$

$\Rightarrow (2)^{3\times4} = 2^{2p}$

$\Rightarrow 2^{12} = 2^{2p}$

$\Rightarrow 12 = 2p$

$\Rightarrow p = \frac{12}{2} = 6$

Hence, $\boldsymbol{p = 6}$.

(*ii*) $27^p = \frac{9}{3^p}$

$\Rightarrow (3^3)^p = \frac{3^2}{3^p}$

$\Rightarrow 3^{3p} = 3^{2-p}$

$\Rightarrow 3p = 2 - p$

$\Rightarrow 3p + p = 2$

$\Rightarrow 4p = 2$

$\Rightarrow p = \frac{2}{4}$

$\Rightarrow p = \frac{1}{2}$

Hence, $\boldsymbol{p = \frac{1}{2}}$.

(*iii*) $5^{p-4} \times 2^{p-5} = 5$

$\Rightarrow \frac{5^{p-4} \times 2^{p-5}}{5} = 1$

$\Rightarrow 5^{p-4-1} \times 2^{p-5} = 1$

$\Rightarrow 5^{p-5} \times 2^{p-5} = 1$

$\Rightarrow (5 \times 2)^{p-5} = 1$

$\Rightarrow 10^{p-5} = 1$

$\Rightarrow 10^{p-5} = 10^{\circ}$

$\Rightarrow p - 5 = 0$

$\Rightarrow p = 0 + 5 = 5$

Hence, $\boldsymbol{p} = 5$.

(*iv*) $\left(\frac{3}{5}\right)^p \left(\frac{5}{3}\right)^{2p} = \frac{125}{27}$

$\Rightarrow \left(\frac{3}{5}\right)^p \left(\frac{5}{3}\right)^{2p} = \left(\frac{5}{3}\right)^3$

$\Rightarrow \left(\frac{5}{3}\right)^{-p} \left(\frac{5}{3}\right)^{2p} = \left(\frac{5}{3}\right)^3$

$\Rightarrow \left(\frac{5}{3}\right)^{2p-p} = \left(\frac{5}{3}\right)^3$

$\Rightarrow \left(\frac{5}{3}\right)^{p} = \left(\frac{5}{3}\right)^3$

$\Rightarrow p = 3$

Hence, $\boldsymbol{p} = 3$.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (*c*) Every rational number is **a real number.**
Real numbers consist of rational and irrational numbers. All natural numbers, whole numbers and integers are rational numbers.

2. (*d*) Decimal representation of a rational number cannot be **non-terminating non-repeating.**
Decimal representation of a rational number is either terminating or non-terminating repeating.

3. (*d*) π is an irrational number because its decimal expansion is **non-terminating non-repeating.**
Decimal expansion of an irrational number is always non-terminating non-repeating.

4. (*a*) Every point an a number line represents a **unique real number.**
Every point on a number line represents a unique number which may be rational or irrational.

5. (*a*) $\mathbf{\frac{-2}{3}}$

Number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is a rational number.

6. (*d*) **0.5015001500015…** is an irrational number because its decimal expansion is non-terminating non-repeating.

7. (*b*) $\mathbf{\frac{6}{34}}$

$\because \quad \frac{3}{17} = \frac{3 \times 2}{17 \times 2} = \frac{6}{34}$

8. (*c*) $\mathbf{\frac{5}{2}}$

$\frac{5}{2}$ = 2.5 is a rational number lying between 2 and 3.

The other options given are all irrational numbers.

9. (*a*) 3.1, 3.2, 3.8, 3.9 are respectively equal to $\frac{31}{10}$, $\frac{32}{10}$, $\frac{38}{10}$ and $\frac{39}{10}$.

So, they are all rational numbers and they lie between 3 and 4.

10. (*a*) $\mathbf{-\sqrt{2}}$

$3 + \sqrt{2} + (-\sqrt{2}) = 3 + \sqrt{2} - \sqrt{2} = 3$ (rational number)

So, $-\sqrt{2}$ is the smallest irrational number that should be added to $3 + \sqrt{2}$ to get a rational number.

11. (*c*) $\mathbf{\frac{1}{3}}$

Let $x = 0.\bar{3}$

$\Rightarrow \quad x = 0.3333\ldots \quad \ldots (1)$

and $\quad 10x = 3.3333\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get $9x = 3$

$\Rightarrow \quad x = \frac{3}{9} = \frac{1}{3}$

Hence, $\quad 0.\bar{3} = \frac{1}{3}$.

12. (*b*) $\mathbf{\frac{29}{90}}$

Let, $\quad x = 0.3\bar{2}$

$\Rightarrow \quad x = 0.32222\ldots;$

$10x = 3.2222\ldots \quad \ldots (1)$

and $\quad 100x = 32.2222\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get $90x = 29$

$\Rightarrow \quad x = \frac{29}{90}$

Hence, $0.3\bar{2} = \frac{29}{90}$.

13. (*d*) $\mathbf{\frac{437}{999}}$

Let $\quad x = 0.\overline{437}$

$\Rightarrow \quad x = 0.437437437\ldots \quad \ldots (1)$

$\Rightarrow \quad 1000x = 437.437437\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$999x = 437$

$\Rightarrow \quad x = \frac{437}{999}$

Hence, $0.\overline{437} = \frac{437}{999}$.

14. (*a*) $\mathbf{\sqrt[3]{25}}$

$$\sqrt[3]{40} = \sqrt[3]{2 \times 2 \times 2 \times 5} = 2\sqrt[3]{5}$$

Now, $\quad 2\sqrt[3]{5} \times \sqrt[3]{5^2} = 2 \times \sqrt[3]{5 \times 5 \times 5} = 2 \times 5 = 10$

$\therefore \quad \sqrt[3]{5^2}$, i.e. $\sqrt[3]{25}$ is the simplest rationalisation factor of $\sqrt[3]{40}$.

15. (*c*) $\mathbf{3\sqrt{5}}$

$$2\sqrt{5} + \sqrt{5} = \sqrt{5}\,(2 + 1) = \sqrt{5}\,(3) = 3\sqrt{5}$$

16. (*b*) $\mathbf{12\sqrt{5}}$

$$3\sqrt{5} + 4\sqrt{5} + 5\sqrt{5} = \sqrt{5}\,(3 + 4 + 5) = \sqrt{5}\,(12) = 12\sqrt{5}$$

17. (*a*) $\mathbf{\sqrt{5} + \sqrt{2} + 5\sqrt{11}}$

$$\left(\frac{2}{3}\sqrt{5} - \frac{1}{2}\sqrt{2} + 6\sqrt{11}\right) + \left(\frac{1}{3}\sqrt{5} + \frac{3}{2}\sqrt{2} - \sqrt{11}\right)$$

$$= \frac{2}{3}\sqrt{5} + \frac{1}{3}\sqrt{5} - \frac{1}{2}\sqrt{2} + \frac{3}{2}\sqrt{2} + 6\sqrt{11} - \sqrt{11}$$

$$= \sqrt{5}\left(\frac{2}{3} + \frac{1}{3}\right) + \sqrt{2}\left(-\frac{1}{2} + \frac{3}{2}\right) + \sqrt{11}\,(6 - 1)$$

$$= \sqrt{5}\left(\frac{3}{3}\right) + \sqrt{2}\left(\frac{2}{2}\right) + \sqrt{11}(5)$$

$$= \sqrt{5} + \sqrt{2} + 5\sqrt{11}$$

18. (*c*) $\mathbf{\sqrt[6]{6125}}$

$$\sqrt[3]{7} \times \sqrt{5} = \sqrt[6]{7^2} \times \sqrt[6]{5^3} = \sqrt[6]{49 \times 125} = \sqrt[6]{6125}$$

19. (*a*) **1**

$$\frac{1}{6}\sqrt{18} \times \frac{1}{3}\sqrt{18} = \frac{1}{6} \times \frac{1}{3} \times \sqrt{18 \times 18} = \frac{1}{18} \times 18 = 1$$

20. (*c*) **105**

$$\sqrt{5} \times \sqrt{7} \times \sqrt{15} \times \sqrt{21} = \sqrt{5 \times 7 \times 5 \times 3 \times 3 \times 7} = 3 \times 5 \times 7 = 105$$

21. (*c*) **6**

$$(3 + \sqrt{3})\ (3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6$$

22. (*b*) **16**

$$(3 + \sqrt{5})^2\ (3 - \sqrt{5})^2 = (9 + 5 + 6\sqrt{5})\ (9 + 5 - 6\sqrt{5})$$
$$= (14 + 6\sqrt{5})\ \ (14 - 6\sqrt{5})$$
$$= (14)^2 - (6\sqrt{5})^2$$
$$= 196 - 180$$
$$= 16$$

23. (*a*) $\mathbf{\sqrt[3]{25}}$

$$\sqrt[3]{250} \div \sqrt[3]{10} = \sqrt[3]{\frac{250}{10}} = \sqrt[3]{25}$$

24. (*c*) $\mathbf{\frac{10}{\sqrt{5}}}$

$$\frac{30}{\sqrt{20}+\sqrt{5}} = \frac{30}{2\sqrt{5}+\sqrt{5}} = \frac{30}{3\sqrt{5}} = \mathbf{\frac{10}{\sqrt{5}}}$$

25. (*c*) $\mathbf{2\sqrt{3}}$

$$\frac{6}{\sqrt{12}-\sqrt{3}} = \frac{6}{2\sqrt{3}-\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

26. (*a*) **2**

$$\frac{2^0+7^0}{5^0} = \frac{1+1}{1} = \frac{2}{1} = 2$$

27. (*d*) $\mathbf{\frac{3}{2}}$

$$\frac{2^{30}+2^{29}}{2^{31}-2^{30}} = \frac{2^{29}(2+1)}{2^{30}(2-1)} = \frac{2^{29}(3)}{2^{30}(1)}$$

$$= \frac{3}{2^{30-29}} = \frac{3}{2}$$

28. (*d*) $\mathbf{\frac{1}{3}}$

$$\sqrt{(3^{-2})} = \sqrt{\frac{1}{3^2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

29. (*c*) $\mathbf{\frac{125}{64}}$

$$\left(\frac{256}{625}\right)^{\frac{-3}{4}} = \left(\frac{625}{256}\right)^{\frac{3}{4}} = \left[\left(\frac{5}{4}\right)^4\right]^{\frac{3}{4}}$$

$$= \left(\frac{5}{4}\right)^{4\times\frac{3}{4}} = \frac{5^3}{4^3} = \frac{125}{64}$$

30. (*c*) $\mathbf{\frac{2}{5}}$

$$(32)^{\frac{1}{5}} \times (125)^{\frac{-1}{3}} = (2^5)^{\frac{1}{5}} \times (5^3)^{\frac{-1}{3}}$$

$$= 2^{5\times\frac{1}{5}} \times 5^{3\times\left(\frac{-1}{3}\right)}$$

$$= 2 \times 5^{-1} = \frac{2}{5}$$

31. (*b*) $\mathbf{-\frac{5}{3}}$

$$\frac{5^{n+2}-6\times 5^{n+1}}{13\times 5^n - 2\times 5^{n+1}} = \frac{5^{n+1}(5^1-6)}{5^n(13-2\times 5^1)}$$

$$= \frac{5^{n+1}(-1)}{5^n(13-10)}$$

$$= \frac{5(-1)}{3}$$

$$= \frac{-5}{3}$$

32. (*a*) $\mathbf{\frac{1}{2}}$

$$\left[8^{\frac{-4}{3}} \div 2^{-2}\right]^{\frac{1}{2}} = \left[\left(\frac{1}{8}\right)^{\frac{4}{3}} \div \left(\frac{1}{2}\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\left(\frac{1}{2^3}\right)^{\frac{4}{3}} \div \frac{1}{2^2}\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2^{3\times\frac{4}{3}}} \div \frac{1}{2^2}\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2^4} \div \frac{1}{2^2}\right]^{\frac{1}{2}} = \left[\frac{1}{2^4} \times 2^2\right]^{\frac{1}{2}}$$

$$= \left(\frac{1}{2^2}\right)^{\frac{1}{2}} = \frac{1}{2^{2\times\frac{1}{2}}} = \frac{1}{2}$$

33. (*b*) $\mathbf{x^{\frac{1}{6}}}$

$$\sqrt[4]{\sqrt[3]{x^2}} = \left[(x^2)^{\frac{1}{3}}\right]^{\frac{1}{4}}$$

$$= (x^{2\times\frac{1}{3}})^{\frac{1}{4}}$$

$$= (x^{\frac{2}{3}})^{\frac{1}{4}}$$

$$= x^{\frac{2}{3}\times\frac{1}{4}}$$

$$= x^{\frac{1}{6}}$$

34. (*b*) **17**

$$x^y + y^x = 2^3 + 3^2 = 8 + 9 = 17$$

35. (*c*) **18**

$$x = 9 - 4\sqrt{5}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}}$$

$$= \frac{9+4\sqrt{5}}{81-80}$$

$$= 9 + 4\sqrt{5}$$

$$\therefore \quad x + \frac{1}{x} = 9 - 4\sqrt{5} + 9 + 4\sqrt{5}$$

$$= 18$$

36. (*c*) $\mathbf{\left(\sqrt{a^5}\right)^{\frac{2}{5}}}$

$$\left(\sqrt{a^5}\right)^{\frac{2}{5}} = \left[(a^5)^{\frac{1}{2}}\right]^{\frac{2}{5}} = (a^{\frac{5}{2}})^{\frac{2}{5}}$$

$$= a^{\frac{5}{2}\times\frac{2}{5}} = a$$

37. (*a*) **–2.125**

$$\frac{17}{8} = 2.125$$

$$\Rightarrow \quad -\frac{17}{8} = -2.125$$

$$\begin{array}{r} 8\,\overline{)\,17\,}(\,2.125 \\ -16 \\ \hline 10 \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline \text{x} \\ \hline \end{array}$$

38. (*b*) **0.714285**

$$\frac{3}{7} = 0.\overline{428571}$$

$$\Rightarrow \quad \frac{1}{7} = 0.\overline{142857}$$

$$\Rightarrow \quad \frac{5}{7} = 0.714285$$

39. (*d*) **0.577**

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

$$= \frac{1.732}{3}$$

$= 0.577$ (*approx*)

40. (*b*) **0.707**

$$\sqrt{3} \div \sqrt{6} = \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2} = \frac{1.414}{2}$$

$= 0.707$

SHORT ANSWER QUESTIONS

1. Let $x = 0.12\overline{3}$

$\Rightarrow \quad x = 0.123333\ldots$

$\Rightarrow \quad 100x = 12.3333\ldots \quad \ldots (1)$

and $\quad 1000x = 123.3333\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$900x = 111$

$$\Rightarrow \quad x = \frac{111}{900} = \mathbf{\frac{37}{300}}$$

2. Let $x = 0.99999\ldots \quad \ldots (1)$

$\Rightarrow \quad 10x = 9.99999\ldots \quad \ldots (2)$

On subtracting (2) from (1), we get

$9x = 9$

$\Rightarrow \quad x = 1$

$\Rightarrow \quad$ **0.99999 = 1**

3. $\sqrt[3]{4}\,,\ \sqrt{3}\,,\ \sqrt[4]{6}$

$$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$$

$$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$$

$$\sqrt[4]{6} = \sqrt[12]{6^3} = \sqrt[12]{216}$$

$\because \quad \sqrt[12]{729} > \sqrt[12]{256} > \sqrt[12]{216}$

Hence, $\mathbf{\sqrt{3} > \sqrt[3]{4} > \sqrt[4]{6}}$.

4. Answers can vary. The answers given below are sample answers.

$-\frac{2}{3}$ and $\frac{1}{4}$

$$\text{1st rational number} = \frac{1}{2}\left(\frac{-2}{3} + \frac{1}{4}\right)$$

$$= \frac{1}{2}\left(\frac{-8+3}{12}\right)$$

$$= \frac{1}{2}\left(\frac{-5}{12}\right)$$

$$= \mathbf{\frac{-5}{24}}$$

$$\text{2nd rational number} = \frac{1}{2}\left(\frac{-2}{3} + \frac{-5}{24}\right)$$

$$= \frac{1}{2}\left(\frac{-16-5}{24}\right)$$

$$= \frac{1}{2}\left(\frac{-21}{24}\right)$$

$$= \frac{1}{2}\left(\frac{-7}{8}\right)$$

$$= \mathbf{\frac{-7}{16}}$$

5.

$$\frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{(\sqrt{7}-\sqrt{6})}{(\sqrt{7}+\sqrt{6})} \times \frac{(\sqrt{7}-\sqrt{6})}{(\sqrt{7}-\sqrt{6})}$$

$$= \frac{(\sqrt{7}-\sqrt{6})^2}{(\sqrt{7})^2-(\sqrt{6})^2}$$

$$= \frac{7+6-2\sqrt{42}}{7-6}$$

$$= \frac{13-2\sqrt{42}}{1}$$

$$= \mathbf{13 - 2\sqrt{42}}$$

6. $a = c^z,\ b = a^x,\ c = b^y$ [Given] $\quad \ldots (1)$

$a = c^z = (b^y)^z = [(a^x)^y]^z \quad$ [Using (1)]

$\Rightarrow \quad a = (a^{xy})^z$

$\Rightarrow \quad a^1 = a^{xyz}$

$\Rightarrow \quad$ $xyz = \mathbf{1}$

7.

$$\sqrt{\left(5^\circ + \frac{2}{3}\right)} = (0.6)^{2-3x}$$

$$\Rightarrow \quad \left(1 + \frac{2}{3}\right)^{\frac{1}{2}} = \left(\frac{6}{10}\right)^{2-3x}$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^{\frac{1}{2}} = \left(\frac{3}{5}\right)^{2-3x}$$

$$\Rightarrow \quad \left(\frac{3}{5}\right)^{-\frac{1}{2}} = \left(\frac{3}{5}\right)^{2-3x}$$

$$\Rightarrow \quad -\frac{1}{2} = 2 - 3x$$

$$\Rightarrow \quad 3x = 2 + \frac{1}{2} = \frac{5}{2}$$

$$\Rightarrow \quad x = \frac{5}{3 \times 2} = \mathbf{\frac{5}{6}}$$

8. $$\frac{1}{7} = 0.\overline{142857}$$

$$= 0.142857142857 \ldots$$

$$\therefore \quad \frac{2}{7} = 0.\overline{285714}$$

$$= 0.285714285714 \ldots$$

In decimal expansion of $\frac{1}{7}$, the first place of decimal is 1 and second place is 4.

In decimal expansion of $\frac{2}{7}$ the first place of decimal is 2 and second place is 8.

So, we can condiser number 0.150150015000150000 … (this is a sample answer).

9. $$\left[\sqrt{\frac{3}{5}}\right]^{x+1} = \frac{125}{27}$$

$$\Rightarrow \quad \left[\left(\frac{3}{5}\right)^{\frac{1}{2}}\right]^{x+1} = \left(\frac{5}{3}\right)^3$$

$$\Rightarrow \quad \left(\frac{3}{5}\right)^{\frac{1}{2}(x+1)} = \left(\frac{3}{5}\right)^{-3}$$

$$\Rightarrow \quad \frac{1}{2}(x+1) = -3$$

$$\Rightarrow \quad x + 1 = -6$$

$$\Rightarrow \quad x = -6 - 1$$

$$\Rightarrow \quad x = -7$$

10. $$64^{\frac{-1}{3}}\left[64^{\frac{1}{3}} - 65^{\frac{2}{3}}\right] = (4^3)^{\frac{-1}{3}}\left[4^{3\times\frac{1}{3}} - 4^{3\times\frac{2}{3}}\right]$$

$$= 4^{3\left(-\frac{1}{3}\right)}[4 - 4^2]$$

$$= 4^{-1}[4 - 16]$$

$$= \frac{1}{4}(-12)$$

$$= -3$$

11. Yes. e.g.

(*i*) $(5 + \sqrt{2}) + (5 - \sqrt{2}) = 10$ (Rational number)

$(5 + \sqrt{2})\ (5 - \sqrt{2}) = 25 - 2 = 23$ (Rational number)

(*ii*) $(3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$ (Rational number)

$(3 + \sqrt{3})\ (3 - \sqrt{3}) = 9 - 3 = 6$ (Rational number)

12. $$\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$$

$$\Rightarrow \quad \frac{3^{2x-8}}{3^2 \times 5^2} = \frac{1}{5^{x-3}}$$

$$\Rightarrow \quad 3^{2x-8} \times 5^{x-3} = 3^2 \times 5^2$$

$$\Rightarrow \quad 2x - 8 = 2 \text{ and } x - 3 = 2$$

$$\Rightarrow \quad 2x = 10 \text{ and } x = 2 + 3$$

$$\Rightarrow \quad x = 5 \text{ and } x = 5$$

Hence, $x = 5$.

13. (*i*) $x^2 = 5$

$\Rightarrow \quad x = \pm\sqrt{5}$ (irrational number)

(*ii*) $y^2 = 9$

$\Rightarrow \quad y = \pm 3$ (rational number)

(*iii*) $z^2 = 0.04$

$\Rightarrow \quad z = \pm 0.2$ (rational number)

(*iv*) $u^2 = \frac{17}{4}$

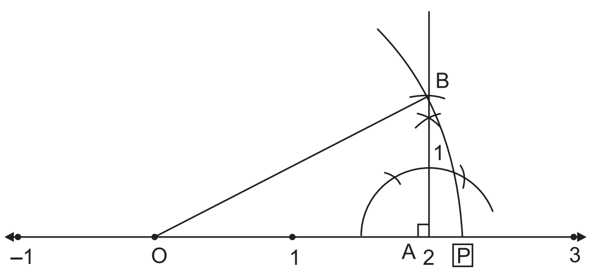
$\Rightarrow \quad u = \pm\sqrt{\frac{17}{4}}$

$= \pm\frac{\sqrt{17}}{2}$ (irrational number)

Rational numbers: ***y*** and ***z***.

Irrational numbers: ***x*** and ***u***.

14. Draw a number line and mark point O, representing zero, on it. Suppose A represents 2 on the number line. Then, OA = 2 units.



Draw AB ⊥ OA such that AB = 1 unit. Join OB. With centre O and radius equal to OB, draw an arc, meeting the number line at P.

Then, OP = OB = $\sqrt{5}$ units

Justification: In right ΔOAB, we have

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow \quad OB = \sqrt{OA^2 + AB^2}$$

$$= \sqrt{(2)^2 + (1)^2}$$

$$= \sqrt{5} \text{ units.}$$

15. $$\left(\frac{2}{3}\right)^x \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$$

$$\Rightarrow \quad \left(\frac{3}{2}\right)^{-x}\left(\frac{3}{2}\right)^{2x} = \left(\frac{3}{2}\right)^4$$

$$\Rightarrow \quad \left(\frac{3}{2}\right)^{-x+2x} = \left(\frac{3}{2}\right)^4$$

$$\Rightarrow \quad \left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^4$$

$$\boldsymbol{x = 4}$$

VALUE BASED QUESTIONS

1. $\dfrac{\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \dfrac{\sqrt{2}}{(\sqrt{5}+\sqrt{3})} \times \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} = \dfrac{\sqrt{2}(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \dfrac{\sqrt{2}(\sqrt{5}-\sqrt{3})}{5-3} = \mathbf{\dfrac{\sqrt{10}-\sqrt{6}}{2}}$

and $\sqrt{28} + \sqrt{98} + \sqrt{147}$

$= \sqrt{4\times 7} + \sqrt{2\times 7\times 7} + \sqrt{3\times 7\times 7}$

$= \mathbf{2\sqrt{7} + 7\sqrt{2} + 7\sqrt{3}}$

Helpfulness.

2. $\dfrac{1}{\sqrt{7}-\sqrt{3}} = \dfrac{1}{(\sqrt{7}-\sqrt{3})} \times \dfrac{(\sqrt{7}+\sqrt{3})}{(\sqrt{7}+\sqrt{3})} = \dfrac{\sqrt{7}+\sqrt{3}}{(\sqrt{7})^2-(\sqrt{3})^2}$

$= \dfrac{\sqrt{7}+\sqrt{3}}{7-3} = \mathbf{\dfrac{\sqrt{7}+\sqrt{3}}{4}}$

Concern for classmate facing difficulty in simplifying the given mathematical expression and helpfulness.

UNIT TEST

1. (*d*) Non-terminating non-recurring.

$\because$ $\sqrt{2}$ is an irrational number and the decimal expansion of an irrational number is non-terminating non-recurring.

2. (*b*) An irrational number

$-7 + 4\sqrt{7} - 3\sqrt{7} = -7 + \sqrt{7}$, which is an irrational number.

3. (*b*) $\mathbf{\dfrac{3}{10}}$

$\dfrac{1}{3} \times \dfrac{3}{10} = \dfrac{1}{10} = 0.1$

4. (*b*) $\mathbf{\dfrac{14}{11}}$

Let $x = 1.\overline{27}$

$\Rightarrow$ $x = 1.272727\ldots$ … (1)

and $100x = 127.272727\ldots$ … (2)

On subtracting (1) from (2), we get

$99x = 126$

$\Rightarrow$ $x = \dfrac{126}{99} = \dfrac{14}{11}$

5. (*c*) $\mathbf{\sqrt{3} + \sqrt{2}}$

$\dfrac{1}{\sqrt{27}-\sqrt{18}} = \dfrac{1}{3\sqrt{3}-3\sqrt{2}}$

$= \dfrac{1}{3(\sqrt{3}-\sqrt{2})} \times \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \dfrac{(\sqrt{3}+\sqrt{2})}{3(3-2)}$

$= \dfrac{(\sqrt{3}+\sqrt{2})}{3}$

6. (*c*) $\mathbf{\dfrac{7}{9}}$

Let $x = 0.\overline{3}$

$\Rightarrow$ $x = 0.3333\ldots$ … (1)

and $10x = 3.3333\ldots$ … (2)

On subtracting (1) from (2) we get

$9x = 3$

$\Rightarrow$ $x = \dfrac{3}{9} = \dfrac{1}{3}$

$\Rightarrow$ $0.\overline{3} = \dfrac{1}{3}$ … (3)

Let $y = 0.\overline{4}$

$\Rightarrow$ $y = 0.4444\ldots$ … (4)

and $10y = 4.4444\ldots$ … (5)

On subtracting (4) from (5), we get

$9y = 4$

$\Rightarrow$ $y = \dfrac{4}{9}$

$\Rightarrow$ $0.\overline{4} = \dfrac{4}{9}$ … (6)

$\therefore$ $0.\overline{3} + 0.\overline{4} = \dfrac{1}{3} + \dfrac{4}{9}$

$= \dfrac{3+4}{9}$

$= \dfrac{7}{9}$ [Using (3) and (6)]

7. (*c*) **2**

$\sqrt[3]{2}\ \sqrt[4]{2}\ \sqrt[12]{32} = \sqrt[12]{2^4}\ \sqrt[12]{2^3}\ \sqrt[12]{2^5}$

$= \sqrt[12]{2^4 \times 2^3 \times 2^5}$

$= \sqrt[12]{2^{12}} = 2.$

8. (*a*) **2**

$\dfrac{14\sqrt{112}}{28\sqrt{7}} = \dfrac{1}{2} \times \dfrac{\sqrt{7\times 16}}{\sqrt{7}}$

$= \dfrac{1}{2} \times \dfrac{4\sqrt{7}}{\sqrt{7}}$

$= 2$

9. (*a*) **3**

$(9)^{0.06} \times (9)^{0.44} = 9^{(0.06+0.44)}$

$= 9^{0.5}$

$= (3^2)^{0.5}$

$= (3^2)^{0.5}$

$= 3^1$

$= 3$

10. (*a*) $\left(\sqrt{a^3}\right)^{\frac{2}{3}}$

$\left(\sqrt{a^3}\right)^{\frac{2}{3}} = \left[(a^3)^{\frac{1}{2}}\right]^{\frac{2}{3}}$

$= \left[a^{3\times\frac{1}{2}}\right]^{\frac{2}{3}}$

$= a^{\frac{3}{2}\times\frac{2}{3}}$

$= a$

11. Let $x = 3.\bar{2}$

$\Rightarrow \quad x = 3.2222\ldots \quad \ldots (1)$

and $\quad 10x = 32.2222\ldots \quad \ldots (2)$

On subtracting (1) from (2), we get

$9x = 29$

$\Rightarrow \quad x = \frac{29}{9}$

$\Rightarrow \quad 3.\bar{2} = \mathbf{\frac{29}{9}}$

12. If x is a rational number, then x^2 may be a rational number or an irrational number.

For example: If $x = \sqrt{3}$, then $x^2 = 3$ which is rational.

If $x = 2 + \sqrt{3}$, then $x^2 = (2 + \sqrt{3})^2 = 4 + 3 + 4\sqrt{3}$

$= 7 + 4\sqrt{3}$ which is irrational.

13. $\frac{5-\sqrt{6}}{5+\sqrt{6}} = \frac{(5-\sqrt{6})}{(5+\sqrt{6})} \times \frac{(5-\sqrt{6})}{(5-\sqrt{6})}$

$= \frac{(5-\sqrt{6})^2}{(5)^2 - (\sqrt{6})^2}$

$= \frac{25 + 6 - 10\sqrt{6}}{25 - 6}$

$= \mathbf{\frac{31 - 10\sqrt{6}}{19}}$

14. $\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{(2-\sqrt{3})}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3} - 3}{3}$

$= \frac{2(1.732) - 3}{3}$

$= \frac{3.464 - 3}{3}$

$= \frac{0.464}{3}$

$=$ **0.1546** (approx)

15. $\frac{2}{3}\sqrt{\frac{144}{64}} = \frac{2}{3} \times \left[\pm\left(\frac{12}{8}\right)\right]$

$= \frac{2}{3} \times \left[\pm\left(\frac{3}{2}\right)\right]$

$= \pm 1$

= 1 or –1 (rational)

16. $2\sqrt[3]{40} - 4\sqrt[3]{320} + 3\sqrt[3]{625}$

$= 2\sqrt[3]{2 \times 2 \times 2 \times 5} - 4\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5}$
$+ 3\sqrt[3]{5 \times 5 \times 5 \times 5}$

$= 2 \times 2\sqrt[3]{5} - 4 \times 2 \times 2\sqrt[3]{5} + 3 \times 5\sqrt[3]{5}$

$= 4\sqrt[3]{5} - 16\sqrt[3]{5} + 15\sqrt[3]{5}$

$= 19\sqrt[3]{5} - 16\sqrt[3]{5}$

$= \mathbf{3\sqrt[3]{5}}$

17. $(4\sqrt{3} - \sqrt{2})\ (3\sqrt{2} + 2\sqrt{3})$

$= 4\sqrt{3}\ (3\sqrt{2} + 2\sqrt{3}) - \sqrt{2}\ (3\sqrt{2} + 2\sqrt{3})$

$= 12\sqrt{6} + 8 \times 3 - 3 \times 2 - 2\sqrt{6}$

$= 24 - 6 + 12\sqrt{6} - 2\sqrt{6}$

$= \mathbf{18 + 10\sqrt{6}}$

18. $\left[5(8^{\frac{1}{3}} + 27^{\frac{1}{3}})^3\right]^{\frac{1}{4}} = \left\{5\left[(2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}}\right]^3\right\}^{\frac{1}{4}}$

$= \left\{5\left[(2)^{3\times\frac{1}{3}} + (3)^{3\times\frac{1}{3}}\right]^3\right\}^{\frac{1}{4}}$

$= \left[5(2+3)^3\right]^{\frac{1}{4}}$

$= \left[5(5)^3\right]^{\frac{1}{4}}$

$= (5^{1+3})^{\frac{1}{4}}$

$= 5^{4\times\frac{1}{4}}$

$= 5$

19. $a = b^{2x},\ b = c^{2y},\ c = a^{2z}$ [Given] … (1)

$a = b^{2x} = (c^{2y})^{2x} = \left[(a^{2z})^{2y}\right]^{2x}$ [Using (1)]

$\Rightarrow \quad a = (a^{4yz})^{2x}$

$\Rightarrow \quad a^1 = a^{8xyz}$

$\Rightarrow \quad 1 = 8xyz$

$\Rightarrow \quad xyz = \mathbf{\frac{1}{8}}$

20. $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{27}+\sqrt{63}-\sqrt{28}-\sqrt{48}}$

$= \frac{\sqrt{3}+\sqrt{7}}{3\sqrt{3}+3\sqrt{7}-2\sqrt{7}-4\sqrt{3}}$

$= \frac{\sqrt{3}+\sqrt{7}}{-\sqrt{3}+\sqrt{7}} \times \frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

$= \frac{3+7+2\sqrt{21}}{4}$

$= \frac{10+2\sqrt{21}}{4}$

$= \frac{5+\sqrt{21}}{2}$

21. $\frac{9^{\frac{1}{3}} \times 27^{\frac{-1}{2}}}{3^{\frac{1}{6}} \times 3^{\frac{-2}{3}}} = \frac{(3^2)^{\frac{1}{3}} \times (3^3)^{\frac{-1}{2}}}{3^{\frac{1}{6}} \times 3^{\frac{-2}{3}}}$

$= \frac{3^{\frac{2}{3}} \times 3^{\frac{-3}{2}}}{3^{\frac{1}{6}} \times 3^{\frac{-2}{3}}}$

$= 3^{\left(\frac{4-9-1+4}{6}\right)}$

$= 3^{\frac{8-10}{6}}$

$= 3^{\frac{-2}{6}}$

$= \mathbf{3^{\frac{-1}{3}}}$

22. $\frac{6}{3\sqrt{2}-2\sqrt{3}} = \frac{6}{(3\sqrt{2}-2\sqrt{3})} \times \frac{(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})}$

$= \frac{6(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$

$= \frac{6(3\sqrt{2}+2\sqrt{3})}{9(2)-4(3)}$

$= \frac{6(3\sqrt{2}+2\sqrt{3})}{18-12}$

$= \frac{6(3\sqrt{2}+2\sqrt{3})}{6}$

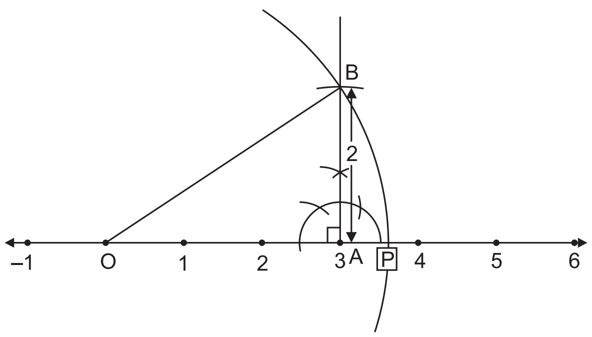
$= 3\sqrt{2}+2\sqrt{3}$ … (1)

$\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2} - a\sqrt{2} = 3\sqrt{2} + 3\sqrt{2}$ [Using (1)]

$= 3\sqrt{2} - (-2\sqrt{3})$

$\Rightarrow$ $\boldsymbol{a = -2}$

23. $\sqrt{13}$ on number line



Draw a number line and mark a point O, representing zero, on it. Suppose point A represents 3 on the number line.
Then, OA = 3 units. Draw AB ⊥ OA such that AB = 2 units. Join OB.
With centre O and radius equal to OB, draw an arc, meeting the number line at P.

Then, OP = OB = $\sqrt{13}$ units.

Justification: In right ΔOAB, we have

$OB^2 = OA^2 + AB^2$

$\Rightarrow \quad OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{(3)^2 + (2)^2}$

$= \sqrt{9+4} = \sqrt{13}$ units.

24. $x = \frac{\sqrt{5}+3}{2}$

$\Rightarrow \quad \frac{1}{x} = \frac{2}{\sqrt{5}+3}$

$= \frac{2}{(\sqrt{5}+3)} \times \frac{(\sqrt{5}-3)}{(\sqrt{5}-3)}$

$= \frac{2(\sqrt{5}-3)}{5-9}$

$= \frac{2(\sqrt{5}-3)}{-4}$

$= \frac{3-\sqrt{5}}{2}$

$\therefore \quad x + \frac{1}{x} = \frac{\sqrt{5}+3}{2} + \frac{3-\sqrt{5}}{2}$

$= \frac{\sqrt{5}+3+3-\sqrt{5}}{2}$

$= \frac{6}{2} = 3$

Now, $x + \frac{1}{x} = 3$

$\therefore \quad \left(x + \frac{1}{x}\right)^2 = 9$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 9$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = 9$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 9 - 2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 7$